Electromagnetic Particle-in-Cell Model with AMR and Application to Current Sheet Evolution in 2D and 3D Systems

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Introduction

MHD scale
~10⁵ km

Electron scale
~10 km

Dynamic range
~ 10⁴

Electromagnetic particle code is required
**Introduction**

Restriction in full particle code: \[ \Delta x \lesssim 3 \lambda_{De} \sim 1 \text{km} \]

[Birdsall & Langdon, 1995]

**Magnetotail Lobe**

\[ \frac{T_i}{T_e} \approx 4.0, \quad n \approx 0.01 \text{ cm}^{-3}, \quad \beta_i \approx 0.1, \quad B = 30 \text{ nT}. \]

(Baumjohann and Treumann, 1997)

\[ \lambda_{De} = 5.6 \times 10^3 \text{ [m]} \]

Grid spacing can be coarser in the lobe region than in the plasma sheet.

**Central Plasma Sheet**

\[ \frac{T_i}{T_e} \approx 8.0, \quad n \approx 0.3 \text{ cm}^{-3}, \quad \beta_i \approx 20, \quad B = 5 \text{ nT}. \]

(Baumjohann and Paschmann, 1989)

\[ \lambda_{De} = 3.1 \times 10^2 \text{ [m]} \]
Introduction

Restriction in full particle code: $\Delta x \lesssim 3\lambda_{De} \sim 1\text{km}$

[Birdsall & Langdon, 1995]
AMR法の使用例

 ✓ 多階層格子を偏微分方程式系に適用

 ✓ MHD + AMR
   Groth et al. (2000)

 ✓ PIC + AMR (N-body code)

 ✓ EMPIC + AMR
   Fujimoto and Machida (2006)
Data Structure

Cells are treated as independent units organized in refinement trees rather than elements of arrays, so that a very flexible cell hierarchy is achieved.

The hierarchical cell structure is supported by a set of pointers, which is basically same as the fully threaded tree (FTT) structure [Khokhlov, 1989].
**Basic Equations**

**Equation of motion**

\[
\frac{dx_s}{dt} = v_s, \quad \frac{dv_s}{dt} = \frac{q_s}{m_s} [E(x_s) + v_s \times B(x_s)] \quad (s = i, e)
\]

**Electromagnetic field**

\[
E = E_L + E_T \quad (\nabla \times E_L = 0, \nabla \cdot E_T = 0),
\]

\[
E_L = -\nabla \phi, \quad \nabla^2 \phi = -\rho/\varepsilon_0,
\]

\[
\frac{\partial E_T}{\partial t} = c^2 \nabla \times B - j_T/\varepsilon_0 \quad (j_T = j + \nabla \eta),
\]

\[
\frac{\partial B}{\partial t} = -\nabla \times E_T
\]

\[
\eta = -\varepsilon_0 \frac{\partial \phi}{\partial t} \quad \text{(Charge continuity equation)}.
\]
Calculation of Electromagnetic Field

EM solver on the coarsest cells

Interpolation to the buffer cells

EM solver on the finer cells

Projection of the solution on the finer cells onto the coarsest cells

Buffer cells  Refined cells
Mesh Refinement Criteria

Electron Debye length alone is really enough?

Refinement meshes are required in the region where the electron scale physics is expected to be significant.
The Number of Meshes and Simulation Cost

The number of meshes

Simulation cost

Walltime

96% of the total simulation time is devoted to the routines related to the particle data.

In order to perform an efficient simulation, it is inevitable to reduce the number of superparticles.
Particle Splitting and Coalescence  (Lapenta, 2002)

\[
\frac{d\tilde{v}_s}{dt} = \frac{q_s}{m_s} \left( \vec{E} + \vec{v}_s \times \vec{B} \right)
\]

Conserving through the splitting

Moment on the grids (\(\rho_c, J\)), Total charge and mass (\(\Sigma \rho_c, \Sigma m\)), Total energy of particles (\(\Sigma m v^2 / 2\)), Distribution function (\(f(v)\))

\(q_i = q_0 / 8, m_i = m_0 / 8\)

\(i = 1, 2, \ldots, 8\)

\(q_1 = q_2; m_1 = m_2\)

\(\vec{x}_0 = (\vec{x}_1 + \vec{x}_2) / 2\)

\(\vec{v}_0 = (\vec{v}_1 + \vec{v}_2) / 2\)

\(q_0 = q_1 + q_2\)

\(m_0 = m_1 + m_2\)

Wall time
Initial Setting for the Test Simulations

Harris-type current sheet

\[ B_x(z) = B_0 \tanh\left(\frac{z}{\lambda}\right) \]

\[ J_y(z) = J_0 \text{sech}^2\left(\frac{z}{\lambda}\right) \]
Test Simulations in the Non-tearing System

Linear dispersion of the lower hybrid drift instability

\[ 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} \left( 1 + \frac{\omega_{pe}^2}{c^2 k^2} \right) - \frac{2 \omega_{pi}^2}{k^2 v_i^2} \left( 1 + \frac{\beta_i}{2} \right) \frac{k V_i}{\omega - k V_e} \]

\[ + \frac{2 \omega_{pi}^2}{k^2 v_i^2} [1 + \xi Z(\xi)] = 0 \]

\[ \xi = (\omega - k V_i)/k v_i \]

[Davidson et al., 1977]
Test Simulations on Magnetic Reconnection

In 3D system with the lower hybrid drift instability (LHDI), the current density is enhanced at the center of the current sheet, which facilitates the onset of a fast reconnection.
Tearing and Kink Modes in 3D System

System size: \( L_x \times L_y \times L_z = 30.7\lambda_i \times 7.7\lambda_i \times 30.7\lambda_i \)

Maximum resolution: \( N_x \times N_y \times N_z = 1024 \times 256 \times 1024 \)

\( m_i/m_e = 25 \)
Parallelization Efficiency

(The test was performed using the FUJITSU HPC2500.)
Summary and Conclusions

We have successfully developed new 2D and 3D electromagnetic particle code with the adaptive mesh refinement technique.

- Refinement meshes are required not only in the region with small Debye length, but also in the region where the electron scale physics is expected to be significant.

- In order to perform an efficient simulation, it is inevitable to reduce not only the number of meshes, but also the number of superparticles.

- Numerical errors associated with cell refinement and particle coalescence are small.

- The code is checked against the LHDI and Tearing instability.

- The computing performance and efficiency are well enhanced by parallelizing the code, using the OpenMP and MPI.