AMR-PIC Simulation of Collisionless Magnetic Reconnection

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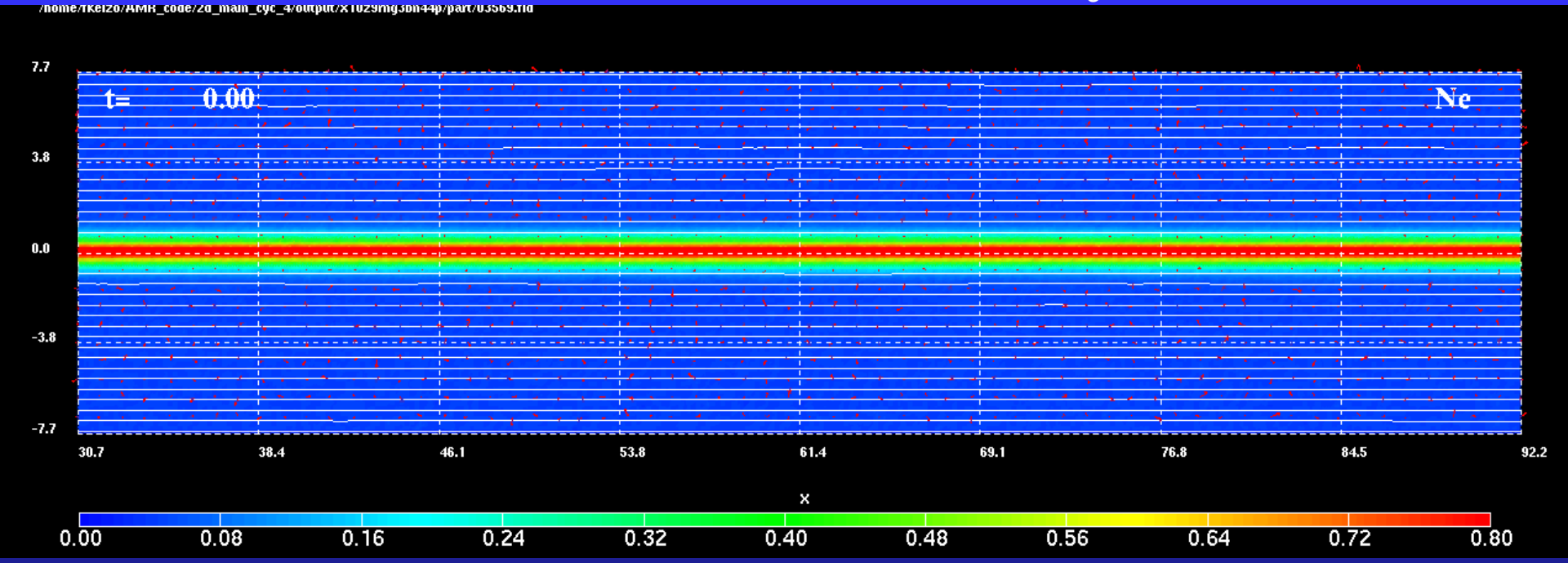
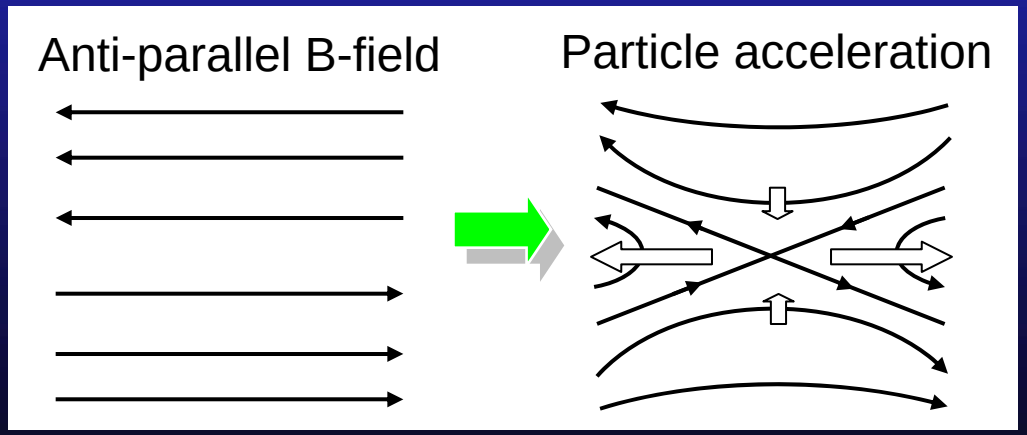
Magnetic Reconnection

Line : Field lines, Contour : n_e
Arrow : V_e

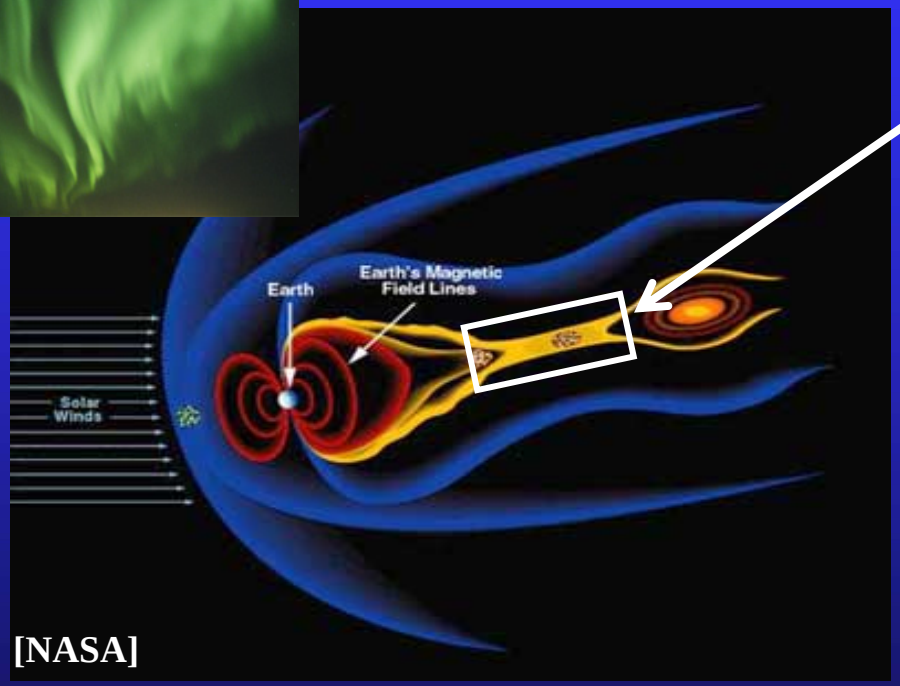


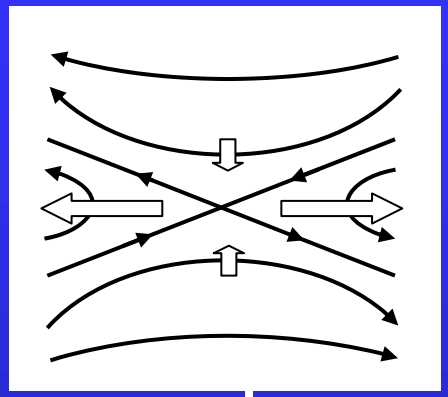


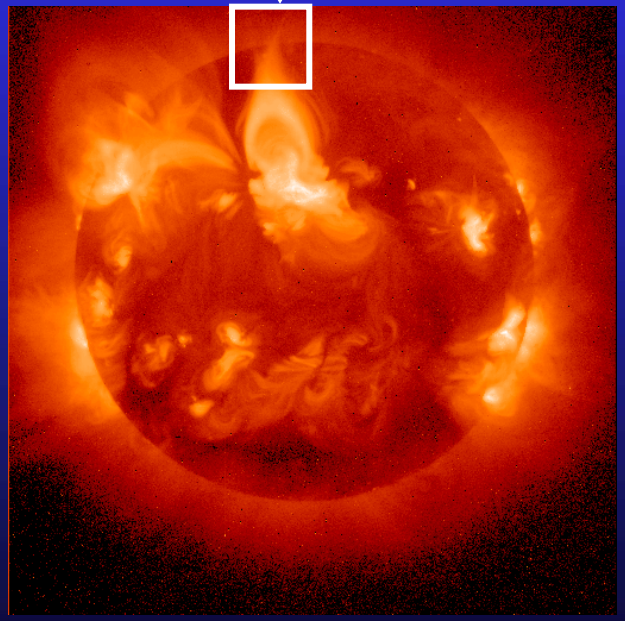
- Change in field line topology
- Plasma acceleration and heating

