AMR-PIC Model and Application to Magnetic Reconnection

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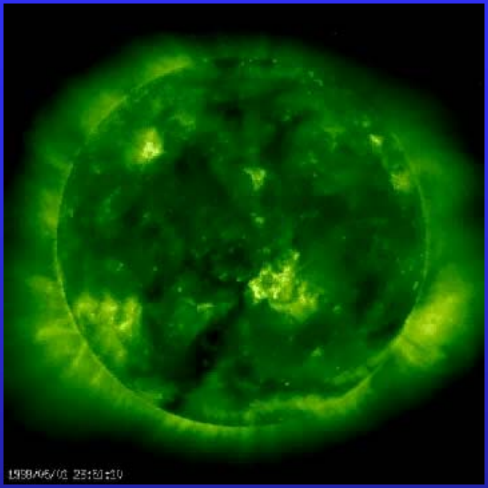
Magnetic Reconnection in Space

[Solar Flares]

[Magnetospheric Substorms]



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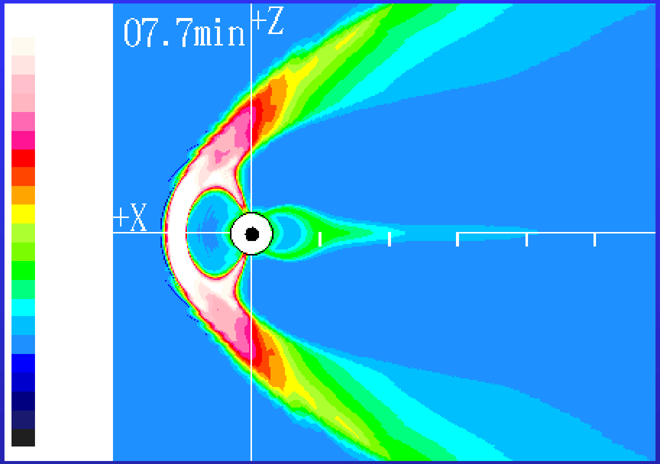

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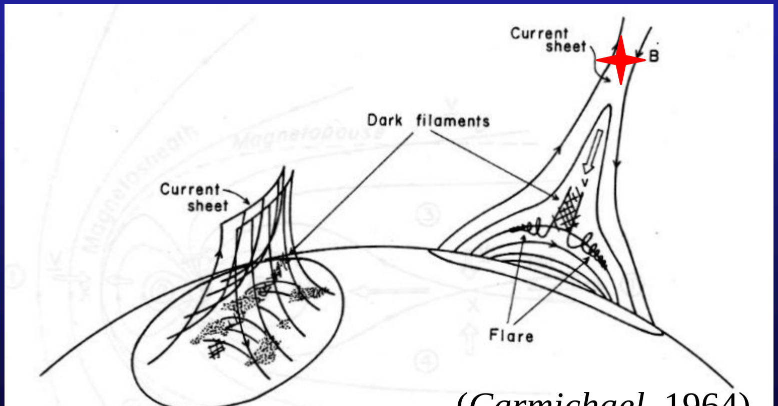
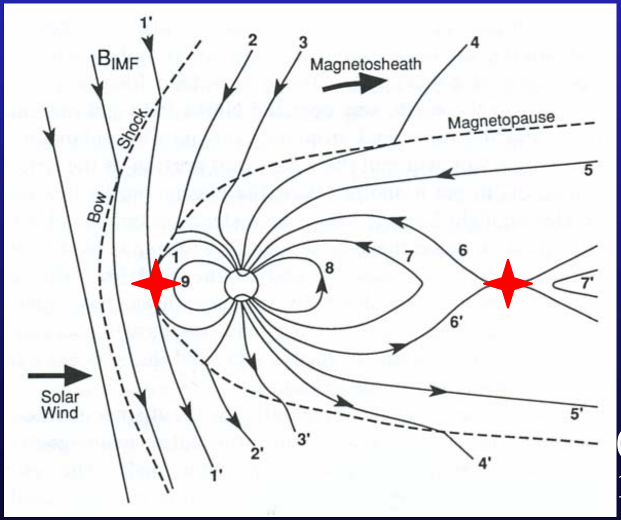
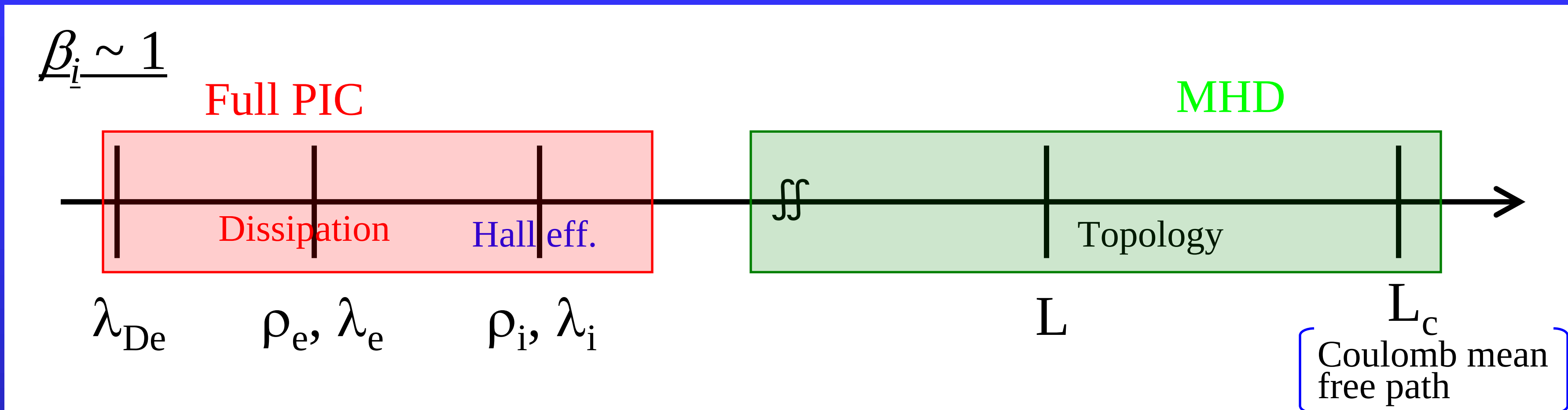


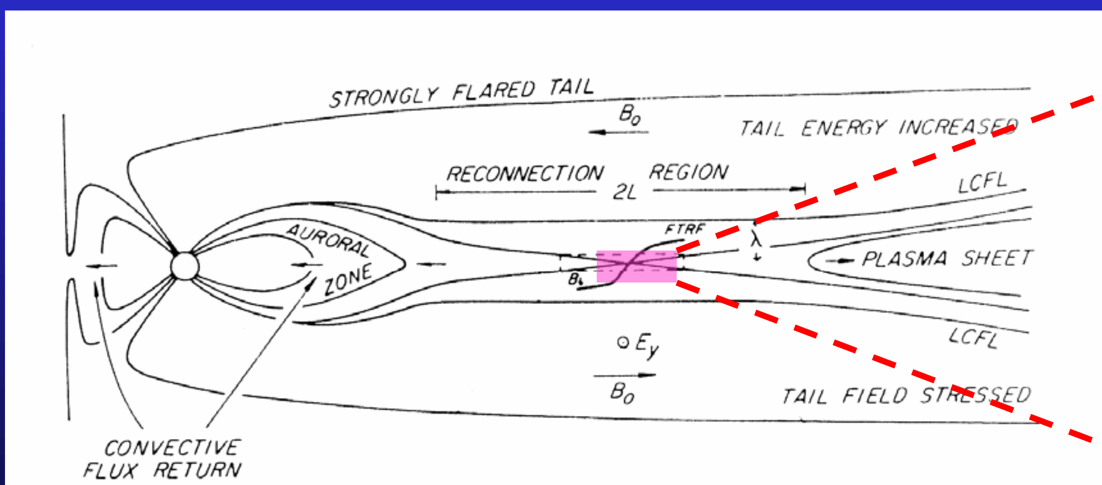
(*Carmichael*, 1964)



