Collapse and Fragmentation of Magnetized Cylindrical Clouds*

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Abstract

Gravitational collapse of the cylindrical elongated cloud is studied by numerical magnetohydrodynamical simulations. In the infinitely long cloud in hydrostatic configuration, small perturbations grow by the gravitational instability. The most unstable mode indicated by a linear perturbation theory grows selectively even from a white noise. The growth rate agrees with that calculated by the linear theory. First, the density-enhanced region has an elongated shape, i.e., prolate spheroidal shape. As the collapse proceeds, the high-density fragment begins to contract mainly along the symmetry axis. Finally, a spherical core is formed in the non-magnetized cloud. In contrast, an oblate spheroidal dense disk is formed in a cloud in which the magnetic pressure is nearly equal to the thermal one. The radial size of the disk becomes proportional to the initial characteristic density scale-height in the r-direction. As the collapse proceeds, a slowly contracting dense part is formed ($\lesssim$ 10% in mass) inside of the fast contracting disk. And this is separated from other part of the disk in which the inflow velocity is accelerated as reaching the slowly-contracting dense part. From arguments on the Jeans mass and the magnetic critical mass, the evolution of the fragments formed in a cylindrical elongated cloud is classified. When the cloud reaches a cylindrical, magnetohydrostatic configuration, if the plasma β (the ratio of thermal pressure to the magnetic pressure) in the center of the cloud is larger then $\gtrsim$0.02, the formed disks can not be supported against the self-gravity and it will eventually collapse. If the cylindrical cloud is supported mainly by the magnetic fields, $\beta\lesssim$0.02, the cloud seems to be fragmented to stable disks.

Subject Headings: ISM: general — MHD — Stars: Formation

1 Introduction

The process of star formation begins in the interstellar clouds as a fragmentation of the clouds. Massive stars are formed only in giant molecular clouds, while less massive stars are born also in less massive dark clouds (for a review, see Larson 1991). This observational fact seems to suggest that the process of massive star formation and that of less massive stars are different. This seems to be related to the difference between the collapse of supercritical clouds and subcritical clouds (Shu, Adams, & Lizano 1987). If a mass of the cloud is larger than a critical mass (for a magnetic cloud, $\sim$ magnetic flux/$G^{1/2}$), there is no equilibrium for the thermal pressure, the Lorentz force, and the centrifugal force to counter-balance the self-gravity. The supercritical clouds begin dynamical collapse. However, the subcritical magnetohydrostatic cloud is thought to evolve slowly only by the plasma drift (ambipolar diffusion; Mouschovias 1977, Nakano 1979) and/or the magnetic braking (Mouschovias 1979). The structure of the cloud changes in a relatively long time scale of the plasma drift and the magnetic braking. Finally, if the cloud becomes supercritical due to the decrease of magnetic flux at the center of the cloud or the decrease of angular momentum to support the cloud, it begins to dynamically collapse, too. However, this is true only when the cloud in a static equilibrium is stable. When the time scale of some dynamical instabilities is shorter than the evolutionary time scale, the cloud may begin a dynamical contraction or fragmentation before it reaches the condition for the supercritical cloud. In a series of papers, the fragmentation process in a magnetohydrostatic cloud is studied by a non-linear magnetohydrodynamical simulation in two-dimensions. This paper is devoted to the filamentary cloud and its gravitational fragmentation.

There are well-known examples for interstellar clouds with filamentary or cylindrical shape. Taurus molecular cloud looks elongated shape as a whole. The cloud has two major condensations Heiles Cloud 2 and L1495, and between these two a filamentary dark cloud (B213) is located. Its size is approximately 20'$\times$ $3^o \simeq$ (1.2pc $\times$ 11)pc (Fukui & Mizuno 1991). Assuming the mass of the filament as $\sim 500 M_\odot$, the line density of this cloud is estimated $\sim 50 M_\odot$/pc. Since a number of IRAS sources and T Tauri stars are found in B213, the cloud is now forming stars. Another example is ρ Ophiuchus cloud, in which two filaments extends from ρ Oph main cloud ($l \simeq 353$, $b \simeq 17$) and ρ Oph East cloud

($l \simeq 354$, $b \simeq 16$), respectively. The filaments have full lengths of 10-17.5 pc, although their widths are as narrow as 0.24 pc (Loren 1989). Dark clouds in the filament has the mass per unit length of 27-49 $M_\odot$/pc. Filaments are found even in giant molecular clouds (such as Orion L1641 cloud: Bally 1989) where massive stars are also formed. The interstellar clouds are often found as filamentary or cylindrical clouds. Collapse and fragmentation of such a cloud is studied.

The process of fragmentation and thus gravitational collapse of the high-density portion of the cloud are studied by linear and non-linear analyses. Linear analyses of the gravitational instability in an isothermal *slab* have been done by many authors (see Spitzer 1978 for a review). The qualitative answer is as follows: the slab is unstable for a perturbation whose wavelength is longer than a critical length and the most unstable perturbation has the wavelength of $\lambda_{\rm max} \simeq 20 c_s/(4\pi G\rho_c)^{1/2}$ and the e-growing time scale of $\tau_{\rm max} \sim 2(4\pi G\rho_c)^{-1/2}$, where ρ_c represents the density on the mid-plane of the disk (Elmegreen & Elmegreen 1978). The *cylindrical cloud* has a similar characteristic wavelength, $\lambda_{\rm max} \simeq 20 c_s/(4\pi G\rho_c)^{1/2}$ and a longer growth time scale as $\tau_{\rm max} \sim 3(4\pi G\rho_c)^{-1/2}$ (Nagasawa 1987). It is concluded that if there is an inhomogeneity with an amplitude of $\Delta\rho/\rho \sim 5\%$, in $3\tau_{\rm max} \lesssim 10(4\pi G\rho_c)^{-1/2}$ the dense part of the cloud grows as a fragment with non-linear density contrast $\Delta\rho/\rho \sim 1$. This time scale is comparable to that for the ambipolar diffusion (Nakano 1988). Therefore, the nonlinear stage of the dynamical instability in the static cylindrical cloud is studied.

The isothermal non-magnetic cylindrical cloud has a critical mass per unit length beyond which no equilibrium is achieved (see Ostriker 1964)

$$\begin{aligned} \lambda_c &= \int_0^\infty 2\pi\rho r dr = 2\frac{c_s^2}{G}, \\ &\sim 42\left(\frac{c_s}{300{\rm m\ s^{-1}}}\right)^2 M_\odot {\rm pc}^{-1} \end{aligned} \tag{1.1}$$

where c_s represents the isothermal sound speed in the cloud. The cylindrical cloud with a mass per unit length, $\lambda = \lambda_c$, has infinite density contrast between the center and the surface. As for the spherical isothermal cloud, the situation is different; the critical cloud has a finite density contrast of $\rho_c/\rho_s = 14$ (Bonnor 1956; Ebert 1955). The non-magnetic critical mass is equal to

$$M_c = 1.18\frac{c_s^4}{p_{\rm ext}^{1/2} G^{3/2}}, \tag{1.2}$$

where $p_{\rm ext}$ represents the external pressure on the surface of the cloud. Finally, in the slab geometry, no critical column density of the self-gravitating disk exists. These differences come from the geometry or the dimension of the system. Generally, the magnetic field and the centrifugal force in rotating clouds have an effect of increasing the critical mass. From studying magnetohydrostatic equilibrium, the critical mass of the magnetized cloud is obtained as

$$M_{c\ {\rm mag}} \simeq 1.4 \left\{ 1 - \left[\frac{0.17}{dm/d(\Phi_B/G^{1/2})} \right]^2 \right\}^{-3/2} \frac{c_s^4}{p_{\rm ext}^{1/2} G^{3/2}}, \qquad (1.3)$$

where $dm/d(\Phi_B/G^{1/2})$ means the mass-to-magnetic flux ratio at the center of the cloud (Mouschovias & Spitzer 1976; Tomisaka, Ikeuchi, &Nakamura 1988). Further the rotating cloud has a larger critical mass as

$$M_{c\ {\rm rot}} \simeq \left[M_{c\ {\rm mag}}^2 + \left(\frac{4.8 c_s j}{G} \right)^2 \right]^{1/2}, \qquad (1.4)$$

where $M_{c\ {\rm mag}}$ and j represent, respectively, the mass which can be supported without any rotation [Eq.(1.3)] and a specific angular momentum of the cloud (Tomisaka, Ikeuchi, & Nakamura 1989). Anyhow, the cloud with $\lambda > \lambda_c$ or $M > M_c$ eventually collapses, unless the excessive mass is eroded by some processes.