Magnetic Reconnection in Space

Auroral Substorms

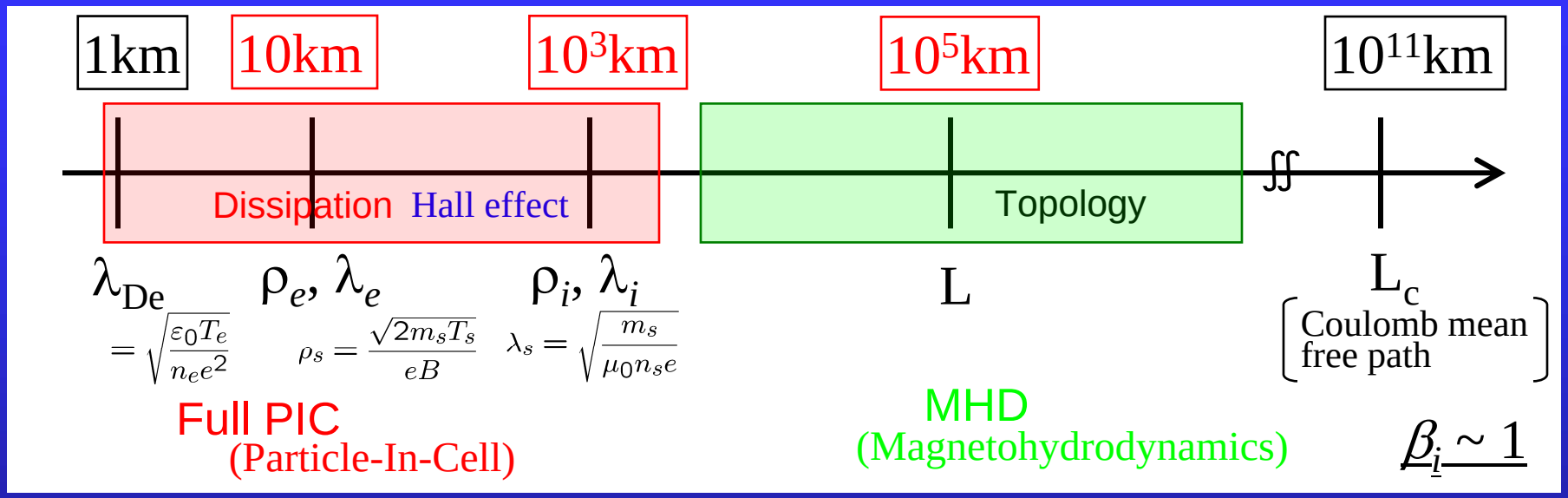


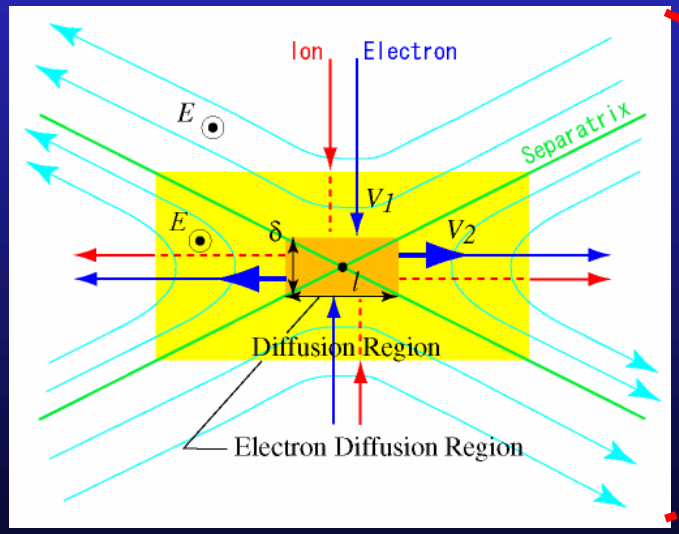


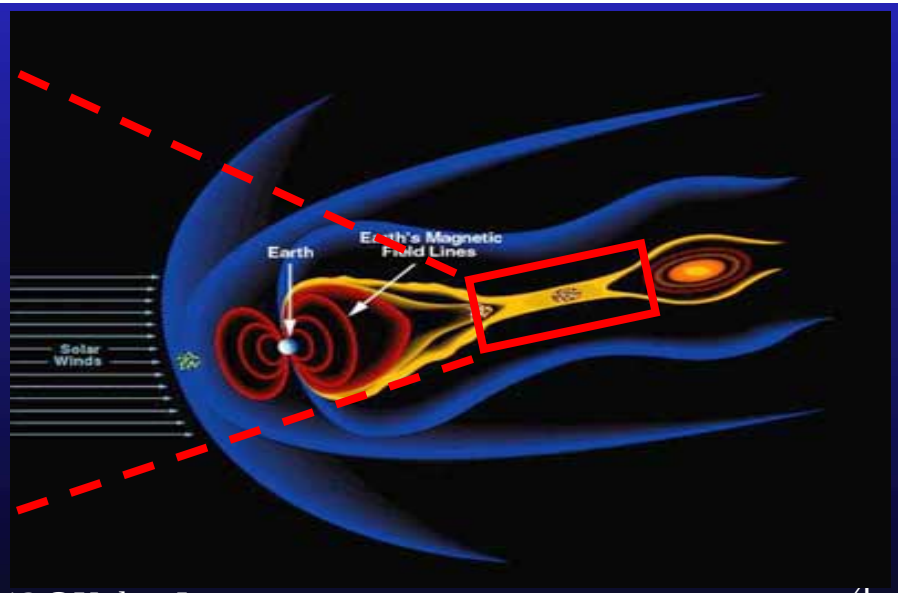


Solar Flares

Multi-Scale Nature of Reconnection



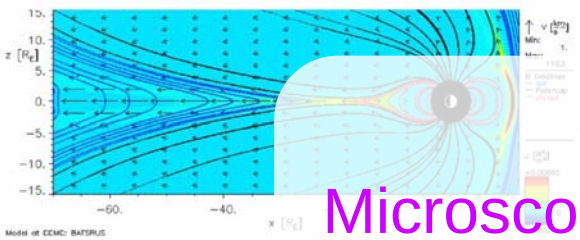




Kuznetsova et al., J. Geophys. Res., 2007

$$\frac{\partial B}{\partial t} = \eta \nabla^2 B / \mu_0$$

Numerical resistivity only

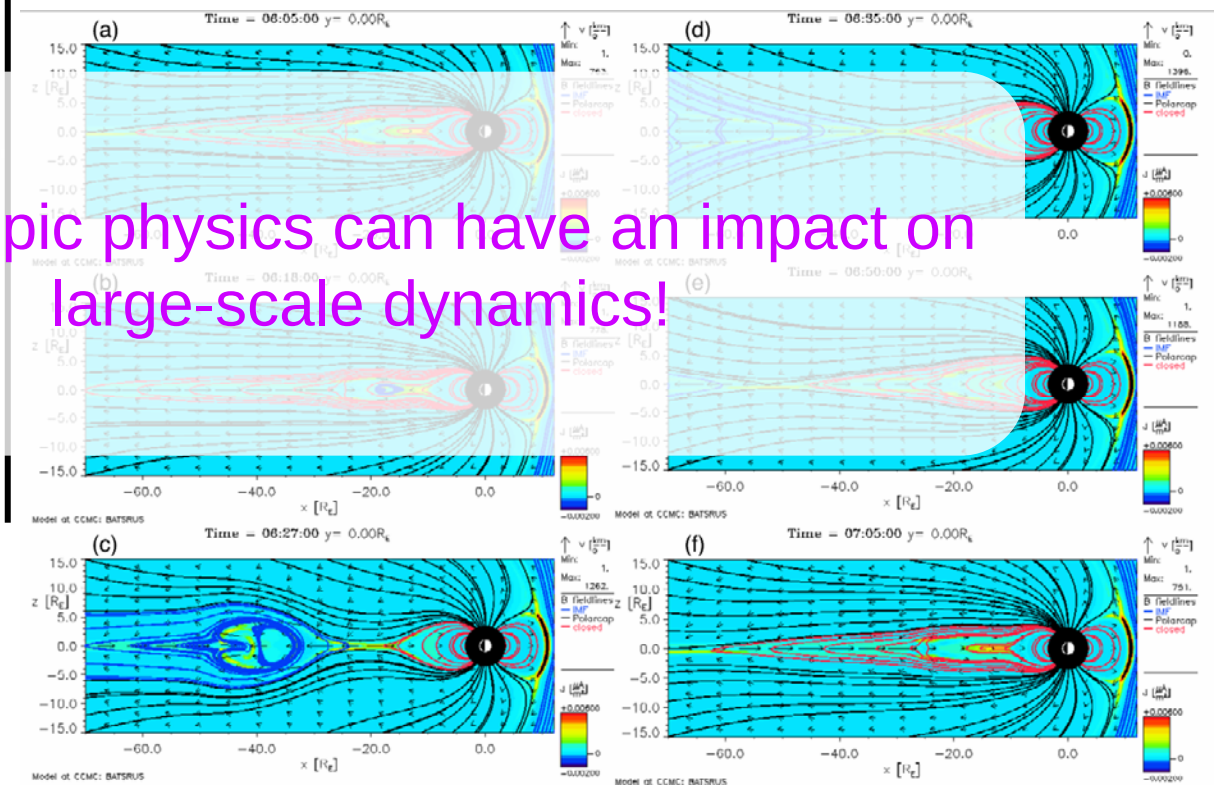


- Slow reconnection
- Quasi-steady configuration

- Fast reconnection
- Quasi-periodic process

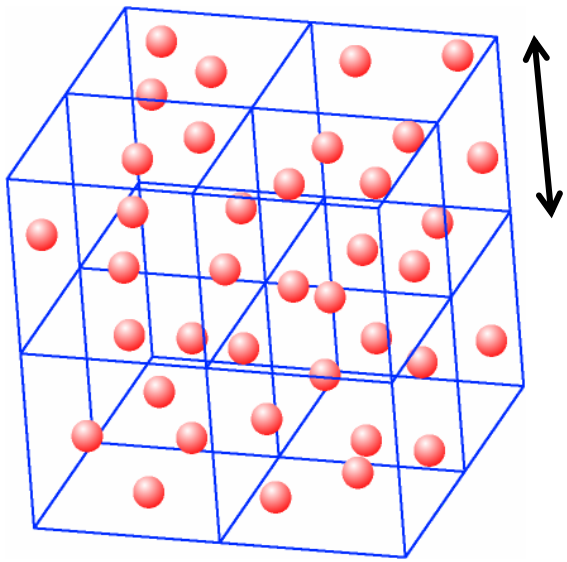
Nongyrotropic correction case

$$E^{\mathrm{ng}} = \frac{1}{ne}\left(\frac{\partial P_{ixy}}{\partial x} + \frac{\partial P_{ixz}}{\partial z}\right) = \frac{m_i}{e}\sqrt{\frac{2P}{\rho}}\frac{\partial V_x}{\partial x}$$