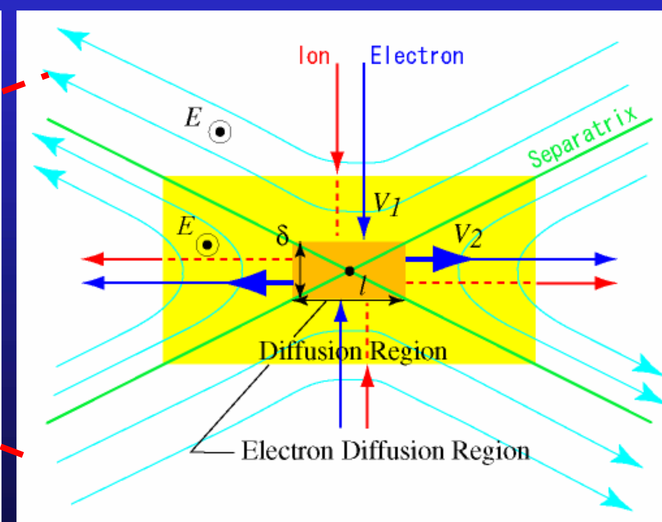
(*Kivelson and Russel*, 1995)

Multi-Scale Nature of Reconnection





Coroniti [1985]

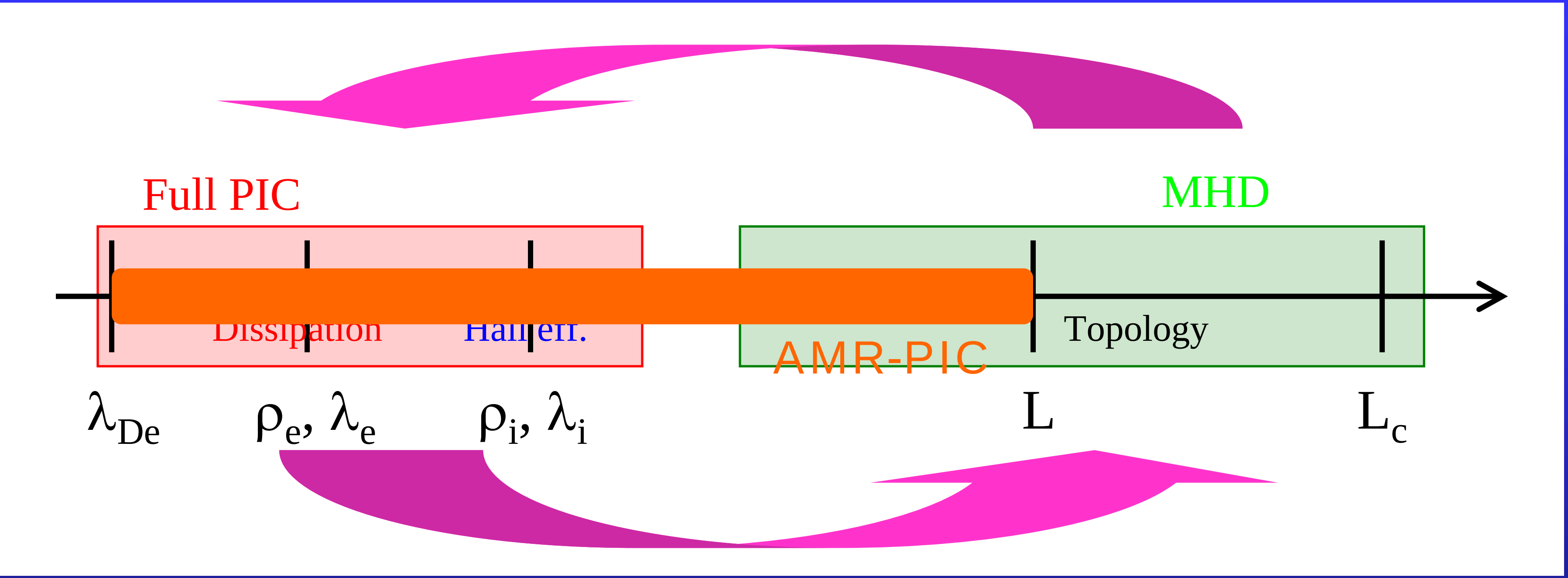


$L \sim 10^5$ km

$\lambda_e \sim 10$ km

$\lambda_i \sim 10^3$ km

Multi-Scale Nature of Reconnection



MHD simulations

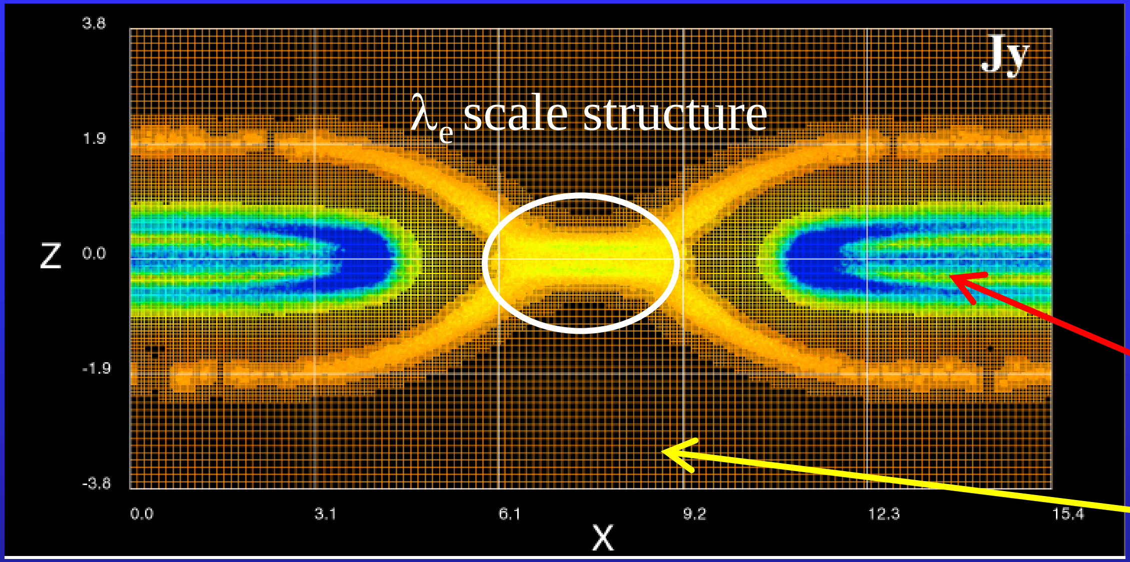
$$\frac{\partial B}{\partial t} = \eta \nabla^2 B$$

- The reconnection rate depends on the resistivity model. (Biskamp,1986; Ugai, 1995)
- Global responses in substorms and flares are sensitive to the parameterization of the resistivity. (Raeder et al.,2001; Kuznetsova et al., 2007)

AMR-PIC Model

[Fujimoto & Machida, JCP, 2006; Fujimoto & Sydora, CPC, 2008]

(Adaptive Mesh Refinement – Particle-in-Cell)