There are a number of papers in which the evolution of the magnetized clouds. Dynamical collapse due to the self-gravity is studied mainly under flux-freezing (Scott & Black 1980; Phillips 1986a, b; Dorfi 1982, 1989). The supercritical cloud contracts as a whole and forms a dense contracting core inevitably. The authors reported that they did not observed any fragmentation in the process of contraction. This seems to correspond to the fact that free-fall time scale is shorter than the growing time scale of the gravitational instability (see above). The situation seems to be much different for subcritical clouds. Subcritical clouds, in which the evolution is driven by the plasma drift, are studied by the quasi-static assumption (for a review, Nakano 1988) and through dynamical calculations (for a review, Mouschovias 1987). The evolution time scale is estimated approximately as 10 times longer than the free-fall time scale.

Here, we consider a cylindrical cloud with poloidal magnetic fields. Its length is assumed infinite and the cloud is in a magnetohydrostatic configuration. From a sense that the cloud is static, it should be called a subcritical cylindrical cloud (cylindrically subcritical[1]). The density fluctuations in the cloud seems to break the symmetry and it grows when the cloud is gravitationally unstable. As a result, dense cores and low-density regions are separated. The formed dense cores may become supercritical ($M > M_{c\ mag}$), since the geometry of the fragment is different from the initial cylindrical configuration. In this case, the formed fragments seem to continue to contract. Contrarily, when the fragments are subcritical ($M < M_{c\ mag}$), we may see dense stable cores are formed in the cylindrical cloud.

As for the non-magnetic cylindrical clouds, Bastien (1983), and Bastien et al. (1991) have studied the contraction of initially uniform cylindrical cloud with a *finite* length. They achieved some conclusions on the fate of the above clouds: for example, for the elongated cloud with the ratio of length to diameter of the cylinder $\gtrsim$2, using the initial Jeans number, J_0, which is defined as the ratio of gravitational to the thermal energies, the evolution is determined. When $J_0 \lesssim J_{2-frag}$, two subcondensations are formed. For more large Jeans number as $J_{2-frag} \lesssim J_0 \lesssim J_{spindle}$, these two subcondensations collapse into one object. For the extreme case $J_0 \gtrsim J_{spindle}$, the cloud contracts onto a line and forms a spindle. However, their simulation has a restriction that the initial state is far from hydrostatic equilibrium. Excess free energy is liberated in the process of the relaxation from the initial state to the equilibrium, which may act an important role. To understand the relatively slow evolution in the subcritical cloud, we should take a (magneto-)hydrostatic configuration as the initial state for the simulation.

2 Model and Numerical Method

2.1 Initial Condition

As described in the preceding section, we assume that the initial state is in a hydrostatic equilibrium. Using the cylindrical coordinate (z, r, ϕ), if the initial cylindrical isothermal cloud is homogeneous in the $z-$direction, the magnetohydrostatic equilibrium configuration is obtained by the equation of

[1]Non-magnetic cylindrical clouds with $\lambda > \lambda_c$ are (cylindrically) supercritical. However, cylindrical clouds with poloidal magnetic fields are all cylindrically subcritical (see Mouschovias & Morton 1991; §4.2 of this paper).

hydrostatic balance and the Poisson equation as

$$-\frac{\partial \psi}{\partial r} - \frac{c_s^2}{\rho}\frac{\partial \rho}{\partial r} - \frac{1}{8\pi\rho}\frac{\partial B_z^2}{\partial r} = 0, \tag{2.1}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) = 4\pi G\rho, \tag{2.2}$$

where ψ, ρ, $\mathbf{B}$, c_s, and G represent, respectively, the gravitational potential, density, magnetic field, isothermal sound speed, and the gravitational constant. To derive equation(2.1), it is assumed there is no helical magnetic field component ($B_\phi = 0$). The density distribution depends upon that of the magnetic field B_z. In this paper, we restrict ourselves to two cases: (i) uniform magnetic field B_z =constant, and (ii) the ratio of the magnetic pressure to the thermal one is constant. In the first model, the density distribution becomes identical to that of nonmagnetic isothermal cylinder as

$$\rho(r) = \frac{\rho_c}{\left(1 + \frac{r^2}{8}\frac{4\pi G\rho_c}{c_s^2}\right)^2}, \tag{2.3}$$

where ρ_c means the density at the center of the cylinder ($r = 0$). As easily seen, the density reaches zero only at infinity ($r = \infty$). Thus an isothermal cloud should be bounded by the external pressure, $p_{\rm ext}$. The cloud has a boundary where $\rho(r_s)$ is equal to $p_{\rm ext}/c_s^2$. The cloud radius is expressed as

$$r_s = 2^{3/2}\left[\left(\frac{\rho_c}{\rho_s}\right)^{1/2} - 1\right]^{1/2}\frac{c_s}{(4\pi G\rho_c)^{1/2}}, \tag{2.4}$$

where ρ_s is the density at the cloud surface and equals to $p_{\rm ext}/c_s^2$. Adopting typical values observed in dark clouds, such as the cloud central density of $\sim 2 \times 10^{-20}$ g cm^{-3} and the sound speed of ~ 100 m s^{-1}, equation (2.4) gives the width of the cylindrical cloud as $\simeq 0.2{\rm pc}(c_s/100{\rm ms}^{-1})(\rho_c/2 \times 10^{-20}{\rm gcm}^{-3})^{-1/2}$, which is comparable to that obsered in the ρ Oph cloud. In the present paper, we use the normalization $c_s = 4\pi G = p_{\rm ext} = 1$. Thus the unit of distance is chosen as $H = c_s/(4\pi G\rho_s)^{1/2}$. The normalized density distribution, $f(r) \equiv \rho(r)/\rho_s$, and the radius of the surface, $\xi_s \equiv r_s/H$, are expressed as

$$f(\xi) = \frac{F}{\left[1 + \frac{F}{8}\xi^2\right]^2}, \tag{2.5}$$

$$\xi_s = \left(\frac{2^3}{F}\right)^{1/2}\left[F^{1/2} - 1\right]^{1/2}, \tag{2.6}$$

where $\xi \equiv r/H$ represents the normalized distance and $F = \rho_c/\rho_s$ denotes the density contrast between the center and the surface.

When the magnetic field plays a role in supporting the cloud, the density distribution is different from equation(2.3). Assuming that the magnetic pressure is proportional to the thermal pressure, i.e., $B_z^2/8\pi \propto c_s^2\rho$, the distribution of the density becomes as

$$f(\xi) = \frac{F}{\left[1 + \frac{F}{8}\frac{\xi^2}{1+\alpha/2}\right]^2}, \tag{2.7}$$

where the parameter α is defined as

$$\alpha \equiv \frac{B_z^2/4\pi}{c_s^2\rho} \equiv \frac{2}{\beta}, \tag{2.8}$$

in terms of the plasma β[2]. This shows that the cloud becomes thick with increasing α, and the density scale-height changes in proportion to $(1+\alpha/2)^{1/2}$. Typical density distributions are shown in Figure 1.

We assumed a hypothetical situation that the cloud is confined in a low-density ambient medium which has no importance as the source of gravity but has a finite pressure $p_{\rm ext}$. Thus, the initial solution of $\rho(r)$ has a boundary where $\rho(r) = p_{\rm ext}/c_s^2$. Beyond this cloud boundary, a tenuous and thus hot medium is assumed to extend. However, as seen in the next subsection, since we assume the isothermal equation of state, it is difficult to calculate two kinds of gases, cold cloud component and warm intercloud component, separately. Therefore, we adopt a one-fluid approximation, where the gas has an identical temperature; but the ambient gas is defined as the portion with $\rho < p_{\rm ext}/c_s^2 \equiv \rho_s$, and the source term of the gravitational field (r.h.s. of eq.2.2) consists of the density $\rho > \rho_s$). This gives a virtual distribution of ambient gas as

$$f(\xi) = \left(\frac{\xi}{\xi_s}\right)^{-\frac{4(F^{1/2}-1)}{F^{1/2}}} \quad (\text{for } \xi > \xi_s), \tag{2.9}$$

to counter-balance the gravity by the cloud as

$$g(\xi) = \frac{\partial \ln \rho}{\partial \xi} = -\frac{4}{\xi}\frac{F^{1/2}-1}{F^{1/2}}. \tag{2.10}$$

[2] $\alpha^{1/2}$ represents the ratio of the Alfvén speed, $B/(4\pi\rho)^{1/2}$ to the isothermal sound speed, c_s.