Microscopic physics can have an impact on large-scale dynamics!

Restrictions of Explicit PIC Model



Cell size

$\Delta x \lesssim 3\lambda_{De}$ $\quad \lambda_{D_e} = \sqrt{\frac{\varepsilon_0 T_e}{n_e e^2}}$

Num of particles per cell

$N_p \gtrsim 10^2$

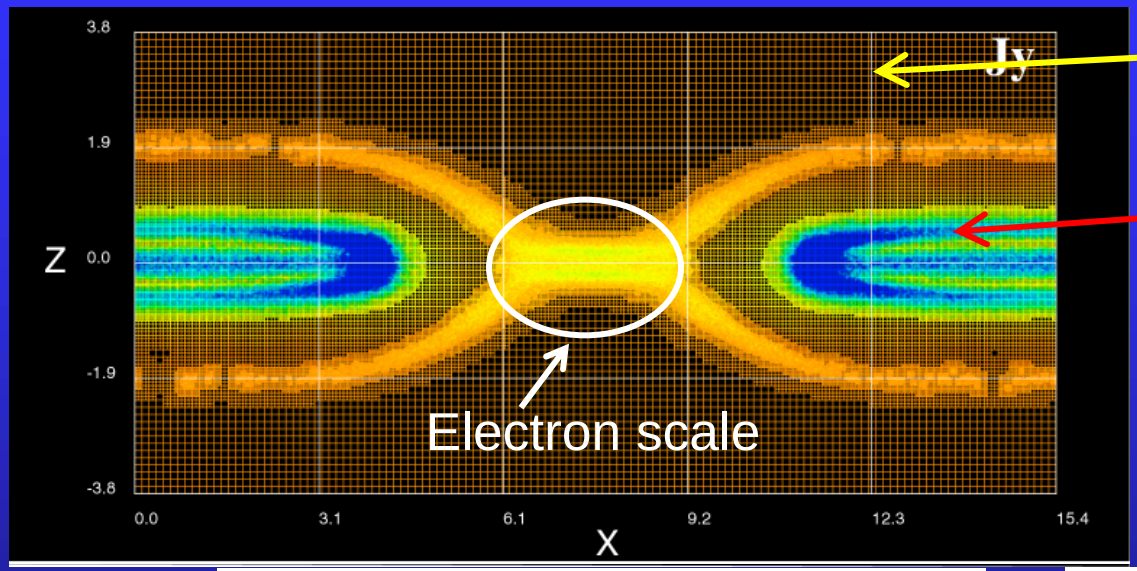
Memory requirement per cell

Field (n_s, J_s, E, B)	<<	Particle (x_s, v_s)
14×4 Byte		$(12 \times 4$ Byte$) \times 10^2$

AMR-PIC Model [Fujimoto, JCP, 2011]

(Adaptive Mesh Refinement – Particle-in-Cell)

$$\Delta x \lesssim 3\lambda_{De} \propto n_e^{-1/2}$$

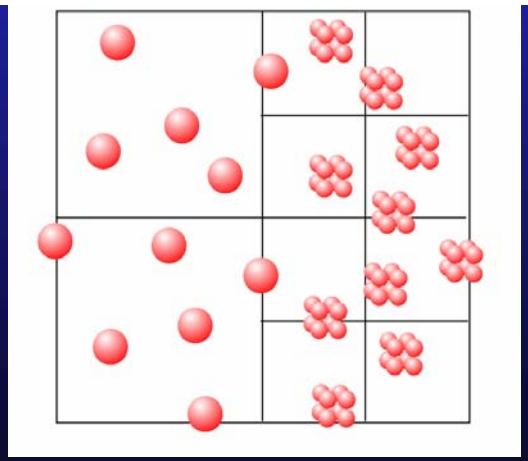


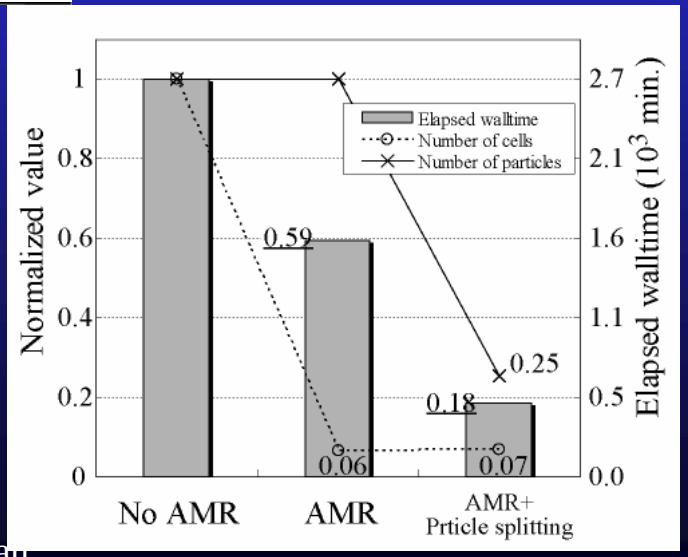
$$\lambda_{De,lobe} \sim 6 \times 10^3 \text{ m}$$

$$\lambda_{De,ps} \sim 3 \times 10^2 \text{ m}$$

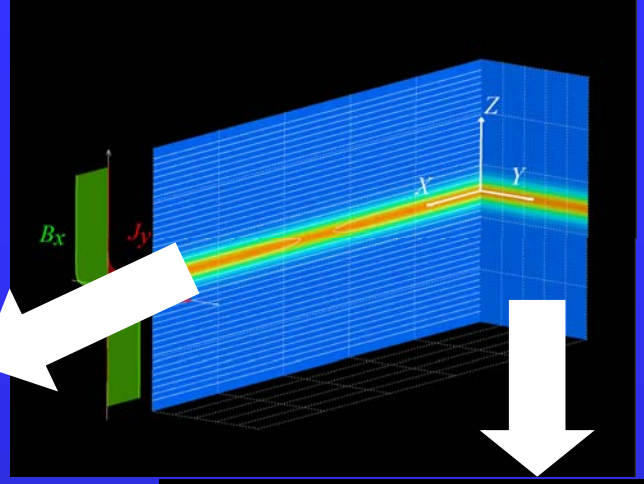
Removing unnecessary cells.