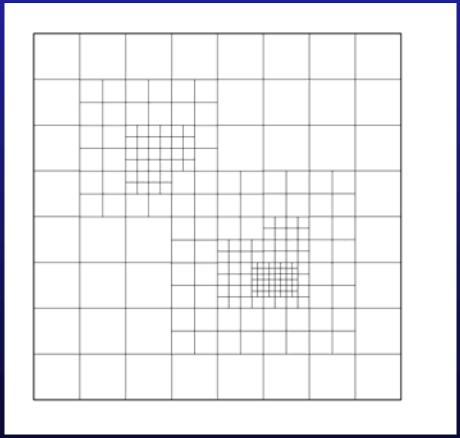
Restriction in explicit method

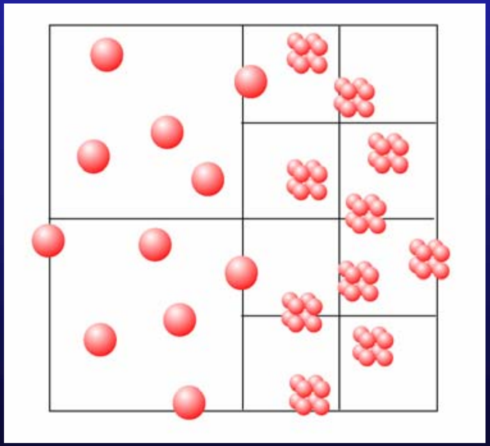
$\Delta x < \lambda_{De}$, $\omega_{pe}\Delta t < 1$

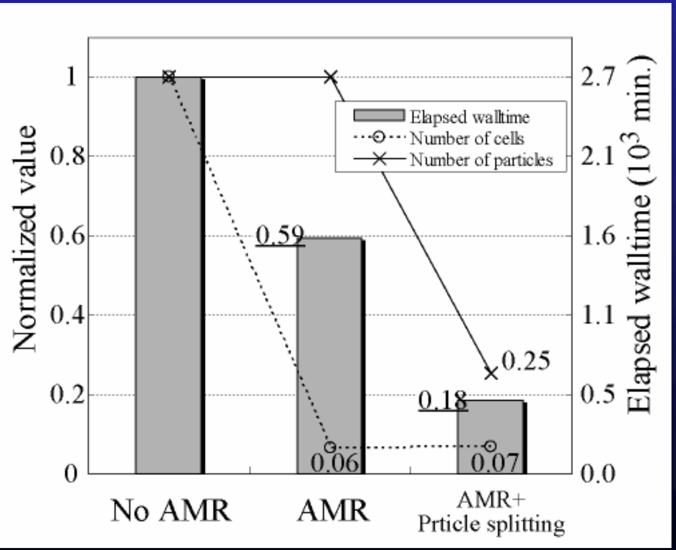
$\Delta x/\Delta t > c$

$\lambda_{De,ps} \sim 3 \times 10^2$ m

$\lambda_{De,lobe} \sim 6 \times 10^3$ m

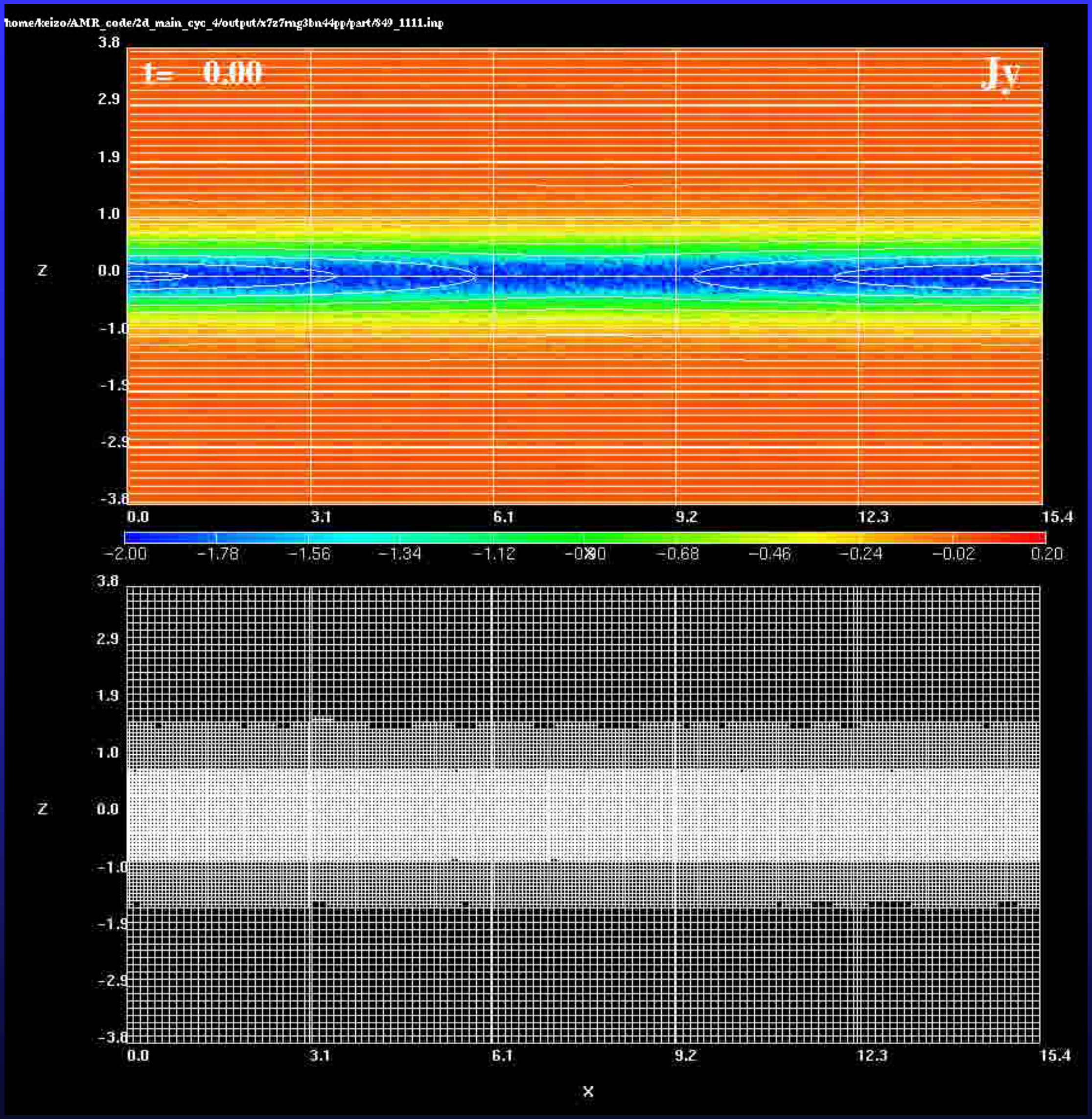


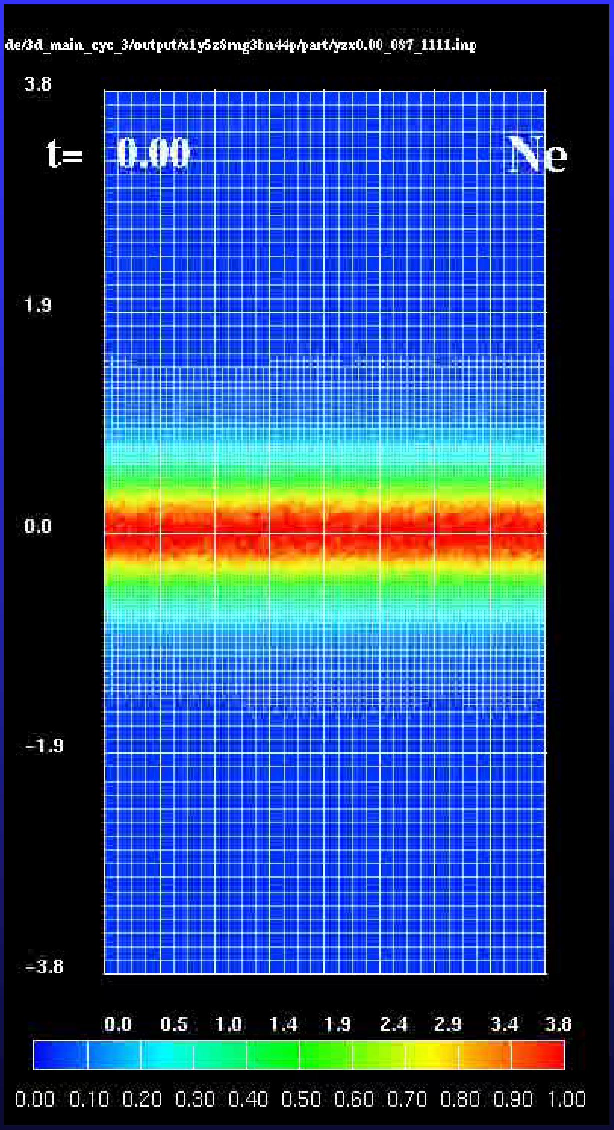




AMR-PIC Model

[Fujimoto & Machida, JCP, 2006; Fujimoto & Sydora, CPC, 2008]