2.2 Basic Equations

Cylindrical symmetry is assumed: $\partial/\partial\phi = 0$. For the dense gas found in interstellar clouds with $\simeq 10$K, the equation of state is well approximated with the isothermal one (Larson 1985). We assume here that the gas obeys the isothermal equation of state. Thus the basic equations become the equations of magnetohydrodynamics for isothermal gases as

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial z}(\rho v_z) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) = 0, \tag{2.11}$$

$$\frac{\partial\rho v_z}{\partial t} + \frac{\partial}{\partial z}(\rho v_z v_z) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_z v_r) = -c_s^2\frac{\partial\rho}{\partial z} - \rho\frac{\partial\psi}{\partial z} - \frac{1}{4\pi}\left(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}\right)B_r, \tag{2.12}$$

$$\frac{\partial\rho v_r}{\partial t} + \frac{\partial}{\partial z}(\rho v_r v_z) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r v_r) = -c_s^2\frac{\partial\rho}{\partial r} - \rho\frac{\partial\psi}{\partial r} + \frac{1}{4\pi}\left(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}\right)B_z, \tag{2.13}$$

$$\frac{\partial B_z}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}[r(v_z B_r - v_r B_z)], \tag{2.14}$$

$$\frac{\partial B_r}{\partial t} = -\frac{\partial}{\partial z}(v_z B_r - v_r B_z), \tag{2.15}$$

$$\frac{\partial^2\psi}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) = 4\pi G\rho, \tag{2.16}$$

where the variables have their ordinary meanings.

We choose the radial size of the numerical box as 2-3 $\times\xi_s$. The density and pressure in equations (2.11-2.13, & 2.16) include those of the cloud material and the ambient component. In the course of numerical simulation, we assume that cloud consists in the gas with $\rho > \rho_s$. To include only the gas density in the cloud as the source of the self-gravity, the right-hand side of equation (2.16) is calculated as follows:

$$\rho(z,r) = \begin{cases} \rho, & \text{if } \rho > \rho_s, \\ 0, & \text{if } \rho < \rho_s. \end{cases} \tag{2.17}$$

As long as the cloud keeps its hydrostatic configuration, the solution of the above equations coincides with the initial configuration, equations (2.5) and (2.9). However, if the ambient matter accretes onto the cloud by the gravity or the cloud material evaporates by the pressure force, the above procedure may give wrong results. We monitored the mass of the cloud in numerical runs, and check the validity of the ambient matter condition. We confirmed that the cloud mass changed by less than $\sim 3\%$.

Nondimensional variables are used as follows: the density, $\rho' \equiv \rho/\rho_s$, the velocity, $\mathbf{v}' \equiv \mathbf{v}/c_s$, the gravitational potential, $\psi' \equiv \psi/c_s^2$, the magnetic fields, $\mathbf{B}' \equiv \mathbf{B}/(4\pi c_s^2\rho_s)^{1/2}$, the time, $t' \equiv$

$t(4\pi G\rho_s)^{1/2}$, and the linear size, $\mathbf{r}' \equiv \mathbf{r}/H$. Using this normalization, equations (2.12), (2.13), and (2.2) become

$$\frac{\partial \rho' v_z'}{\partial t'} + \frac{\partial}{\partial z'}(\rho' v_z' v_z') + \frac{1}{r'}\frac{\partial}{\partial r'}(r' \rho v_z' v_r') = -\frac{\partial \rho'}{\partial z'} - \rho' \frac{\partial \psi'}{\partial z'} - \left(\frac{\partial B_r'{}'}{\partial z} - \frac{\partial B_z'}{\partial r'}\right) B_r', \tag{2.18}$$

$$\frac{\partial \rho' v_r'}{\partial t'} + \frac{\partial}{\partial z'}(\rho' v_r' v_z') + \frac{1}{r'}\frac{\partial}{\partial r'}(r' \rho' v_r' v_r') = -\frac{\partial \rho'}{\partial r'} - \rho' \frac{\partial \psi'}{\partial r'} + \left(\frac{\partial B_r'}{\partial z'} - \frac{\partial B_z'}{\partial r'}\right) B_z', \tag{2.19}$$

$$\frac{\partial^2 \psi'}{\partial z'^2} + \frac{1}{r'}\frac{\partial}{\partial r'}\left(r' \frac{\partial \psi'}{\partial r'}\right) = \rho'. \tag{2.20}$$

Other equations are identical even when these normalized variables are used. Hereafter, we will use these nondimensional variables and omit the prime unless otherwise mentioned.

2.3 Numerical Method

The basic equations are solved by the finite difference method. The mesh spacing (Δz, Δr) is constant spatially except for models CB and CB2 in table 1. Unequal spacing is used for model CB and CB2 to see the fragment more closely. Cell numbers in one dimension are chosen $N = 200 - 800$ (table 1). The "Monotonic Scheme" (van Leer 1977; Norman & Winkler 1986) is adopted to solve the hydrodynamical equations, i.e., equations (2.11)–(2.13), and the "Constrained Transport" method (Evans & Hawley 1988) is adopted to solve the induction equations of the magnetic field, i.e., equations (2.14) and (2.15). Although the equation of state is different, this scheme is essentially the same as that used by Tomisaka (1992), For numerical stability at the shock front, an artificial viscosity is included in equations (2.12) and (2.13). A tensor artificial viscosity term is used as shown in the Appendix of Tomisaka(1992). The program for adiabatic gas was checked by several test problems: Sedov solution, spherical stellar wind bubble solution (Weaver et al. 1977), one-dimensional magnetic Rieman problem (Brio & Wu 1988). Further, the contraction of a spherical isothermal cloud is compared with solutions of Larson (1969) and Penston (1969). The radial density distribution is well fitted by $\propto r^{-2}$ and derived velocity profiles agree with their results by one-dimensional calculation.

To solve the Poisson equation is reduced to find the solution of simultaneous linear equations with a dimension of N^2. The linear equations are solved by the "Conjugate Gradient Squared method" (CGS method, Dongarra et al. 1991) preconditioned by the modified incomplete LU decomposition

(MILUCGS) (Meijerink & van der Vorst 1977; Gustafsson 1978). A brief description of this method and its performance are described in Appendix.

On the upper ($z = l_z$) and lower ($z = 0$) boundary, the periodic boundary conditions are applied. Due to the boundary condition, the Fourier modes which can be included are restricted, that is, the Fourier modes with the wavelength such as $\lambda > l_z$ and $l_z/2 < \lambda < l_z$ can not be included. On the outer boundary the fixed boundary condition is applied.

3 Result

3.1 A Typical Evolution

Model A corresponds to $\alpha = 1$ and $F \equiv c_s^2 \rho_c / p_{\rm ext} = \rho_c/\rho_s = 100$. The size of the calculated region is taken as $1.93H$, which coincides with the wavelength which has the maximum growth rate in the gravitational instability (Nakamura, Hanawa, & Nakano 1993). The density fluctuation $\delta\rho$ is added to the initial density in each meshes $\rho(z, r)$ (Eq.2.7). In model A we assume a white noise, in which $\delta\rho/\rho$ is subject to the Gaussian distribution whose average $\overline{\delta\rho/\rho}$ is equal to zero and the standard deviation is chosen as $\sigma = 10^{-4}$. Figure 2 shows the evolutions of density (left panel) and magnetic fields (right panel). In $t = 1.2$, no prominent fragmentation appears. In the stage shown in Figure 2b, a high-density region elongating in the z-direction appears. The shape of this high-density region is prolate, i.e., if it fits to an ellipsoid the major axis coincides with the symmetric axis (the z-axis). This agrees with the result of linear analyses (Nakamura et al. 1993; Nagasawa 1987); The wavelength of the most unstable perturbation was predicted from the linear analysis as $\simeq 2 \times H$. And the global shape of the high-density region in Figures 2b-c coincides with an expected shape from the eigen-function of the linear analysis. From this figure, it is confirmed that the perturbation with a shorter wavelength than that of the most unstable mode does not appear at all, before the most unstable mode grows into the non-linear region. However, a perturbation with a longer wavelength than the numerical box can not be properly calculated. We have calculated in model A2 (not shown) the same problem with a larger spacing and thus with a numerical box twice as large as model A. In model A2, two high-density regions appear and each fragment shows the identical evolution as model A, which confirms the most unstable mode expected from the linear analysis grows first.

After the stage in Figure 2c, collapse along the symmetry axis proceeds as well as the radial direction. The final structure is a disk contracting towards the center seen in Figure 2d. Since the gas easily falls down along the magnetic field lines, this disk is perpendicular to the symmetry axis of the cylinder. The magnetic fields are squeezed by the effect of radial contraction near the disk (right panel). Since this configuration is unstable against the Parker instability (1979 and references therein), the gas is accumulated flowing along the magnetic fields. The critical wavelength of the Parker instability with $\alpha \simeq 1$ is estimated as long as πH_0, where H_0 represents the scale-height in the center of the cylindrical cloud and this corresponds to $\sim F^{-1/2}H \sim 0.3H$. Since this is much smaller than the size our numerical box, it is natural that the Parker instability plays a role even in the self-gravitating configuration. Contracting speed in the z-direction is $\simeq$ twice as fast as the radially contracting speed. In the central region, a high-density core is formed, on to which accretion of extra mass continues. Finally, near the high-density core the infalling speed exceeds the isothermal sound speed at this stage. It is concluded that a typical product of the self-gravitational instability in the cylindrical cloud is oblate spheroidal disks separated by $\lambda_{\rm max}$.