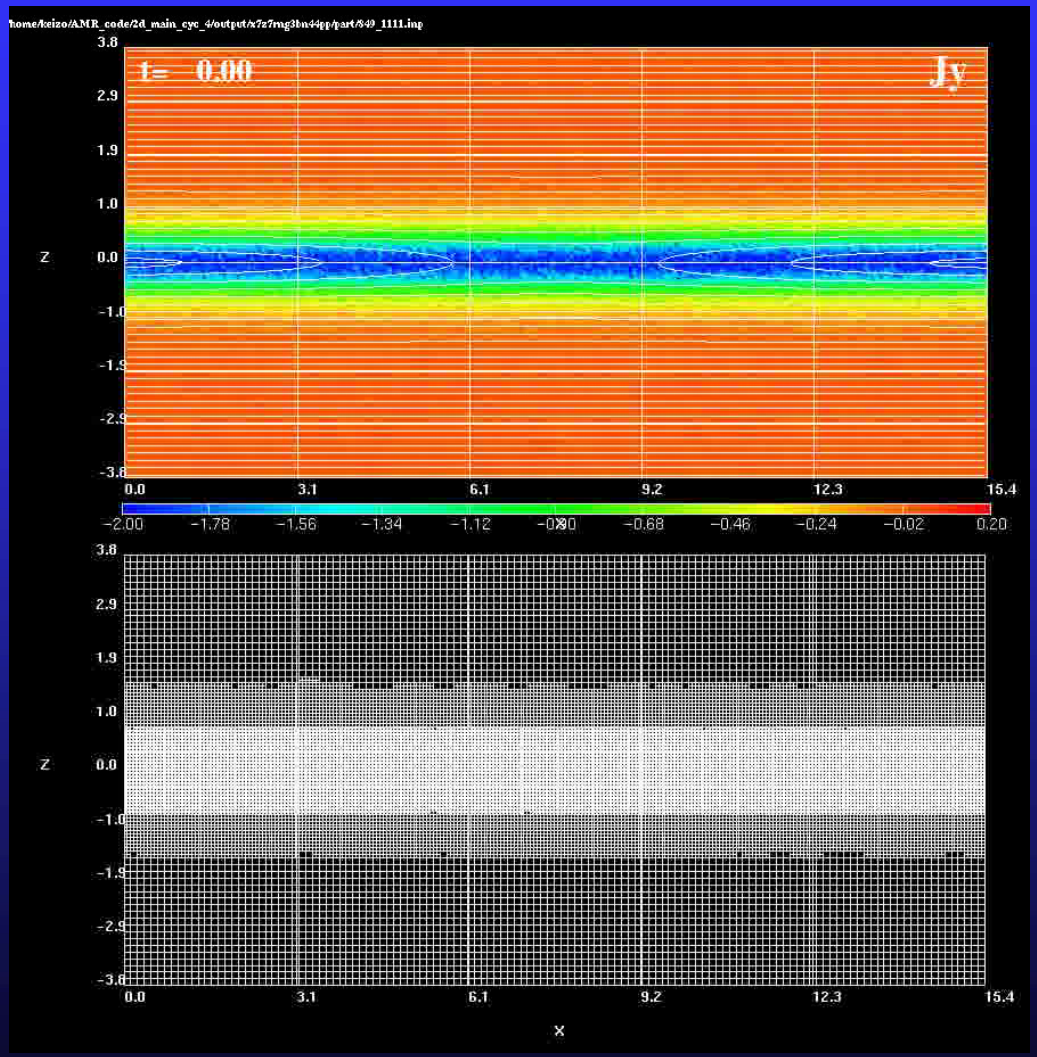
Particle splitting-coalescence

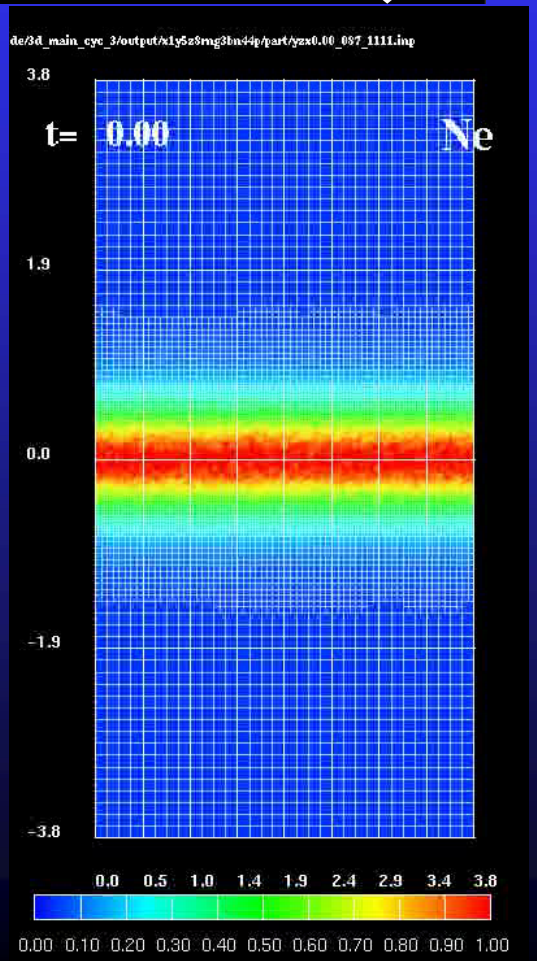




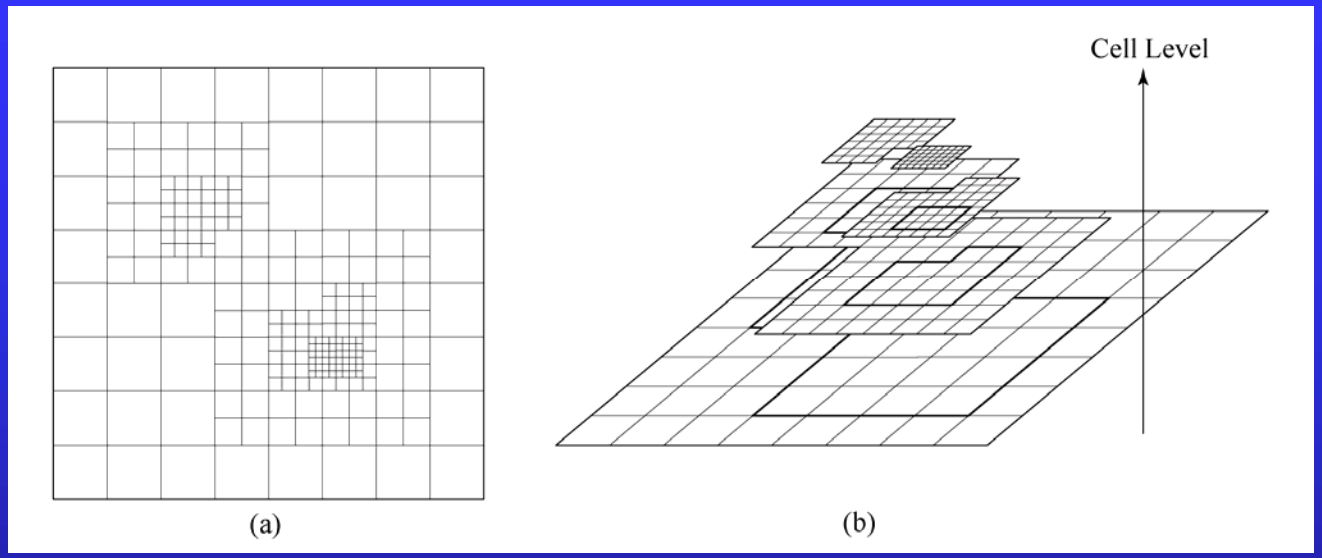
AMR-PIC Simulations





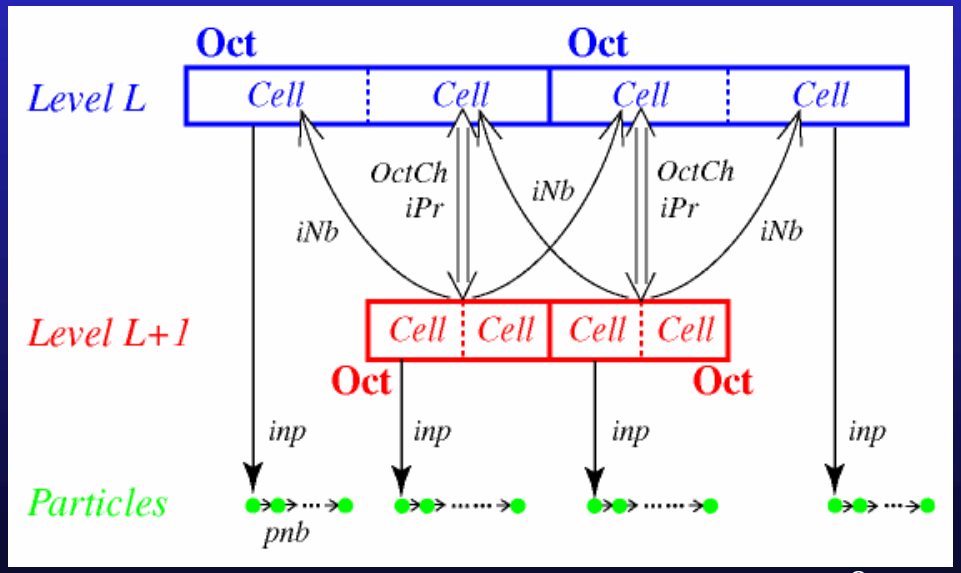


Data Structure



Similar to a fully threaded tree (FTT) structure (Khokhlov, 1998).