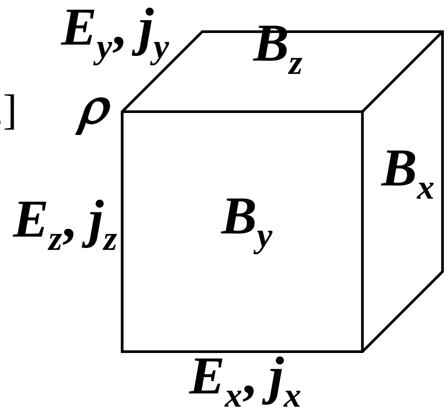
Finite-Difference Equations

Yee-Buneman scheme (Staggering grid scheme)

$$\frac{\vec{B}^{n+1/2}-\vec{B}^{n-1/2}}{\Delta t} = -\nabla\times\vec{E}^n$$

$$\frac{\vec{E}^{n+1}-\vec{E}^{n}}{\Delta t} = c^2\nabla\times\vec{B}^{n+1/2} - \frac{1}{\varepsilon_0}\boxed{\vec{j}^{n+1/2}}$$

Charge Conservation Method (CCM)
[Villasenor & Buneman, 1992]



- Local operations → Facilitates parallel computation

Buneman-Boris method

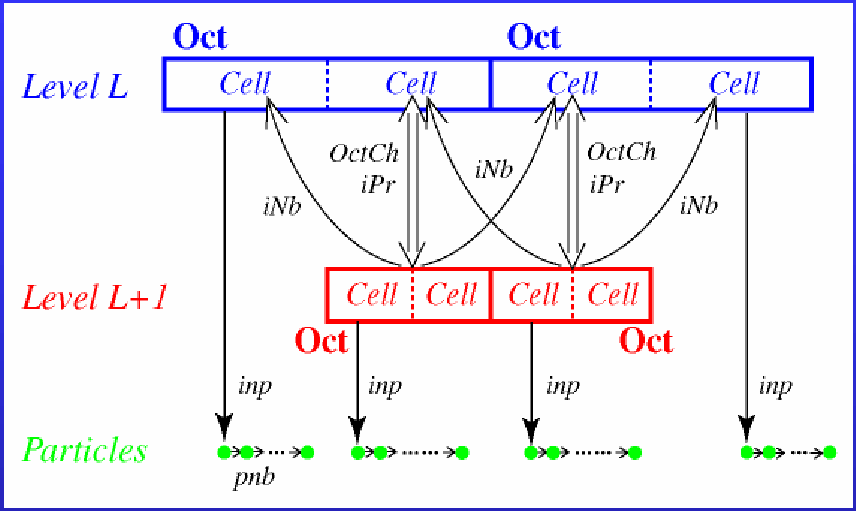
$$\frac{\vec{v}_{sj}^{n+1/2}-\vec{v}_{sj}^{n-1/2}}{\Delta t} = \frac{q_{sj}}{m_{sj}}\left[\vec{E}^n(\vec{x}_{sj}) + \frac{\vec{v}_{sj}^{n-1/2}+\vec{v}_{sj}^{n+1/2}}{2}\times\vec{B}^n(\vec{x}_{sj})\right]$$

$$\frac{\vec{x}^{n+1}-\vec{x}^{n}}{\Delta t} = \vec{x}_{sj}^{n+1/2}$$

Inter-Level Communications

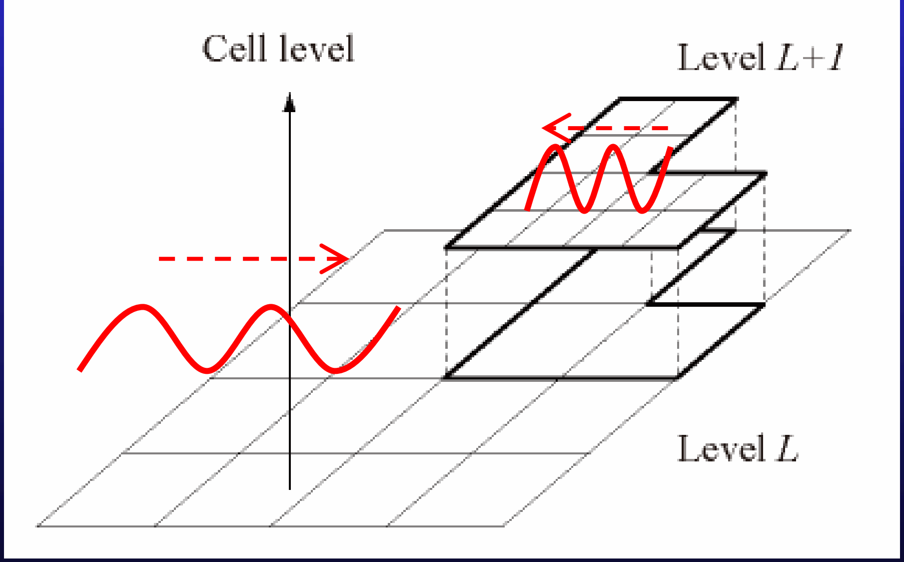
Fine-to-coarse operations

- Deliver the data from fine cells to coarser cells,
- Give appropriate smoothing which removes the aliasing.

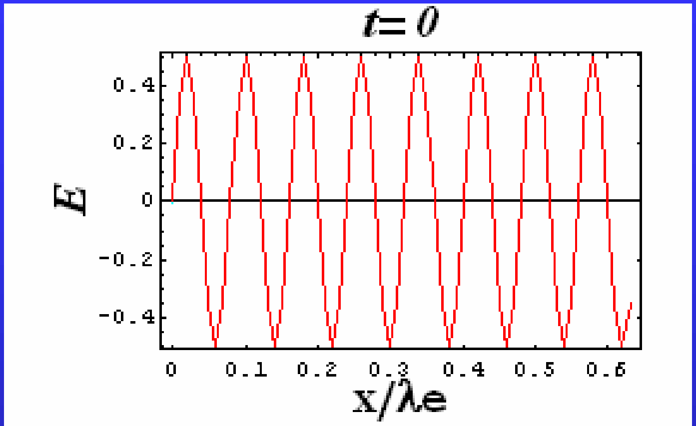


Coarse-to-fine operations

- Deliver the data from coarse cells to finer cells,
- Give the boundary conditions for the refinement regions.