The position where the disk is finally formed (z_c) is not determined a priori, since the assumed fluctuation has a spectrum of the white noise. The perturbation with the wavelength which coincides with that of the maximum growth grows from the white noise. It seems that the nonlinear irregularity grows by the effect of gravitational instability and finally it forms fragments.

To confirm the idea, another model B is calculated, whose parameters are identical with model A except for the point that the irregularity has a scale. That is, in model B, a perturbation with relative contrast $\delta_{\rm ini} \equiv \Delta\rho/\rho = 10^{-2}$ is added as

$$\rho(z,r) = \rho_0(r)\left(1 - \delta_{\rm ini}\cos\frac{2\pi z}{l_z}\right), \qquad (3.1)$$

where ρ_0 and l_z represent, respectively, the initial density determined by $f(\xi)$ of equation(2.7) and the z-length of numerical box. The wavelength of this perturbation is again taken identical with that of the most unstable mode in the linear analysis. The structure at $t = 1.26$ is shown in Figure 3. This shows that the final structure of this model is identical with that of the model A (Fig.2d), although the time necessary for the fragment to grow is shortened much (from $t = 3.5$ to $t = 1.26$). It is clear that this quick evolution is due to a finite amplitude of the perturbation added to the initial state of model B, compared with that of the irregularities added with a spectrum to model A. However, the evolution itself is very similar with each other.

To confirm this, the time evolution of the amplitude of the density irregularity is plotted for both models A and B in Figure 4. Using the average $< \rho(r = 0) >$ and the maximum $\rho(r = 0)|_{\rm max}$ density on the z-axis, the relative density contrast

$$\delta_{\rm max} \equiv \frac{\rho(r = 0)|_{\rm max}}{< \rho(r = 0) >} - 1, \tag{3.2}$$

is plotted against the time passed after the calculation begins. This shows that in model A the perturbation grows in accordance with $\propto \exp[\omega t]$ for $2.27 \lesssim t \lesssim 3.26$, from 10^{-2} to 1. Model B shows the similar growth but the region in which it shows the exponential growth is much restricted (for $0 < t < 1.01$, the density grows as from $\delta\rho/\rho = 10^{-2}$ to 1). If the origin of time for model B is moved, two curves for models A and B coincides and both curves is fitted by $\delta\rho/\rho \propto \exp(4.6t)$. This agrees with the growth rate predicted by the linear perturbation analysis $\exp[4.2t]$ (Nakamura et al. 1993)[3]. This indicates that the evolution is similar whether the most unstable mode is overlaid on to the initial density distribution (model B) or not (model A). This confirms the expectation that the most unstable mode grows selectively even from the irregularity of white noise. Thus, we add the perturbation of the expected most unstable mode to the initial density distribution as model B.

3.2 Non-Magnetic Cylinder

To see the effect of the magnetic field, we calculated the non-magnetic isothermal cylinder as model C. From the linear analysis, the wavelength of the most unstable mode is equal to $\simeq 2.21H$ (Nakamura et al. 1993). Since the magnetic field has an effect of supporting the cloud, the radial size of the cloud becomes more compact than that of models A and B.

In Figure 5, the evolution of model C is shown. In $t = 1.2$, the fragment, whose density contrast δ reaches ~ 3, has entered the nonlinear regime (Fig.5a). However, comparing with Figure 3 (these

[3] Small difference in the growth rates seems to come from a slight difference in the models. That is, Nakamura et al. assumes $\rho_s << \rho_c$.

two models are assumed to have irregularities with the same initial amplitude), it is shown that this model evolves slower than model B. This difference is understood as follows: (1) since the magnetic fields of model B has a configuration unstable for the Parker instability, the system becomes more unstable including the magnetic fields, or (2) the mass per unit length of the cylinder and thus the effect of the self-gravity becomes larger when the magnetic fields are supporting the cloud.

Figure 5b shows the structure at the age of $t = 1.4$. In this phase, the maximum density at the center of the core reaches 10^5. The envelope with low-density $\simeq 10^{1.5}$ seems to have the shape of prolate spheroid as well as the magnetic models. In contrast, the high-density central region appears to be almost spherically symmetric. The difference between structures appearing in the magnetic cloud and the non-magnetic one is apparent. Since there is no lateral restoring force by the magnetic fields, the high density fragment shows spherical shape, while in the magnetic cloud the fragment is formed from the matter flowing along the magnetic fields as a disk perpendicular to the magnetic fields.

3.3 Central Core

To see the structure of the central core, the density profile is plotted against $\log|z - z_c|$ and $\log r$ in Figure 6 for models B and C, where z_c represents the position of the core center.

There exist two characteristic power law solutions. One is a singular isothermal sphere solution (Chandrasekhar 1939):

$$\rho_{\rm sing} = \frac{c_s^2}{2\pi G r^2}, \tag{3.3}$$

or in a non-dimensional form:

$$\frac{\rho_{\rm sing}}{\rho_s} = \frac{2}{(r/H)^2}. \tag{3.4}$$

The other is an asymptotic solution of equation(2.5) for $\xi \gg 1$ or $r \gg H$:

$$\rho_{\rm out} = \frac{\rho_c}{\dfrac{r^4}{8^2}\dfrac{(4\pi G\rho_c)^2}{c_s^2}}, \tag{3.5}$$

or in a non-dimensional form:

$$\frac{\rho_{\rm out}}{\rho_s} = \frac{1}{F}\left(\frac{8}{\xi^2}\right)^2. \tag{3.6}$$

As seen in Figure 6, the radial density distribution in the *outer envelope* does not evolve much which is fitted by equation (3.6). As collapse proceeds, the radial density distribution in the inner part of the cloud becomes similar to that of the unstable hydrostatic solution [Fig. 1 of Shu (1977)] which approaches the singular isothermal sphere solution [equation (3.3)] in the limit of $\rho_c/\rho_s \to \infty$, especially for the non-magnetized cloud.

As for the z-distribution, in the *non-magnetic cloud* (model C), the density distribution in the core ($|z-z_c|\lesssim 0.1$) is rather similar to that in the r-direction. However, in the *magnetic cloud* (model B), the r- and z-distributions are much different. This difference comes from the effect of magnetic fields, that is, in models A and B a disk is finally formed, although in model C an almost spherical core is formed. Since the gravities by a disk and a sphere are quit different, the resultant density distributions are different.

When the spherical cloud has the density distribution like equation (3.3), the total mass inside of the sphere R is written as

$$\begin{aligned} M(R) &= \int_0^R \rho_{sing} 4\pi r^2 dr = \frac{2c_s^2}{G}R, \\ &= \left(\frac{2}{\pi}\right)^{1/2} \frac{c_s^4}{[\rho_{sing}(R)c_s^2]^{1/2}G^{3/2}}, \end{aligned} \tag{3.7}$$

using the density $\rho(R)$ at $r=R$. This represents the Jeans mass for the spherical cloud with infinite center-to-surface density contrast, $M_J(R)$. Thus, when the density increases more rapid than equation (3.3) toward the center, the actual mass is larger than this Jeans mass and contraction never stops. In contrast, when the density distribution is flat, the actual mass $M(r)$ never surpasses $M_J(r)$ in any region of $r<R$. Considering this, since the radial density distribution $\rho \gtrsim \rho_{sing}$, the spherical core formed in the center of the non-magnetic cloud seems to continue to contract.

From the studies on magnetohydrostatic equilibrium, the mass-to-flux ratio at the *center* of the cloud, $M/(\Phi/G^{1/2})$, has a crucial role to divide the 3-dimensional clouds into super- and subcritical clouds (Mouschovias & Spitzer 1976; Tomisaka et al. 1988); that is, considering a cloud whose mass is larger than the non-magnetic critical mass ($\gtrsim M_{\rm cr}$ of eq.[1.2]), when the mass-to-flux ratio is larger than $\simeq 1/(2\pi)$ the cloud has no equilibrium solutions; while the cloud has at least one hydrostatic solution, when the ratio is smaller than this value. Since the mass-to-flux ratio remains constant during the contraction, the mass-to-flux ratio *in the fragment* can be written using quantities in the initial stage as

$$\frac{M}{\Phi/G^{1/2}} = \frac{\rho_c l_z G^{1/2}}{B_c} = \frac{\rho_c^{1/2} l_z G^{1/2}}{(4\pi\alpha)^{1/2} c_s}, \tag{3.8}$$

where l_z represents the z-length of the numerical box and is taken equal to the wavelength of the most unstable perturbation, $\simeq 20c_s/(4\pi G\rho_c)^{1/2}$. This ratio is approximately equal to

$$\frac{M}{\Phi/G^{1/2}} \simeq \frac{1.59}{\alpha^{1/2}}, \tag{3.9}$$

(The Mouschovias & Spitzer 1976 criterion for the central mass flux tube is $1.54/\alpha^{1/2}$.) Thus, for the fragment with the mass appreciably larger than that of equation (1.2), only when the magnetic field is extremely strong as $\alpha \gtrsim 100$, the magnetized fragment may be subcritical (in 3D). The contraction of the core can not be stopped by a magnetic field, which is assumed ordinarily, $\alpha \simeq 1$. This is valid until the wavelength of the most unstable perturbation is much longer than $20c_s/(4\pi G\rho_c)^{1/2}$ in a cloud with low density contrast F.