[Fujimoto & Machida, JCP, 2006]

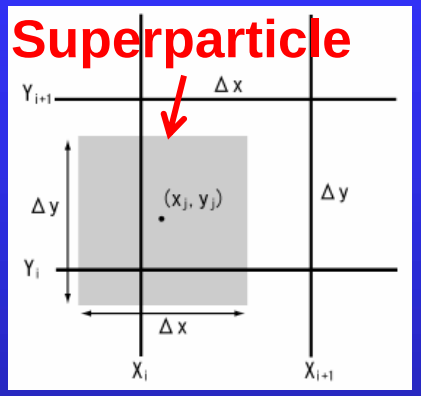


Basic Equations of PIC model

[Birdsall and Langdon, IOP, 1991]

$$\rho_{l,m,n} = \sum_s \sum_j q_{sj} S(\vec{x}_{sj} - \vec{X}_{l,m,n})$$
$$A(\vec{x}_{sj}) = \sum_l \sum_m \sum_n A_{l,m,n} S(\vec{x}_{sj} - \vec{X}_{l,m,n})$$

S: Shape function



- Superparticles (Buneman-Boris method)

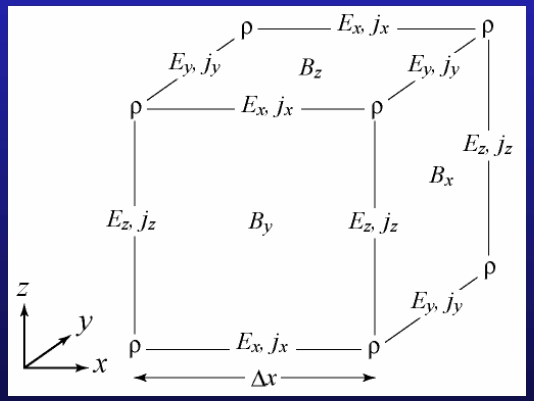
$$\frac{\vec{v}_{sj}^{n+1/2} - \vec{v}_{sj}^{n-1/2}}{\Delta t} = \frac{q_{sj}}{m_{sj}} \left[\vec{E}^n(\vec{x}_{sj}^n) + \frac{\vec{v}_{sj}^{n-1/2} + \vec{v}_{sj}^{n+1/2}}{2} \times \vec{B}^n(\vec{x}_{sj}^n) \right]$$
$$\frac{\vec{x}_{sj}^{n+1} - \vec{x}_{sj}^n}{\Delta t} = \vec{v}_{sj}^{n+1/2}$$

- EM Field (Yee-Buneman scheme)

$$\frac{\vec{B}^{n+1/2} - \vec{B}^{n-1/2}}{\Delta t} = -\nabla \times \vec{E}^n$$
$$\frac{\vec{E}^{n+1} - \vec{E}^n}{\Delta t} = c^2 \nabla \times \vec{B}^{n+1/2} - \frac{1}{\varepsilon_0} \boxed{\vec{j}^{n+1/2}}$$

Charge Conservation Method

[Villasenor & Buneman, 1992]



von Neumann Stability Analysis

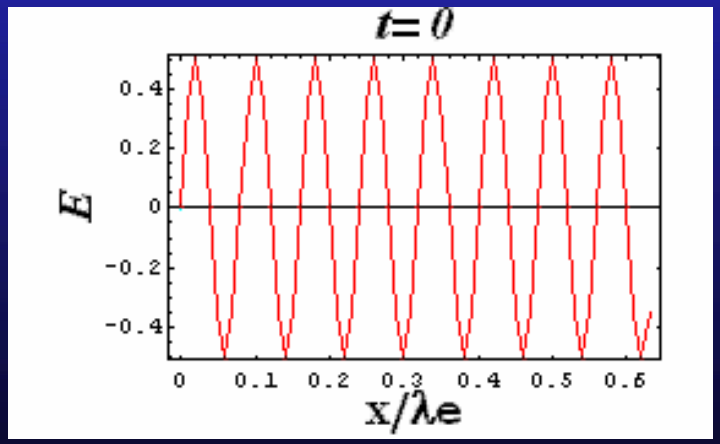
$$E_j^n = g^n \exp[ik(j\Delta x)]$$
$$g = |g| \exp[-i\omega\Delta t]$$

$$\frac{\vec{B}^{n+1/2} - \vec{B}^{n-1/2}}{\Delta t} = -\nabla \times \vec{E}^n$$
$$\frac{\vec{E}^{n+1} - \vec{E}^n}{\Delta t} = c^2 \nabla \times \vec{B}^{n+1/2}$$

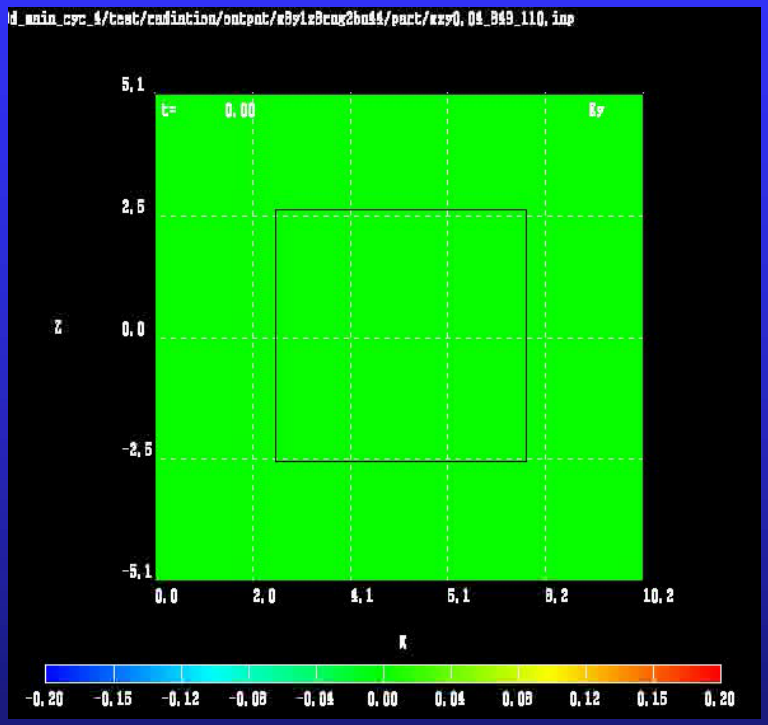
$$g = 1 - \frac{(\kappa c \Delta t)^2}{2} \pm i(\kappa c \Delta t)\sqrt{1 - \frac{(\kappa c \Delta t)^2}{4}} \quad (\Delta x/\Delta t > c)$$

$$|g| = 1$$

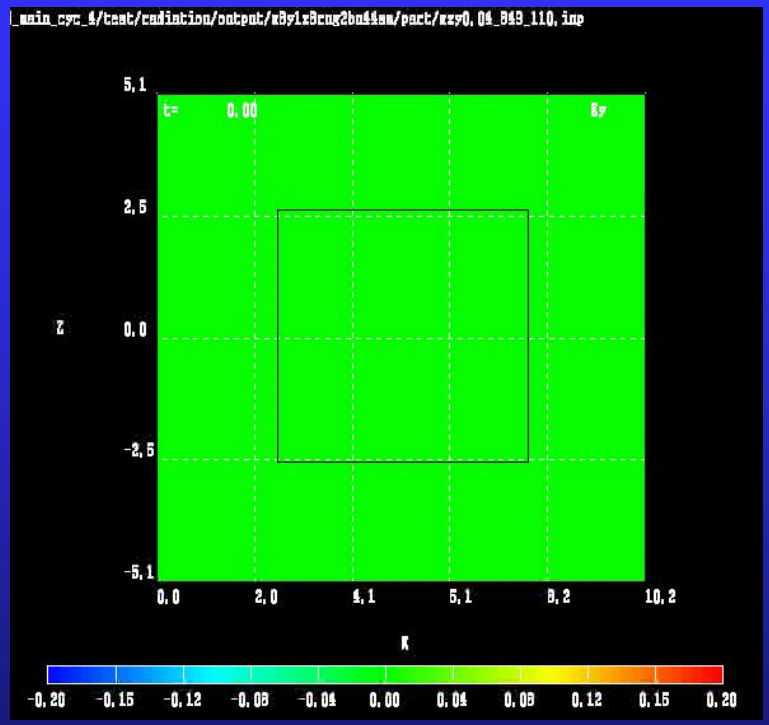
$$\kappa = \frac{\sin\left(\frac{k\Delta x}{2}\right)}{\left(\frac{k\Delta x}{2}\right)} k$$



Radiation of EM Waves



$E_{SM}=f_{SM}(E)$
$B_{SM}=f_{SM}(B)$

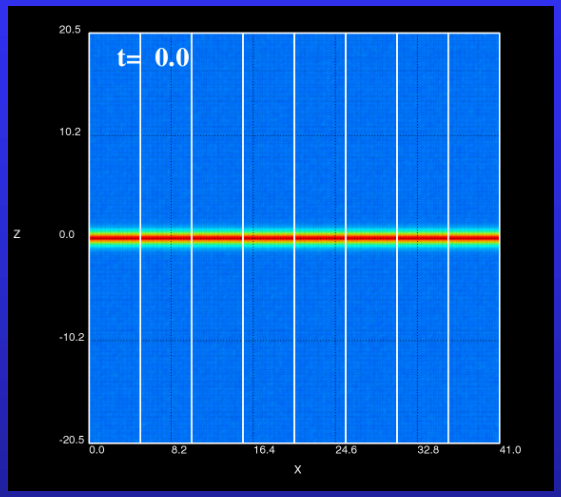


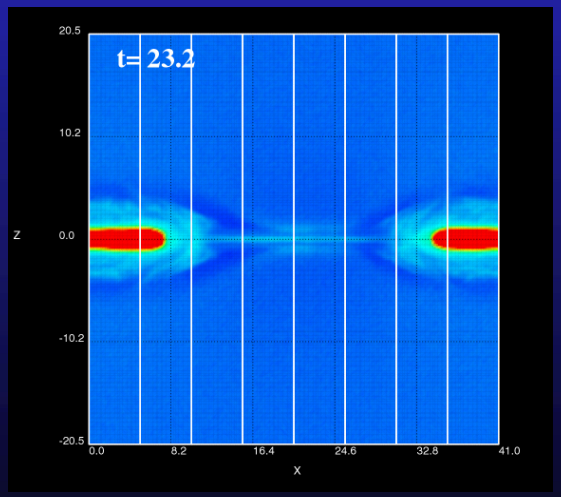
$$A_{SM,j} = f_{SM}(A_j) = \frac{\alpha A_{j-1} + A_j + \alpha A_{j+1}}{1+2\alpha} \quad (\alpha = 0.002)$$