Electromagnetic Wave in Vacuum



Staggering grid scheme

No numerical damping
for any wave numbers

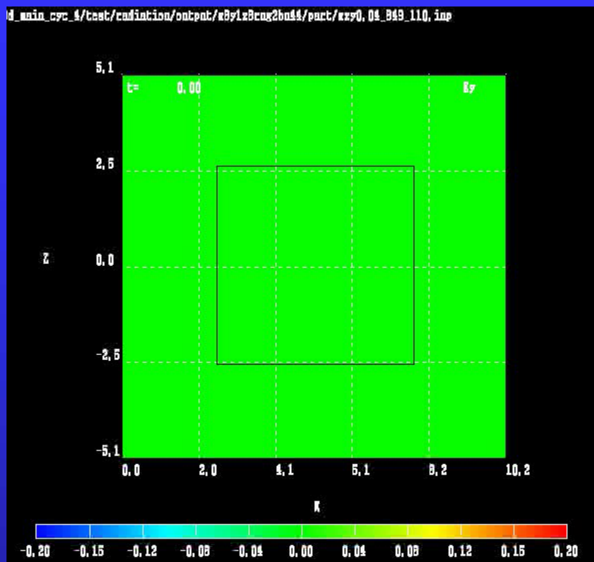
Von Neumann stability analysis

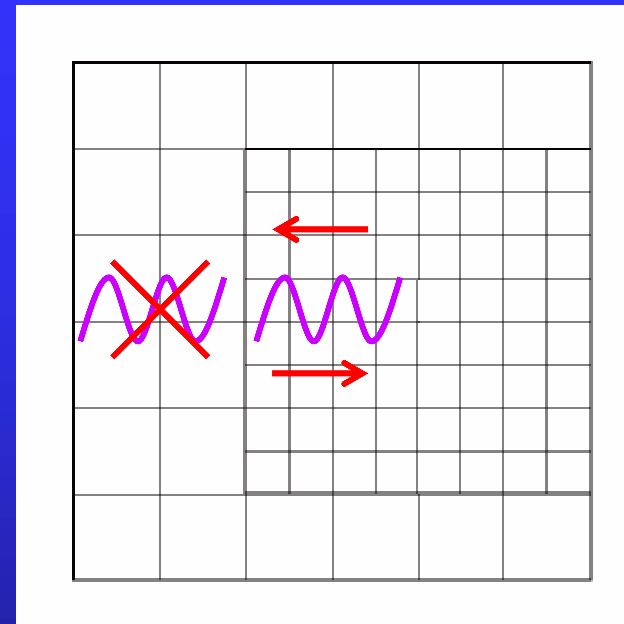
$$(E_l^n, B_l^n) \propto g^n e^{ik(l\Delta x)}$$ g: Amplification factor

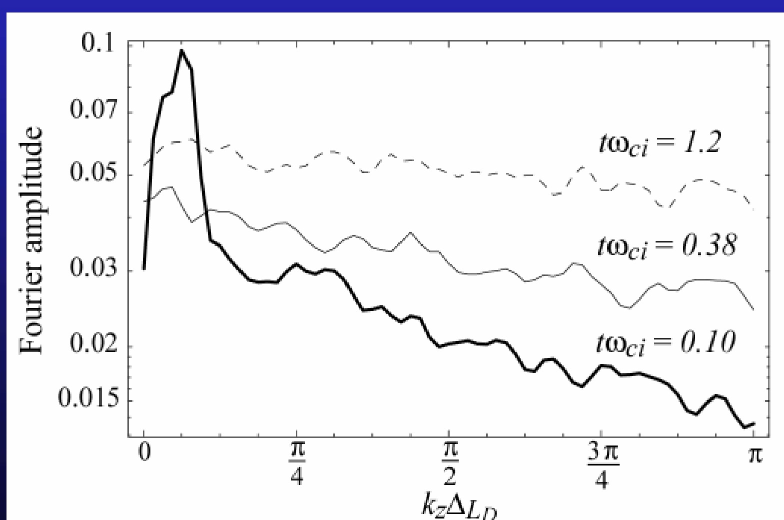
$$g = 1 - \frac{(\kappa c \Delta t)^2}{2} \pm i(\kappa c \Delta t)\sqrt{1 - \left(\frac{\kappa c \Delta t}{2}\right)^2}$$

$|g| = 1$ ($\Delta x/\Delta t > c$; Courant condition)

Electromagnetic Radiation Test







It is very difficult to apply the AMR to the staggering grid scheme!

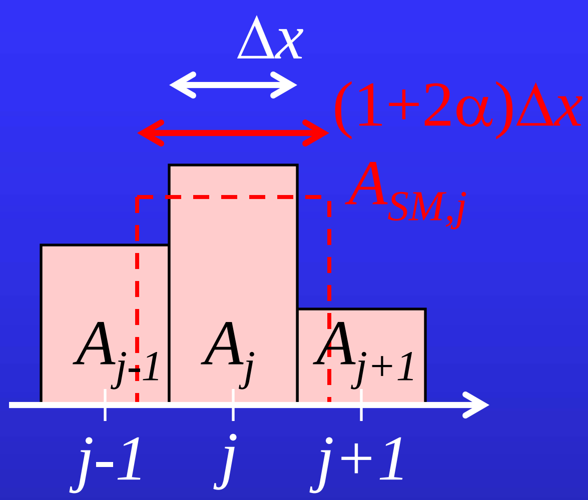
Smoothing Function: f_{SM}

For $A = E$ and B,

$$A_{SM,j} = f_{SM}(A_j) = \frac{\alpha A_{j-1} + A_j + \alpha A_{j+1}}{1+2\alpha}$$

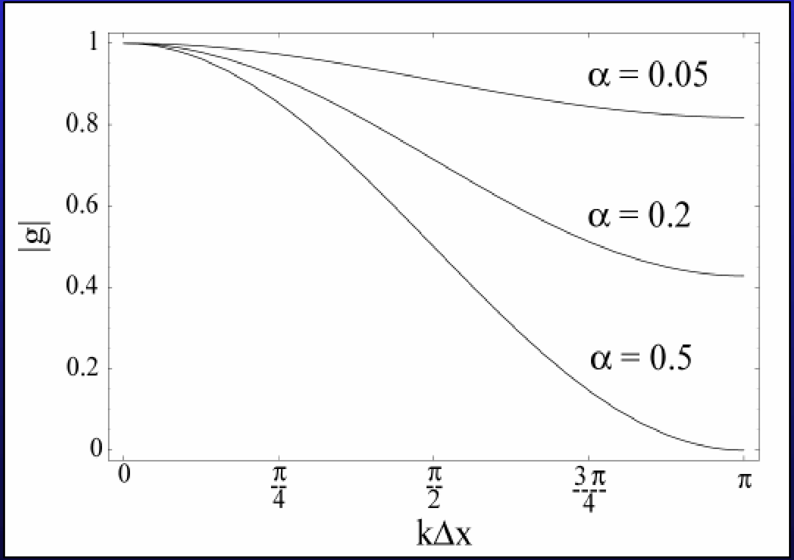
$$(0 \le \alpha \le 0.5)$$

$$g = \frac{1 + 2\alpha \cos k\Delta x}{1+2\alpha} \le 1$$



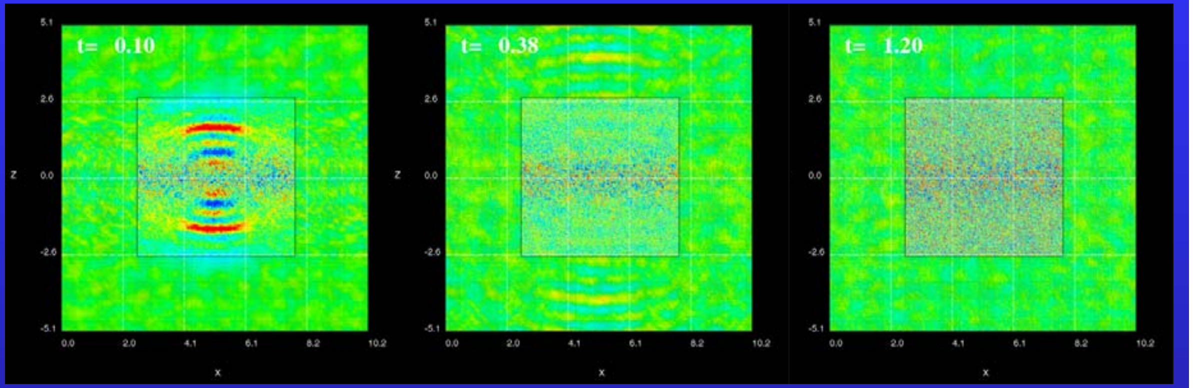
- Selectively damps the short wavelength modes.