In conclusion, if $\alpha \simeq 1$ for a magnetohydrostatic configuration, the cores formed in the fragment of the cylindrical cloud seem to continue to contract, until other effects not considered here may work in supporting the fragments, such as, the equation of state of the gas changes from isothermal to a more hard one.

3.4 Diffuse Cloud

Here, we compare the cloud with high central density and that with low density contrast calculated in models D. In Figure 7a, the evolution of model DB is plotted. Both models B and DB have the same parameters except for the center-to-surface density ratio, F. Difference between models B and DB are apparent: although a disk is formed for both models by the effect of the magnetic field, the disk perpendicular to the magnetic fields of model DB is much larger than that of model B. The disk has a radial size of 0.16 (model B) and 0.71 (model DB) (the size is measured as a radius of the largest contour line which has an oblate shape).

The radial extent of the finally formed disk seems to correspond to the size of the initial central cylindrical core, i.e., in equation (2.3) the initial density is almost uniform irrespective of the radial distance for $r \lesssim r_c = 2^{1.5}(1+\alpha)^{1/2} c_s/(4\pi G\rho_c)^{1/2}$. Since this gives

$$r_c \simeq 0.4 \left(\frac{F}{100}\right)^{-1/2} \left(\frac{1+\alpha}{2}\right)^{1/2} \frac{c_s}{(4\pi G\rho_s)^{1/2}}, \tag{3.10}$$

the above result shows that a disk threaded by magnetic fields whose radial extent is equal to ~ 40 – 55 % of the initial core size is formed.

If this is true, does the cloud with a large α form a large disk? Yes. In model DD with $\alpha = 4$ (Fig. 7b), the disk with a radial size of ~ 1.2 is formed, while $r_c \simeq 2(F/10)^{1/2}[(1+\alpha)/5]^{1/2}$. Thus, it can be concluded that in the magnetized cylindrical cloud a disk is formed perpendicularly threaded by the magnetic fields and its radial extent is proportional to r_c.

The next characteristic point of the evolution of diffuse clouds is a low growth rate of fragmentation. A typical time scale for the perturbation to evolve into the nonlinear stage is $t \sim 4.72$ for $\alpha = 0$ (model DC), $t \sim 4.12$ for $\alpha = 1$ (model DB), and $t \sim 3.31$ for $\alpha = 4$ (model DD). This is much longer than that of models B and C with $F = 100$. This apparently comes from the fact that the growth time-scale is proportional to the free-fall time for the central density. Finally, comparing the non-magnetic clouds of models C and DC, Figures 4 (model C) and 7c (model DC) look very similar. These figures are scaled in proportion to l_z, and l_z is chosen as $\lambda_{\max} \simeq 20 c_s/(4\pi G\rho_c)^{1/2}$. This means that the actual size of the spherical core of model DC is just $10^{1/2}$ times larger than that of model C. If we scale the size by $c_s/(4\pi G\rho_c)^{1/2}$ instead of $H = c_s/(4\pi G\rho_s)^{1/2}$, the structures of these models becomes almost identical. Therefore, the structure of the fragment is mainly determined by the initial highest density at the center of the cylinder cloud for the non-magnetic cloud.

3.5 Further Evolution

To see the structure of the contacting central core in more detail, we have to execute larger calculation with finer resolution using a plenty of meshes. However, this approach will meet an inevitable limit of available memories on any computers[4]. Therefore, we avoid this difficulty by varying mesh sizes, as a

[4]The adaptive or nested grid method is applied to this kind of problem (Fiedler & Mouschovias 1992, 1993; Yorke, Bodenheimer, & Laughlin 1993), in which mesh points are moved or generated in accordance with the density achieved.

finer mesh is used around the expected position where the fragment is formed and a coarser mesh is used far from the cloud. We apply so-called log-mesh: the size of the grid is chosen like $\Delta z_i = F_z \Delta z_{i-1}$ for $i > i_c$, $\Delta z_i = F_z \Delta z_{i+1}$ for $i < i_c$, and $\Delta r_j = F_r \Delta r_{j-1}$ (F_z, $F_r > 1$), where i and j represent the sequential numbers of the cell in the z-direction and in the r-direction, respectively, and subscript c means the the midpoint of the z-axis. Models CB and CB2 have the same physical parameters as model B but they have $\simeq$ 2 - 4 times closer spatial resolution than model B, respectively. Although these two models use different initial amplitude of perturbations (table 1), qualitative evolutions are similar with each other.

Due to a finer resolution, the evolution can be traced till the maximum density reaches $\simeq 10^6 = 10^4\times$ initial central density. In Figure 8, we show the structure at $t = 0.705$ for model CB2 (due to a larger amplitude of $\delta_{\rm ini} = 10^{-1}$ in model CB2, irregularities grow faster than models CB and B. This time scale is equivalent to $t = 1.26$ for models in which smaller initial perturbation is assumed as $\delta_{\rm ini} = 10^{-2}$). As shown in Figure 8c and d, the collapse speed ($-v_r$) becomes faster as reaching the fragment center in the region $r \gtrsim 0.07$. And the maximum falling speed increases as collapse proceeds. From Figure 8c, at that time, the maximum infall velocity in the z-direction agrees with that observed in the asymptotic solution for the spherical isothermal collapse by Larson(1969) and Penston(1969), $3.28c_s$. It is shown that high-density region with a thickness of $\sim 10^{-2}$, i.e., $|z - z_c| \lesssim 10^{-2} H = 10^{-1} c_s/(4\pi G \rho_{c\ \rm init})^{1/2}$, is formed, in which contraction speed is decreased as reaching z_c. In the r-direction, in contrast, very smooth distributions of density, magnetic field strength, and velocity are seen. This numerical result qualitatively agrees with the calculation by Hanawa, Nakamura, & Nakano (private communication) using a different numerical scheme.

Thin dense object which is contracting slowly now seems to be separated from the outer inflow region seen in $|z - z_c| \gtrsim 10^{-2}$. The fraction of mass which form this relatively static high-density disk is estimated as 6% - 10 %. So, if stars are formed from the relatively static matter, the ratio of the mass of newly formed protostars to that of a parent cloud becomes as 6 % - 10 %.

Calculation with these methods is now preparing.

3.6 A Model with Strong Magnetic Fields

In model S, we study the cylindrical cloud with strong magnetic fields ($\alpha = 300$). Due to the Lorentz force, the motion in the radial direction is blocked and the gas moves only in the z-direction. Figure 9 shows the cross-cut view along the z-axis (a) and the time evolution of the maximum density (b). The density outside the dense disk is much smaller than the initial density, $\rho_{init} = 10$. Comparing with models DB, DC, & DD, the density outside of the dense disk is much smaller. This indicates that the cylindrical cloud does not shrink radially and the irregularities grow by a motion in the z-direction. The velocity field shows that a static balance is achieved in the cloud ($v_z \simeq 0$ in the cloud). Further, Figure 9(b) shows that the maximum of the density increases due to the gravitational instability ($t \lesssim 1$) but it keeps constant after it reaches $\simeq 500$. This indicates that a magnetohydrostatic *disk* configuration is achieved from a uniform *cylindrical* configuration by the gravitational instability. This is reasonable considering the mass-to-flux ratio in the center of the cloud,

$$\frac{M}{\Phi/G^{1/2}} \simeq 0.092 \left(\frac{\alpha}{300}\right)^{1/2}, \tag{3.11}$$

is smaller than the critical value ($1/2\pi$). It is concluded that the cylindrical cloud with strong magnetic fields fragments into stable disks perpendicular to the magnetic fields. According to the initial mass-to-flux ratio in the cylindrical cloud, the fragment becomes supercritical or subcritical.

4 Discussion

4.1 A Model with Uniform Magnetic Fields

In the preceding section, we investigated the models that the cloud is supported by magnetic fields ($\partial B_z/\partial r < 0$). In contrast, here, we study the model with uniform magnetic field ($\partial B_z/\partial r = 0$) and compare the structure of fragments. Instead of $B_z^2/8\pi \propto c_s^2\rho$, we assume that the strength of B_z is constant. Radial density distribution is given by equation (2.3). Relative strength of the magnetic fields is expressed by a parameter α_c which is equivalent to the ratio of $B_z^2/4\pi$ to $c_s^2\rho$ at the center of the cloud.