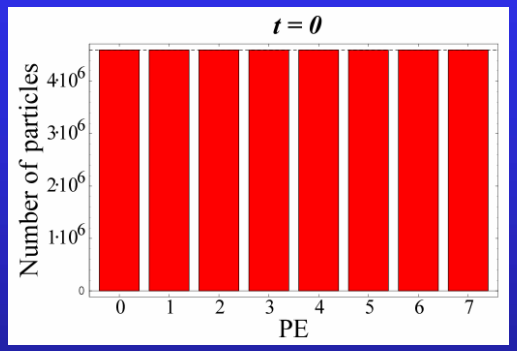
Parallelization on Distributed Memory System

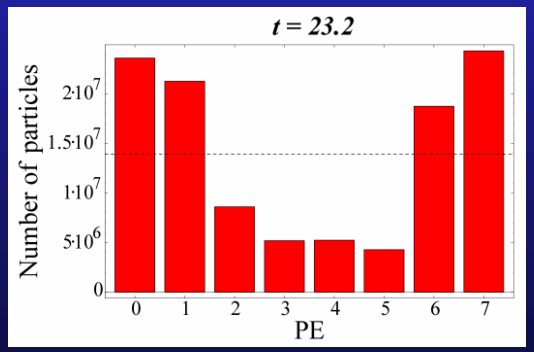
Case with 8 nodes

More than 99% of the total computation is devoted to the particle calculations.

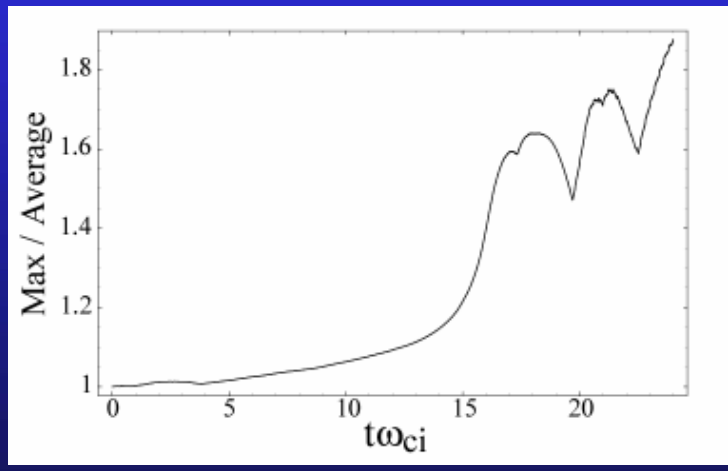








Imbalance of the number of particles per cell.

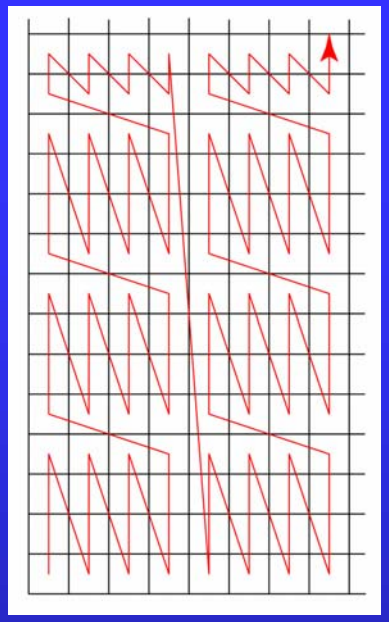


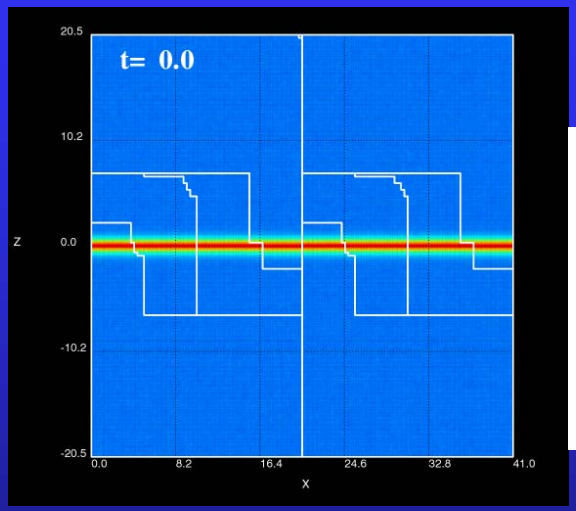
Adaptive Block Technique

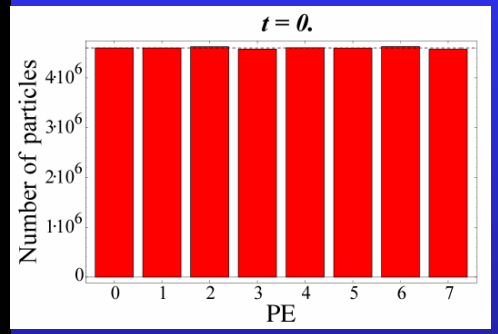
[Fujimoto, JCP, 2011]

Case with 8 nodes

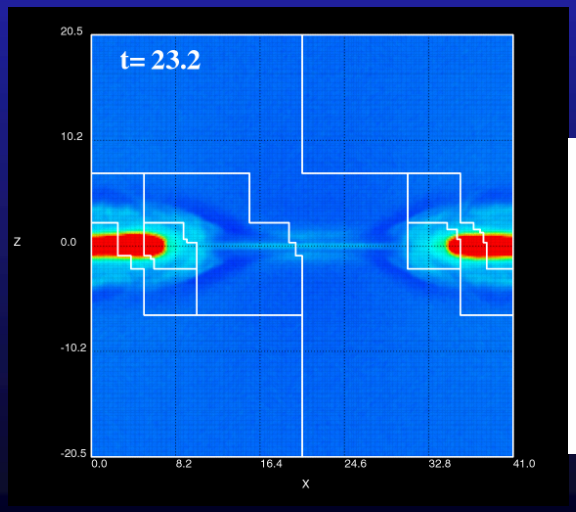
Morton-like ordering for the base octs.

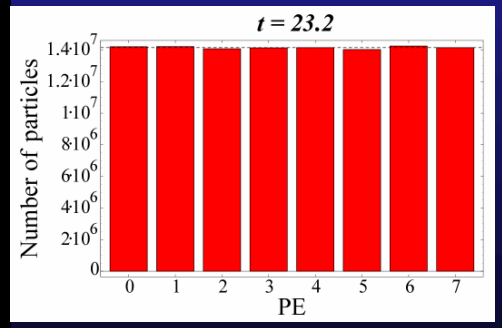


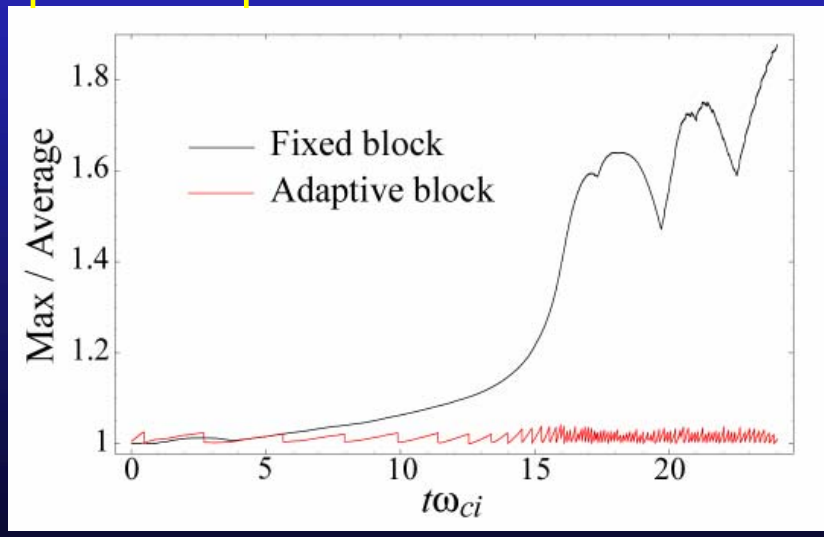




Imbalance of the number of particles per cell.