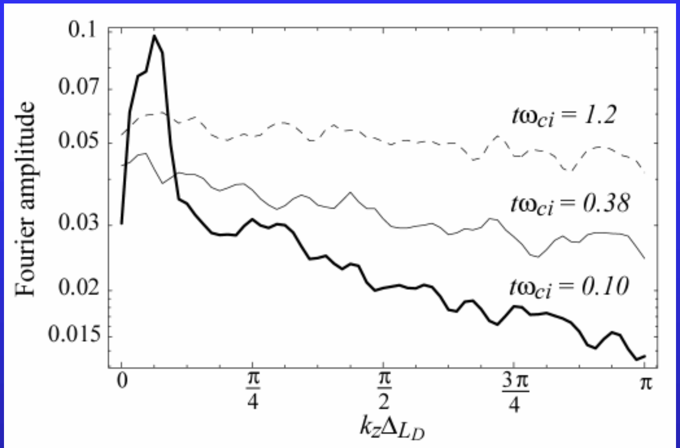
- Very simple (Very fast)
- Easy parallelization
- No wave dispersion changes
- Flexible about damping rate



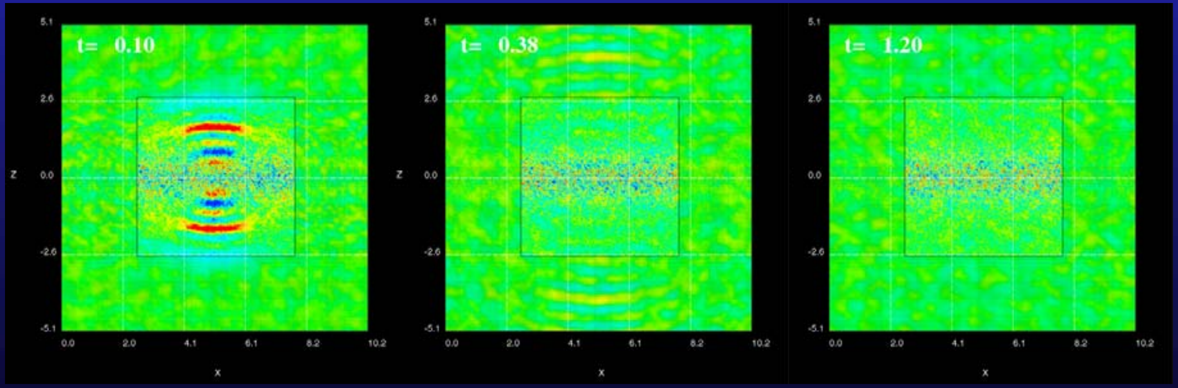
Electromagnetic Radiation Test 2

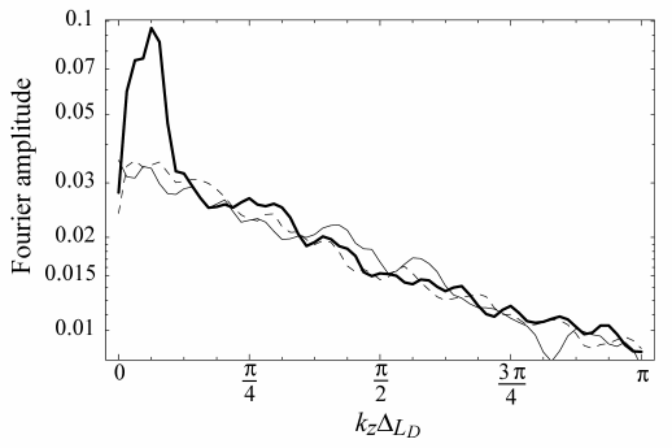
- The case without the smoothing





- The case with the smoothing ($\alpha = 0.002$)