In Figure 10, the structure at $t = 1.74$ is shown. This figure shows several differences with Figures 1 and 2 (constant α model). (i) the cloud outer boundary: the distance from the z-axis increases near

$z \simeq l_z/2$, and decreases far from the fragment. This seems to come from the fact that the magnetic fields becomes more important far from the z-axis and the gas moves toward $z \simeq l_z/2$ without any effects on the magnetic fields (Fig.10b). As a result, the density contours indicates a convex shape. Contrarily, in models with constant α, the gas falls down along the valley of magnetic fields. Thus, the cylindrical tube of the cloud shrinks near $z \simeq l_z/2$ and inflates near $z \simeq 0$ and $z \simeq l_z$. (ii) the magnetic field line: as seen in Figure 10b the field line breaks weaker than previous models. This seems to come from the fact that the configuration is not unstable against the Parker instability, that is, the magnetic valley does not necessarily promote the instability in this case. That is, there is no positive feedback. The second factor is: since the ratio of magnetic pressure to the thermal one, $\alpha/2$, increases outwardly, the magnetic field becomes relatively important for the dynamics with increasing r, even if α_c is taken $\simeq 1$. Thus the field line is almost locked in the initial configuration.

We studied two models for the magnetic field configuration, α =constant and B_z =constant, which correspond two extremes which are realized in the real interstellar space.

4.2 Classification of the Cylindrical Cloud

How does a cylindrical cloud with a finite line density λ and a magnetic field strength evolve? In the preceding section, it is shown that even in a cloud in the hydrostatic equilibrium, fragments are formed. Dense part of the fragment falls into a runaway collapse or becomes a stable disk, depending upon the magnetic field strength (equation 3.9). From equation (2.7), the line density of a cloud with $F = \rho_c/\rho_s$, is written as

$$\lambda(F) = \frac{2c_s^2}{G}\frac{F^{1/2}-1}{F^{1/2}}\left(1+\frac{\alpha}{2}\right). \tag{4.1}$$

This leads to the maximum line density which can be supported by the thermal and magnetic pressure *as long as α is constant* (Mouschovias & Morton 1991, eq.[16]):

$$\lambda_{\max} = \frac{2c_s^2}{G}\left(1+\frac{\alpha}{2}\right), \qquad \text{for} \quad F \to \infty, \tag{4.2}$$

which include the critical line density: equation(1.1).

Therefore, if the cloud has a line density smaller than $\lambda_{\max}$, which is related to the isothermal sound speed, c_s, and the Alfvén speed, $\alpha^{1/2}c_s$, the cloud has a magnetohydrostatic equilibrium with constant α. In such a cloud, irregularity grows in several $\times\tau_{\rm max}$ depending on the amplitude of irregularities. Adopting $3\tau_{\rm max}$ as a typical growth time, this gives

$$\tau_{\rm grow} \simeq 2.4 \times 10^6 {\rm yr} \left(\frac{F}{100}\right)^{-1/2} \left(\frac{\rho_s}{2\times 10^{-22}{\rm g\ cm^{-3}}}\right)^{-1/2}. \tag{4.3}$$

Final phase of the contraction, that is, the phase when the maximum density in the high-density core increases rapidly, continues only in a short duration $\sim 0.1 \times \tau_{\rm grow}$ (see Fig. 4).

On the other hand, since there is no magnetohydrostatic configurations with α =constant for a cloud with a line density $\lambda > \lambda_{\rm max}$. Thus, the cloud contracts with flux-freezing as a whole, and forms a thin cloud (see §3.3.3 of Mouschovias & Morton 1991). Recently, hydrodynamical simulation has been done for such a massive contracting cylindrical clouds by Inutsuka and Miyama (private communication) for non-magnetic clouds. They found that in such clouds, the irregularity grows relatively slowly compared with the contraction time-scale $\simeq$ free-fall time-scale and the cloud does not fragment but forms a string as long as the gas obeys the isothermal equation of state. However, since B/ρ-ratio keeps constant during the radial contraction, the magnetic pressure force becomes dominant and a different hydrostatic configuration will appear (Mouschovias & Morton 1991). It should be noted that there is no other configuration in non-magnetic cloud with $\lambda > \lambda_c$. Therefore, in the magnetized cylindrical cloud, the gravitaional instability grows after a hydrostatic configuration is achieved, in which α is no more constant (α decreases with increasing r). Assuming the wavelength for maximum growth rate is $\gamma 20 c_s/(4\pi G\rho_c)^{1/2}$ (a numerical factor γ calculated by the linear perturbation theory shows almost constant as $\gamma \simeq 1$ under $\alpha < 20$ [Nakamura et al 1993]), the mass-to-flux ratio in the center of the fragment,

$$\frac{M}{\Phi/G^{1/2}} \simeq \gamma \frac{1.59}{\alpha_c^{1/2}}, \tag{4.4}$$

where α_c represents the α at the center of the cylindrical cloud. Even if α is initially equal to $\simeq 1$, α_c reaches ~ 100 when the central density increases by a factor of 100. In the case that the mass-to-flux ratio of equation (4.4) is smaller than the critical value, $1/(2\pi)$, stable disks seem to appear with a separation of $\gamma 20 c_s/(4\pi G\rho_c)^{1/2}$ similar to model S. These disks evolve by the effect of ambipolar diffusion (plasma drift). Contrally, when $G^{1/2}M/\Phi$ is larger than the critical value, the disks contract dynamically similar to the models studied in the present paper.

From this, it is concluded that the disks formed by the gravitational instability from the hydrostatic configuration with relatively weak magnetic fields become supercritical (models except for S in the present paper). On the other hand, that from the hydrostatic configuration with strong magnetic fields seems a (stable) subcritical disk (like model S). For both cases, a typical mass of the fragment formed by the gravitational instability is estimated

$$M_F \sim \lambda l_z = 62 M_\odot \left(\frac{c_s}{300\mathrm{m\ s^{-1}}}\right)^3 \left(1+\frac{\alpha}{2}\right)\left(\frac{\rho_c}{2\times 10^{-20}\mathrm{g\ cm^{-3}}}\right)^{-1/2}. \tag{4.5}$$

5 Summary

We studied the process of fragmentation in an isothermal cylindrical cloud with infinite length. By a numerical magnetohydrodynamical method, the evolution of the fragment was investigated from the linear stage to nonlinear stage under the flux freezing assumption. It is shown that the fastest growing perturbation has the same wavelength as predicted with the linear theory. In a linear stage the fragment appears as a prolate spheroidal shape. However, in a nonlinear stage, the fragment threaded by the magnetic field forms a disk perpendicular to the field line. At the center of the fragment there forms a high-density core, which continues to collapse, when α is larger than ~ 100. In the cloud supported by strong magnetic fields $\alpha > 100$, static disks are formed by the gravitaional instability. Non-magnetized cloud only forms collapsing spherical cores which are separated by $\lambda_{\rm max}$. For magnetic clouds, it is shown that when the collapsing velocity reaches $\sim 3.5c_s$, relatively slowly contracting dense inner part is formed in the contracting disk, which also extends perpendicular to the magnetic fields. This inner disk component is in almost static and is separated from the accreting flow which is accelerated as reaching the center of the core.

From this, the evolution in the flux-freezing regime of the cylindrical cloud is predicted. If the initial configuration is not in the hydrostatic balance in the radial direction, the cloud contracts and it will be supported by the magnetic fields and the thermal pressure. The cloud seems to reach a static configuration. If the plasma $\beta = 2/\alpha$ in the center of the equilibrium cloud is as large as $\beta \gtrsim 0.02$, the disks formed by the gravitational instability continue to contract. When the static balance is achieved mainly by the magnetic fields ($\beta \lesssim 0.02$), stable disks seem to be formed.

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Tables

TABLE 1: Model Parameters

Model	α	F	$\delta_{\rm ini}$	l_z	cell number
A[a]	1	100	white noise $\sigma = 10^{-4}$	1.935	400×400
A2[a]	1	100	white noise $\sigma = 10^{-4}$	3.87	400×400
B	1	100	10^{-2}	1.935	400×400
C	0	100	10^{-2}	2.21	400×400
DB[b] ...	1	10	10^{-2}	1.935	400×400
DC[b] ...	0	10	10^{-2}	2.21	400×400
DD[b] ...	4	10	10^{-2}	1.935	400×400
CB[c] ...	1	100	10^{-2}	1.935	400×280
CB2[c] ..	1	100	10^{-1}	1.935	800×560
S	300	10	10^{-1}	1.811	100×500
UB[d] ...	1	100	10^{-2}	1.935	200×200

[a]In models A and A2, random fluctuations with the standard deviation of $\sigma = 10^{-4}$ were added to Eq.(2.7)
[b]In a series of models with character D, diffuse clouds are considered.
[c]Spatially varying spacing grid is used. The physical parameters are taken as identical as model B.
[d]Uniform magnetic fields are assumed. In this model, α represents the value at the center of the cloud ($r = 0$), that is α_c.