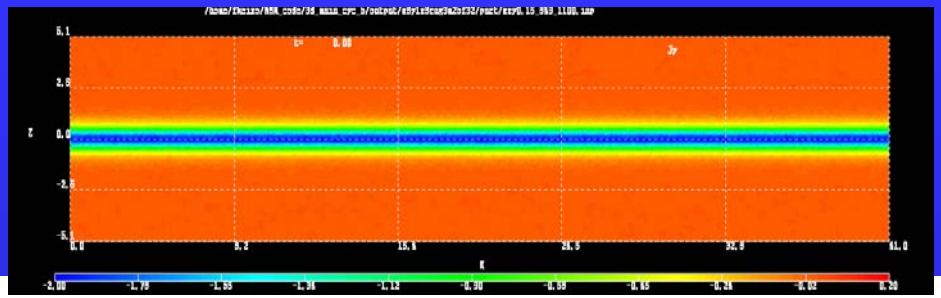


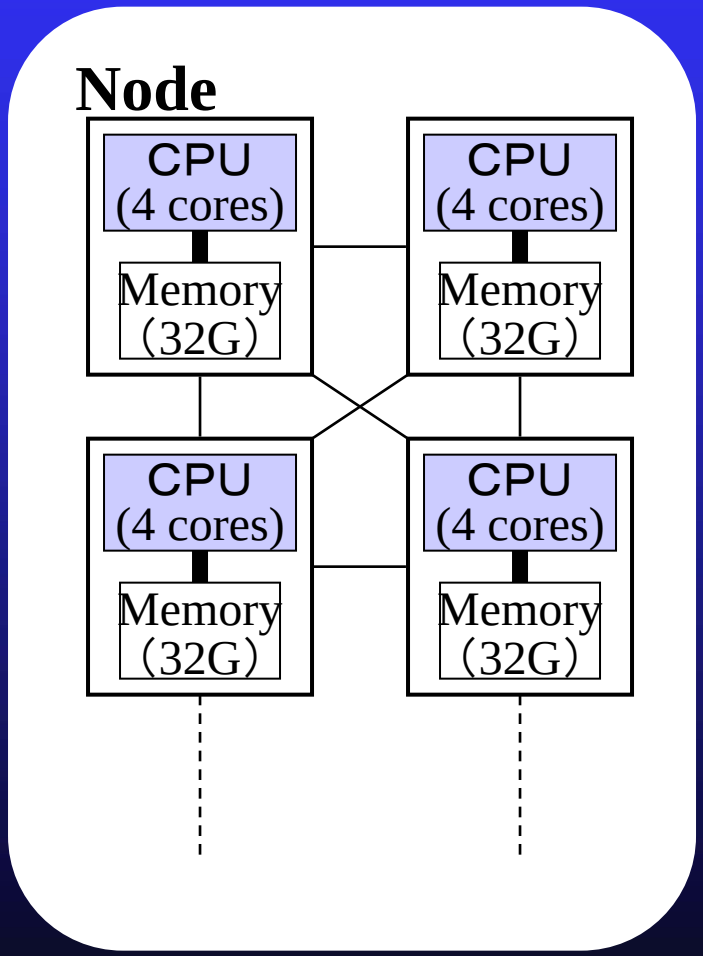


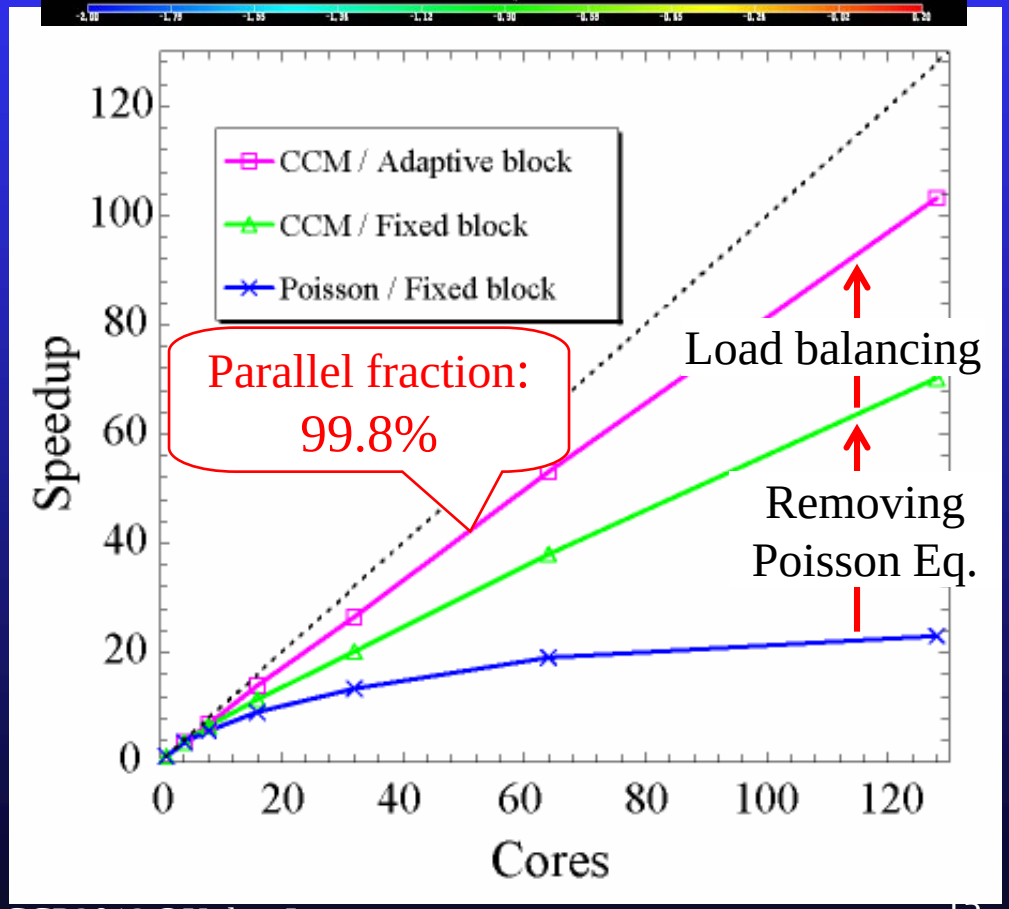
Performance of the AMR-PIC Model

Fujitsu FX1

@Nagaya Univ.



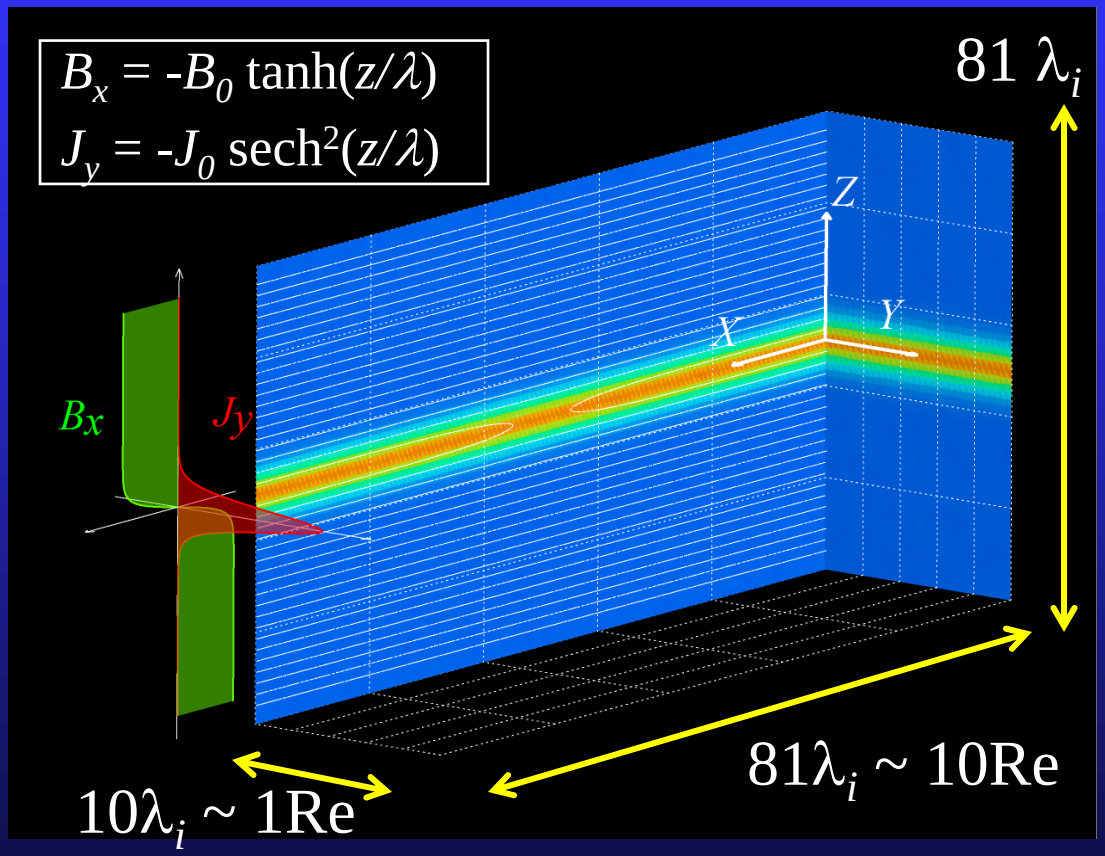




Large-Scale 3D Simulation

[Fujimoto & Sydora, PRL, submitted]