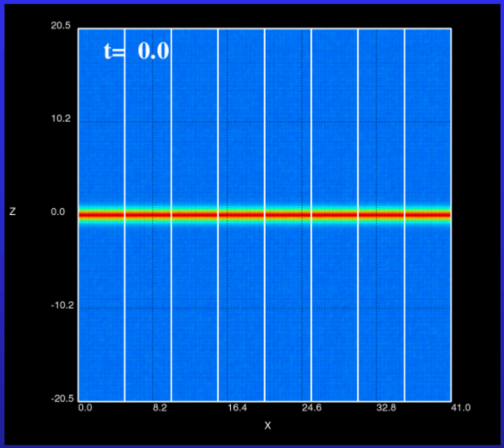
Load Balancing

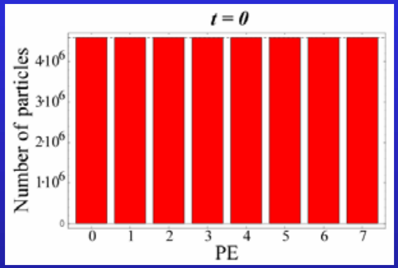
* Block = Decomposition domain

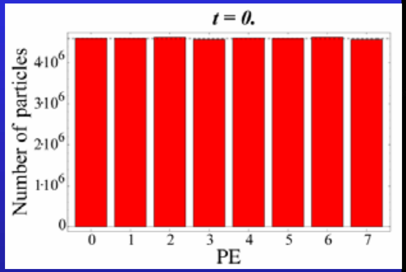
Example using 8 nodes

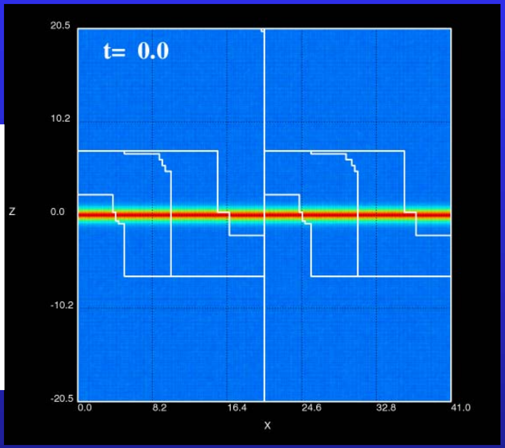
Fixed block case

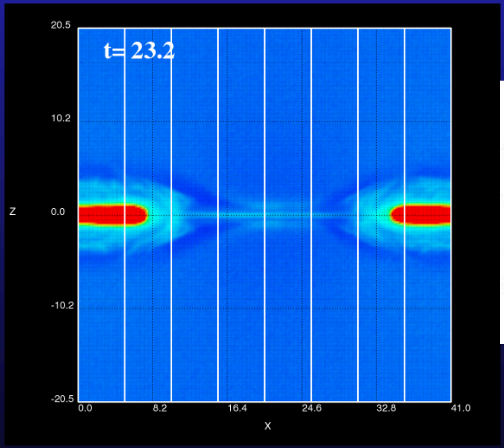
Adaptive block case

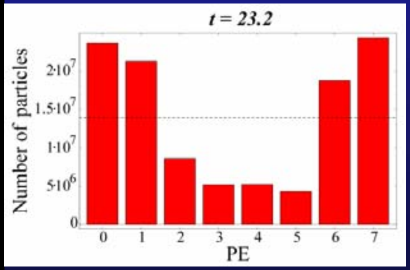


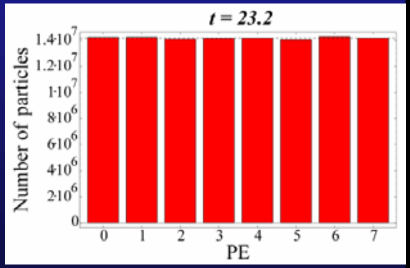


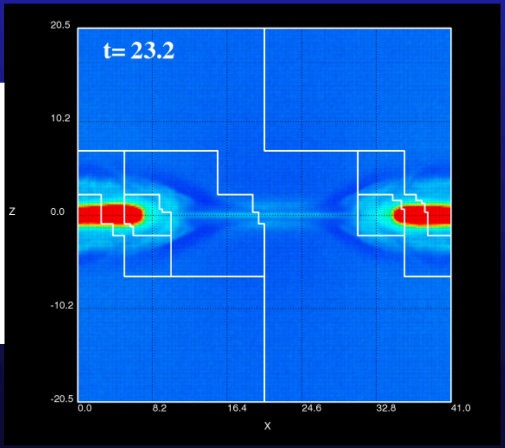








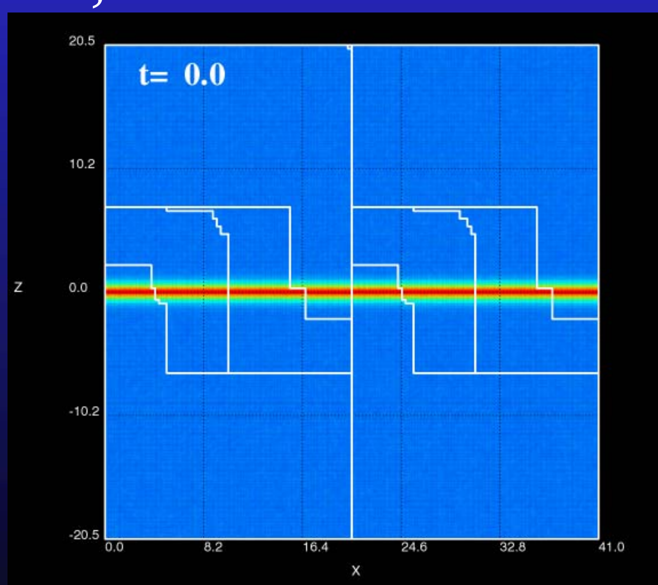


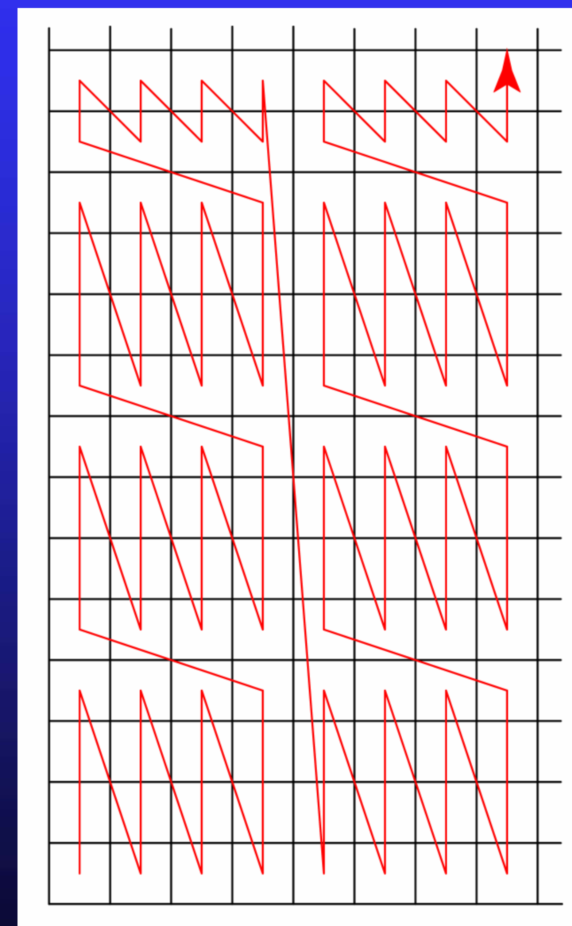


Adaptive Block Technique

Base-level cells in the entire domain are sorted in an appropriate order:

- That is similar to Morton order,
- So that the block surface is as small as possible,
- Especially in the central current sheet, the surface must be small.