TABLE A1: Necessary CPU time to solve the Poisson equation once

Machine		CGS (sec)	MILUCGS (sec)	N
Sparc IPX	(Workstation)	505	143	200
M-682H	(General Purpose Computer)	43.1	14.1	200
S-820/80	(Supercomputer)	4.1	2.4	400

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A MILUCGS

Here, a brief description of MILUCGS and its performance are described. The finite difference scheme for the Poisson equation is written

$$
\begin{pmatrix}
a_1 & ub_1 & & & uc_1 & & & \\
db_2 & a_2 & ub_2 & & & uc_2 & & \\
& \ddots & \ddots & \ddots & & & \ddots & \\
& & \ddots & \ddots & \ddots & & & \ddots \\
dc_{m+1} & & & db_{m+1} & a_{m+1} & ub_{m+1} & & \\
& dc_{m+2} & & & \ddots & \ddots & \ddots & \\
& & \ddots & & & \ddots & \ddots & \ddots \\
& & & \ddots & & & \ddots & \ddots
\end{pmatrix}
\begin{pmatrix}
\psi_1 \\ \psi_2 \\ \psi_3 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \psi_{nm}
\end{pmatrix}
=
\begin{pmatrix}
\rho_1 \\ \rho_2 \\ \rho_3 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \rho_{nm}
\end{pmatrix}, \tag{A.1}
$$

where two-dimensional expression of $\psi_{i,j}$ and $\rho_{i,j}$ ($1 \le i \le m$ and $1 \le j \le n$) are converted to a one-dimensional expression ψ_k and ρ_k with $k = (j-1) \times m + i$. m and n represent, respectively, the cell numbers in the z- and r-direction. The coefficients db and ub correspond to the derivatives with respect to the z-direction, and the coefficients of dc and uc correspond to those of r-direction. If the problem is a simple Neuman or Dirichret boundary condition, the matrix to be solved is like equation (A.1). However, since we apply here a periodic boundary in the z-direction, we have to add two coefficients expressing this boundary condition. That is, the term of $\psi_{m,j} = \psi_{jm}$ should connect with $\psi_{m-1,j} = \psi_{jm-1}$ and $\psi_{1,j} = \psi_{jm-m+1}$ instead of ψ_{jm+1}. The equation for periodic boundary becomes

$$
\begin{pmatrix}
a_1 & ub_1 & & ud_1 & uc_1 & & & \\
db_2 & a_2 & ub_2 & & ud_2 & uc_2 & & \\
& \ddots & \ddots & \ddots & & \ddots & \ddots & \\
dd_m & & \ddots & \ddots & \ddots & & \ddots & \ddots \\
dc_{m+1} & dd_{m+1} & & db_{m+1} & a_{m+1} & ub_{m+1} & & \ddots \\
& dc_{m+2} & dd_{m+2} & & \ddots & \ddots & \ddots & \\
& & \ddots & \ddots & & \ddots & \ddots & \ddots \\
& & & \ddots & \ddots & & \ddots & \ddots
\end{pmatrix}
\begin{pmatrix}
\psi_1 \\ \psi_2 \\ \vdots \\ \vdots \\ \psi_{m+1} \\ \vdots \\ \vdots \\ \psi_{nm}
\end{pmatrix}
=
\begin{pmatrix}
\rho_1 \\ \rho_2 \\ \vdots \\ \vdots \\ \rho_{m+1} \\ \vdots \\ \vdots \\ \rho_{nm}
\end{pmatrix}, \tag{A.2}
$$

where only $dd_{j\times m}$ as well as $ud_{(j-1)m+1}$ has non-zero components and the others have the value of zero. This linear simultaneous equation is solved by conjugate gradient squared method (Dongarra et al. 1991). If we write equation(A.2) as $A\hat{\psi} = \rho$, the algorithm is as follows:

1. Prepare the initial guess $\hat{\psi}_0$

   $\hat{r}_0 = \rho - A\hat{\psi}_0$

   $\hat{p}_0 = e_0 = \hat{r}_0$

   $l = 0$

2. while $\|\hat{r}_l\| > \epsilon\|\rho\|$ do

   (a) $\alpha_l = (\hat{r}_0, \hat{r}_l)/(\hat{r}_0, A\hat{p}_l)$

   (b) $h_{l+1} = e_l - \alpha_l A\hat{p}_l$

   (c) $\hat{r}_{l+1} = \hat{r}_l - \alpha_l A(e_l + h_{l+1})$

   (d) $\hat{\psi}_{l+1} = \hat{x}_l + \alpha_l(e_l + h_{l+1})$

   (e) $\beta_l = (\hat{r}_0, \hat{r}_{l+1})/(\hat{r}_0, \hat{r}_l)$

   (f) $e_{l+1} = \hat{r}_{l+1} + \beta_l h_{l+1}$

   (g) $\hat{p}_{l+1} = e_{l+1} + \beta(h_{l+1} + \beta_l\hat{p}_l)$

   (h) $l = l + 1$.

The convergence of this method is improved much by preconditioning. In the present paper, we apply the incomplete LU-decomposition. The incomplete LU-decomposition is a method that matrix A is decomposed as $LDU - R$, where L, D, U, and R are a lower triangle matrix, a diagonal matrix, an upper triangle matrix, and a residual matrix. Instead of $A\hat{\psi} = \rho$, we apply the CGS method to the reduced equation of

$$(LDU)^{-1}A\hat{\psi} = (LDU)^{-1}\rho. \tag{A.3}$$

Since $(LDU)^{-1}A$ is nearly equal to the identity matrix I and the eigen values of this matrix gather around unity, the convergence of the CGS method is much accelerated. The way to decompose $A = LDU - R$ is arbitrary. Here, we use the method by Meijerink & van der Vorst (1977) and

Gustafsson (1978). Describing briefly, L, D, and U are chosen as

$$L = \begin{pmatrix} 1/\tilde{d}_1 & & & & & & \text{\Large 0} & \\ db_2 & 1/\tilde{d}_2 & & & & & & \\ \vdots & \ddots & \ddots & & & & & \\ dd_m & & \ddots & \ddots & & & & \\ dc_{m+1} & dd_{m+1} & & db_{m+1} & 1/\tilde{d}_{m+1} & & & \\ & dc_{m+2} & dd_{m+2} & & \ddots & \ddots & & \\ & & \ddots & \ddots & & \ddots & \ddots & \\ & & & \ddots & \ddots & & db_{mn} & 1/\tilde{d}_{mn} \end{pmatrix}, \tag{A.4}$$

$$D = \begin{pmatrix} \tilde{d}_1 & & & & & \text{\Large 0} & & \\ & \tilde{d}_2 & & & & & & \\ & & \ddots & & & & & \\ & & & \ddots & & & & \\ & & & & \ddots & & & \\ & & & & & \ddots & & \\ & \text{\Large 0} & & & & & \tilde{d}_{mn-1} & \\ & & & & & & & \tilde{d}_{mn} \end{pmatrix}, \tag{A.5}$$

$$U = \begin{pmatrix} 1/\tilde{d}_1 & ub_1 & & ud_1 & uc_1 & & & \\ & 1/\tilde{d}_2 & ub_2 & & ud_2 & uc_2 & & \\ & & \ddots & \ddots & & \ddots & \ddots & \\ & & & \ddots & \ddots & & \ddots & \ddots \\ & & & & 1/\tilde{d}_{m+1} & ub_{m+1} & & \ddots \\ & & & & & \ddots & \ddots & \\ & \text{\Large 0} & & & & & \ddots & \ddots \\ & & & & & & & \ddots \end{pmatrix}. \tag{A.6}$$

If we choose the diagonal element of D as

$$\tilde{d}^{-1} = a_k - db_k \tilde{d}_{k-1} ub_{k-1} - dc_k \tilde{d}_{k-m} uc_{k-m} - dd_k \tilde{d}_{k-m+1} ud_{k-m+1}, \tag{A.7}$$

the diagonal part of the matrix A becomes equal to that of LDU. However, when the matrix LDU is operated on $\hat{\psi}$, it generates extra terms (fill-in) such as

$$db_k \tilde{d}_{k-1} uc_{k-1} \hat{\psi}_{k+m-1} + dc_k \tilde{d}_{k-m} ub_{k-m} \hat{\psi}_{k-m+1} \tag{A.8}$$

as well as the ordinary terms:

$$a_k \hat{\psi}_k + ub_k \hat{\psi}_{k+1} + db_k \hat{\psi}_{k-1} + uc_k \hat{\psi}_{k+m} + dc_k \hat{\psi}_{k-m} + ud_k \hat{\psi}_{k+m-1} + dd_k \hat{\psi}_{k-m+1}. \tag{A.9}$$