Fujitsu FX1, 1024 cores



$m_i/m_e = 100$

Max resolution:

$4096 \times 512 \times 4096 \sim 10^{10}$

Max number of particles

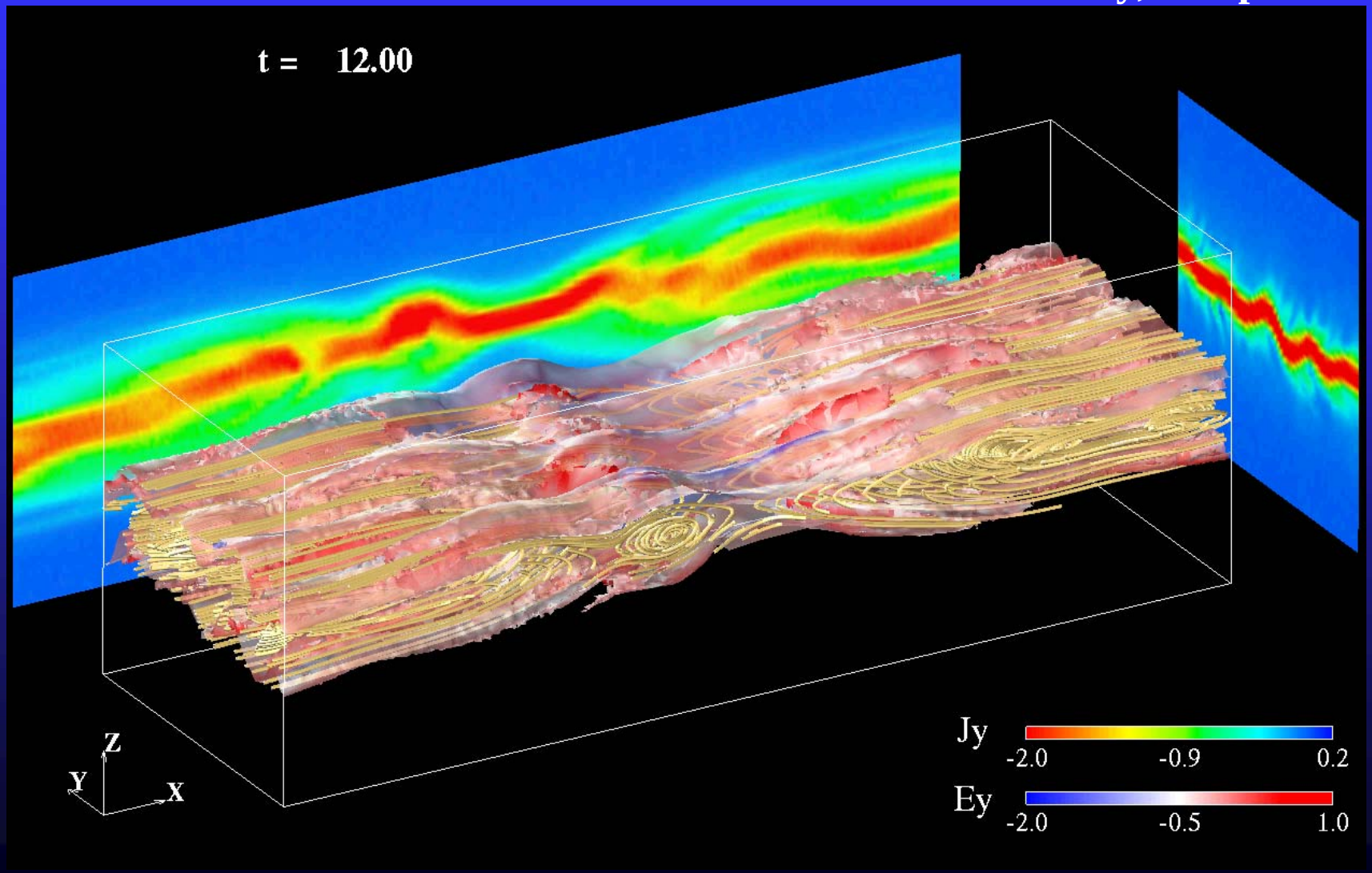
Ion + Electron $\sim 10^{11}$

Max memory used ~ 6TB

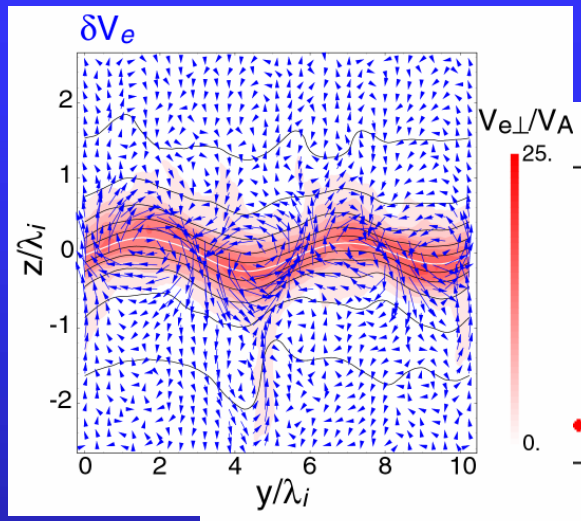
Time Evolution of the Current Sheet

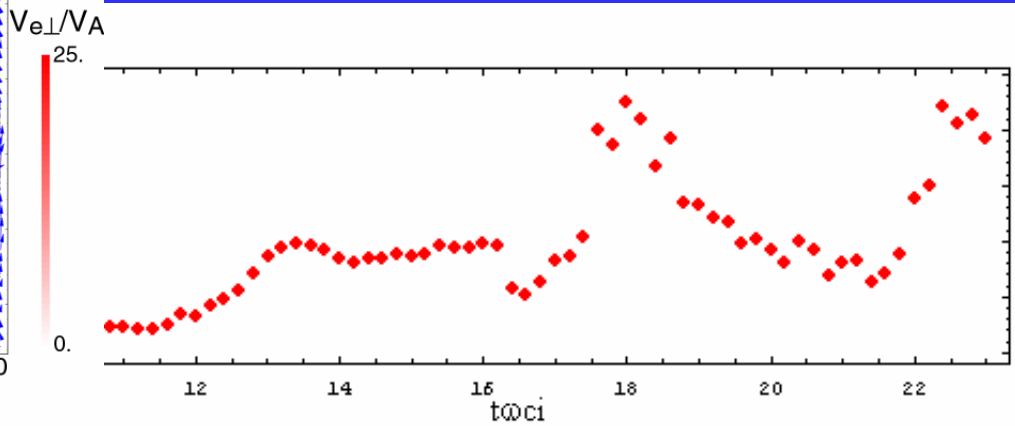
Surface: |J|, Line: Field line

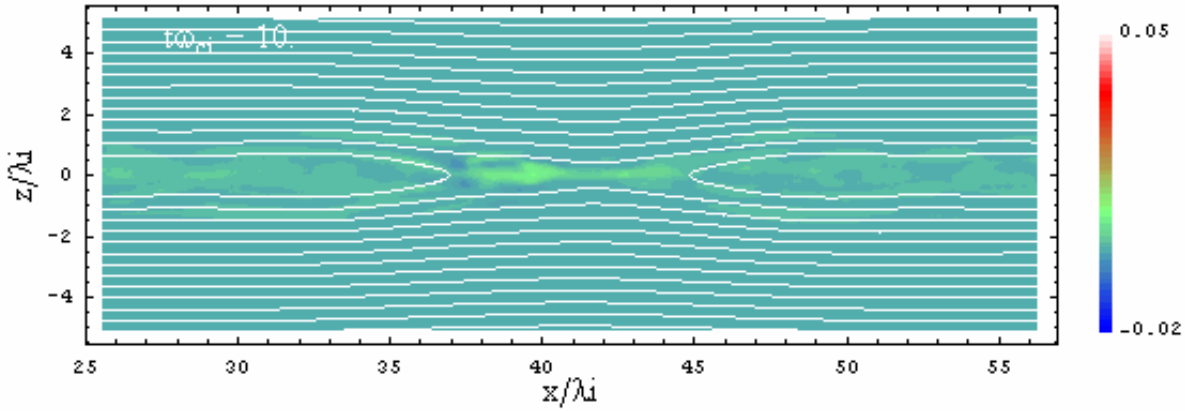
Color on the surface: Ey, Cut plane : Jy



EM Turbulence at the X-line







Summary

Adaptive mesh refinement (AMR) has been implemented in the electromagnetic particle-in-cell (PIC) model to achieve large-scale simulations of magnetic reconnection.

Main differences from the usual PIC models are

- Tree-type data structure,
- Smoothing of the EM fields to avoid the wave reflection,
- Particle splitting-coalescence to control the number of particles per cell (Lapenta, JCP, 2002),
- Adaptive block technique to keep load balancing.