To suppress the effect of these extra terms, the diagonal term $\tilde{d}$ is modified as

$$\tilde{d}^{-1} = a_k - db_k\tilde{d}_{k-1}ub_{k-1} - dc_k\tilde{d}_{k-m}uc_{k-m} - dd_k\tilde{d}_{k-m+1}ud_{k-m+1} - \gamma(db_k\tilde{d}_{k-1}uc_{k-1} + dc_k\tilde{d}_{k-m}ub_{k-m}), \tag{A.10}$$

where the factor γ is chosen as the way by Gustafsson (1978). Using the LDU-decomposition, in the CGS algorithm the product of Av is replaced with $(LDU)^{-1}Av$. The operation multiplying $(LDU)^{-1}$ is done as follows: $y = (LDU)^{-1}x$ is equivalent to $x = (LDU)y$, i.e.,

$$Lz = x \tag{A.11}$$

$$DUy = z. \tag{A.12}$$

The equation(A.11) is solved as: for $k = 1, 2, \cdots, mn$

$$z_k = \tilde{d}_k(x_k - db_kz_{k-1} - dc_kx_{k-m} - dd_kx_{k-m+1}), \tag{A.13}$$

and then equation (A.12) is solved as: for $k = mn, mn-1, \cdots, 1$

$$y_k = z_k - \tilde{d}_k(ub_ky_{k+1} + uc_ky_{k+m} + ud_ky_{k+m-1}). \tag{A.14}$$

Necessary CPU time is proportional to the cell number, mn, and to loop count. A numerical experiment was done and we compared CGS with MILUCGS. The adopted test problem is as follows: the gravitational potential for a uniform cylinder is solved with square grids of $m = n = N$. The same problem was solved by a supercomputer Hitachi S-820/80, a general purpose main frame machine Hitachi M-682H, and a workstation Sun Sparc IPX. The allowable maximum relative error is chosen as $\epsilon = 10^{-7}$ and calculations are done using 64 bits. As a result, about $80 \times (N/200)$ cycles are necessary to achieve a solution for MILUCGS, while the simple CGS requires $410 \times (N/200)$. Since the number of cycles necessary for convergence is proportional to N, CPU time is proportional to N^3 irrespective with the preconditioning or not.

In table A1, we summarize the CPU time necessary for solving the Poisson equation by respective machines. Comparing CPU time, the preconditioning reduces the CPU time about $\lesssim$1/3 for scalar machines (M-682 and Sparc IPX). Since the amount of operations in MILUCGS is larger than that of CGS, the factor is not equal to $\simeq 1/5$ (the ratio of necessary loops) but to $\lesssim 1/3$. However, this clearly shows the advantage of MILUCGS over the simple CGS. The difference for the supercomputer is relatively small, for in order to vectorize the operation of multiplying $(LDU)^{-1}$ [equations (A.11, A.12)] a more complicated algorithm with list vectors is necessary.

Figure Captions

Fig.1: The initial radial density distributions. A solid line corresponds to models A and B, i.e., a magnetized cloud with $\alpha = 1$. A dashed line shows the density of model C which has no magnetic fields $\alpha = 0$. Both correspond to the models with the center-to-surface density ratio, $F = 100$. The dotted part of the curves represents the ambient medium $f < 1$ which is assumed to have no effect as a source of gravity.

Fig.2: The time evolutions of density (left panel) and magnetic fields (right panel) for model A. The z-axis and r-axis are directed horizontally and vertically, respectively, in contrast to an ordinary fashion. The number attached to the density contour lines represent the logarithm of the density: $\log_{10} \rho$. The step of the contour lines is taken constant $= 0.25$. Four snapshots, i.e., $t = 1.2$ (a), $t = 2.4$ (b), $t = 3.2$ (c), and $t = 3.5$ (d) are shown. In $t = 1.2$, no prominent fragmentations appear. The surface of the cloud $\log \rho = 0$ is running near $r \simeq 1$. In the stage shown in (b), a high-density region elongating in the z-direction appears. After (c), contraction along the symmetry axis proceeds. The final structure is a disk contracting towards the center seen in (d). The disk is perpendicular to the symmetry axis of the cylinder.

Fig.3: The structure of the cloud for model B. The upper-left panel shows the density distribution (a) and the upper-right does the magnetic field lines (b). It is shown that a fragment contracts and forms a disk perpendicular to the symmetric axis. Cross-cut views along the z-axis (c) and the r-axis (d) are plotted. In panel c, a logarithmic plot of $\rho(z, 0)$ and $B_z(z, 0)$ and a linear plot of $v_z(z, 0)/10$ are illustrated. Similarly, in panel d, a logarithmic plot of $\rho(z_c, r)$ and $B_z(z_c, r)$ and a linear plot of $v_r(z_c, r)/10$ are shown, where z_c represents the position of the center of the fragment $= l_z/2$. Physical quantities outside the cloud ($\rho < 1$) is also plotted for reference. This snapshot corresponds to the state of $t = 1.26$.

Fig.4: Time evolutions of the relative density enhancement, $\delta_{\rm max}$, for models A and B. This is defined as $\delta_{\rm max} \equiv \rho(r = 0)|_{\rm max} / < \rho(r = 0) > -1$. The curve for model B is similar to the late phase ($t \gtrsim 4$) of the model A. This shows that the structure and evolution of the fragment can be calculated from the initial state which has the most unstable perturbation with a finite amplitude.

Fig.5: The time evolution of density for Model C. In this model, the fragmentation in non-magnetic cloud is studied. Since the magnetic field, which has the effect of supporting cloud laterally, is not included, the radial extent of the cloud is thinner than that of models A and B. Two snapshots of density distribution at $t = 1.2$ (a) and $t = 1.4$ (b) are plotted.

Fig.6: The cross-cut views of the cloud of models B (a) and C (b). The left panel shows the variation of $\rho(z, 0)$ against $\log|z - z_c|$, and the right panel shows that of $\rho(z_c, r)$ against $\log r$. Two characteristic solutions [a dotted line (3.4), and a dashed line (3.6)] are also shown in right panels. Five snapshots at $t = 0$, 1.08, 1.19, 1.23, and 1.25 are plotted for model B. For model C, the density distributions at $t =$0, 1.23, 1.35, 1.40, and 1.42 are shown.

Fig.7: The density distributions of models DB (a), DD (b), and DC (c). Model DB ($\alpha = 1$) has the same parameter as model B except for the density contrast between the center and the surface, $F = 10$. Model DD corresponds to the cloud with stronger magnetic fields ($\alpha = 4$). Model DC ($\alpha = 0$) has the same parameter as model C except for the density contrast between the center and the surface, $F = 10$. For panel c, comparing how the density contour lines are running with that for Fig.5, it can be seen that the density distributions are very similar with each other. Since the length of numerical box in the z-direction l_z is taken as proportional to ${\rho_c}^{-1/2}$, it is seen that the fragments of models C and F have an almost similar structure, if the size is scaled with the "scale-height" at the center, $c_s/(4\pi G\rho_c)^{1/2}$ instead of H. The number attached to the contour line, k, indicates that the value of the contour is equal to $\rho = 10^{k/4}$.

Fig.8: The structure of the fragment of model CB2 at $t = 0.705$. Physical parameters of this model are chosen identical with model B but this model has 4.28 times higher spatial resolution than model B. In this model a perturbation with a larger amplitude $\delta_{\rm ini} = 10^{-1}$ is assumed initially to reduce a computing time. This time corresponds to 1.26 for model CB with $\delta_{\rm ini} = 10^{-2}$. The panel a shows the density and velocity fields and b shows the magnetic field lines. To see the structure near the fragment clearly, in panel a, only a region of $0 < z < l_z/2$ and $r < l_z/2$ is plotted. Velocity vectors are plotted every 16 grids. Panels c and d are respectively, the cross-cut views along the z-axis and r-axis. A logarithmic plot of $\rho(z, 0)$ and $B_z(z, 0)$ and a linear plot of

$v_z(z,0)$ are illustrated against $\log|z-z_c|$ in panel c. Similarly, in panel d, a logarithmic plot of $\rho(z_c,r)$ and $B_z(z_c,r)$ and a linear plot of $v_r(z_c,r)$ are shown against $\log r$.

Fig.9: (a) The cross-cut view along the z-axis for model S. (b) similar to Fig. 4 but for model S.

Fig.10: The structure of the fragment formed in a cylindrical cloud threaded by a uniform magnetic field (model UB). The initial radial distribution of the density is as the same as model C (non-magnetic model), since the magnetic fields do not play a role in supporting the cloud. The density contour (a) and the magnetic fields (b) are illustrated. Panels c and d are, respectively, the cross-cut views along the z-axis and the r-axis. That is, a logarithmic plot of $\rho(z,0)$ and $B_z(z,0)$ and a linear plot of $v_z(z,0)$ are illustrated in panel c. Similarly, in panel d, a logarithmic plot of $\rho(z_c,r)$ and $B_z(z_c,r)$ and a linear plot of $v_r(z_c,r)$ are shown. This snapshot corresponds to the state of $t=1.26$.