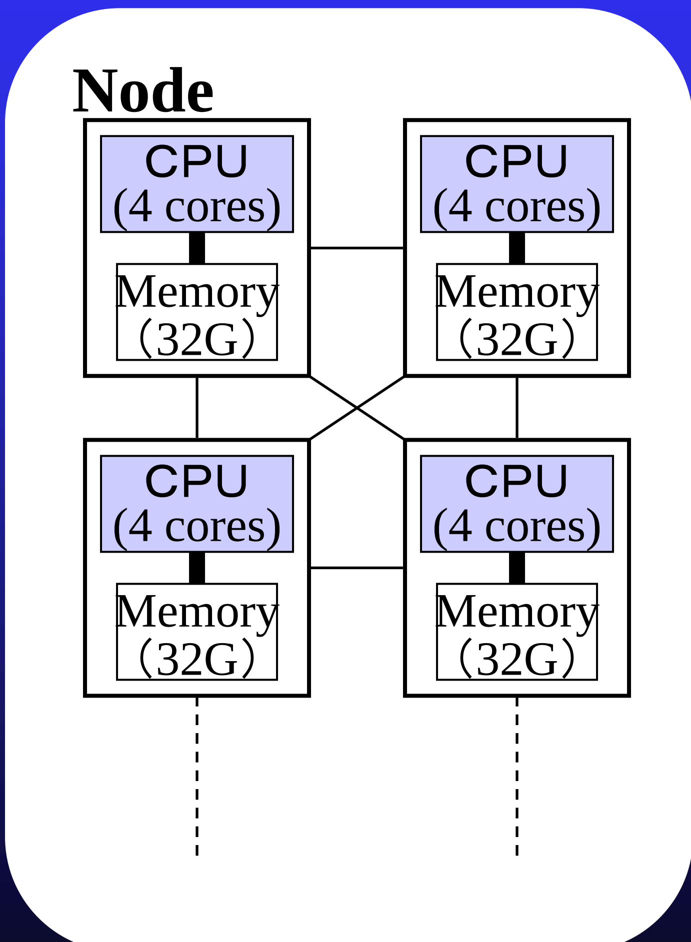


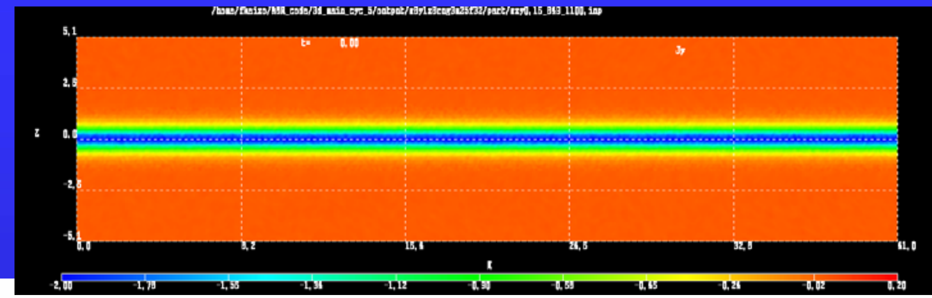


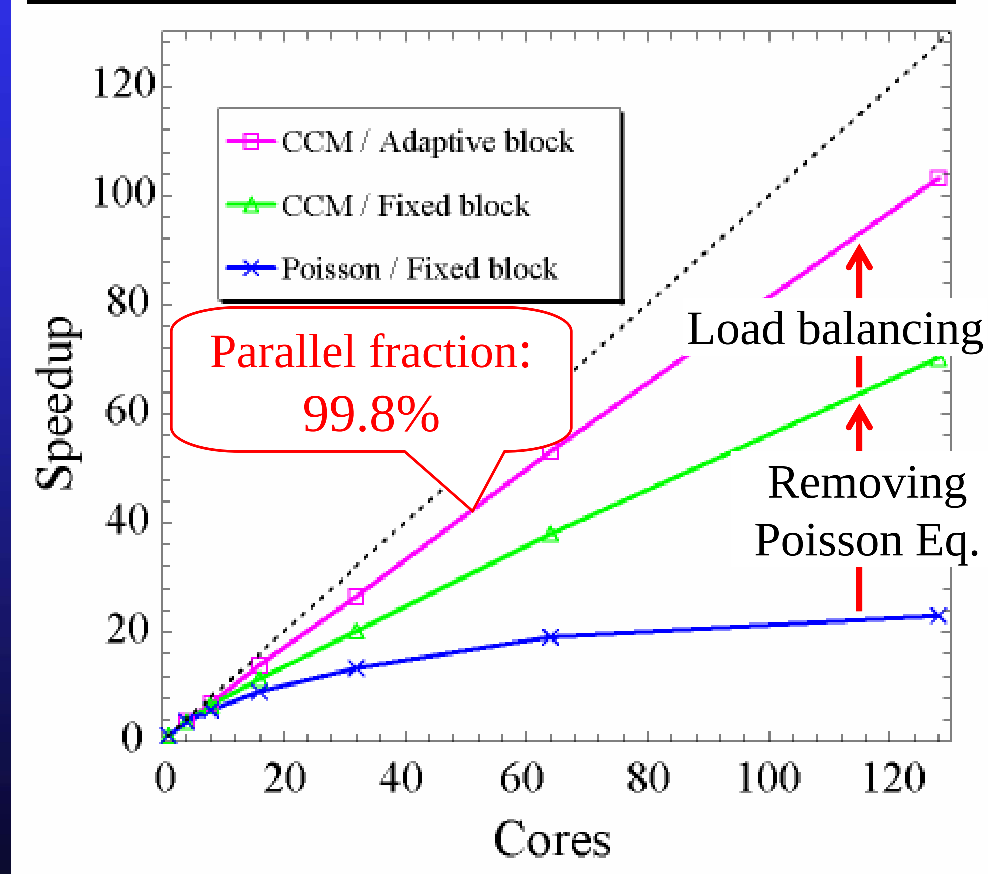
Performance of the AMR-PIC Model

Fujitsu FX1

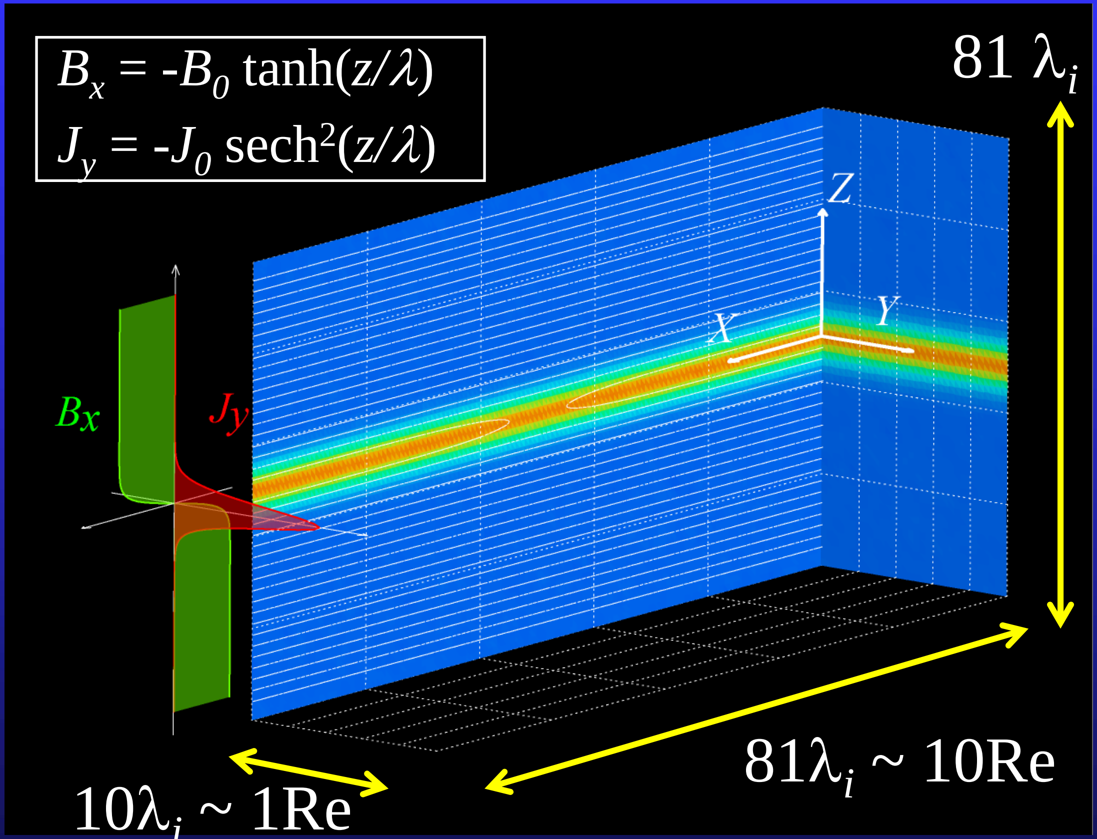
@Nagaya Univ.







Large-Scale 3D Simulation



$m_i/m_e = 100$

Max. resolution:
$4096 \times 512 \times 4096 \sim 10^{10}$

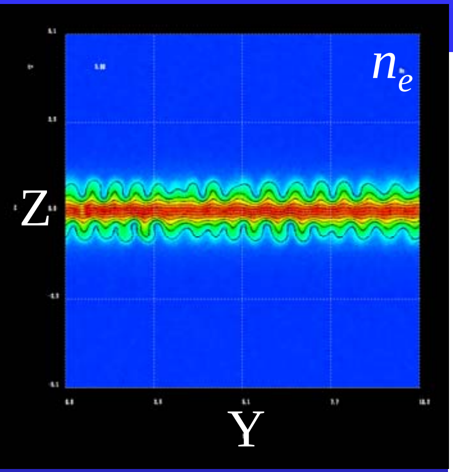
Max. number of particles
Ion + Electron ~ 10^{11}

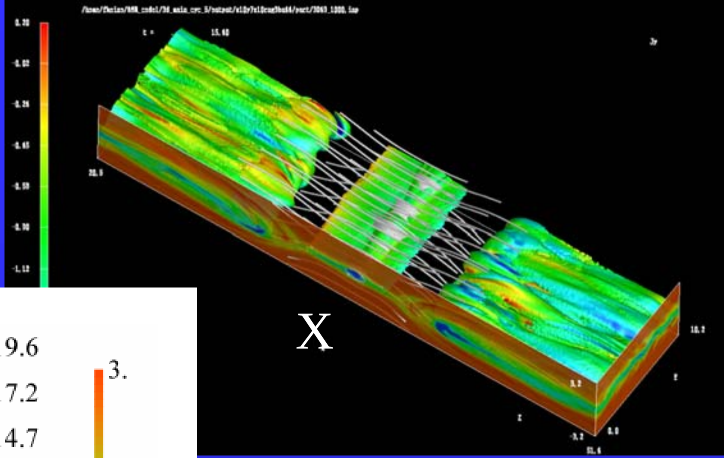
Max. memory ~ 5TB

Large-Scale 3D Simulation

Side surface : J_y Surface contour: $|J|$ Solid curves : Field lines

Wave Spectra (Y Direction)

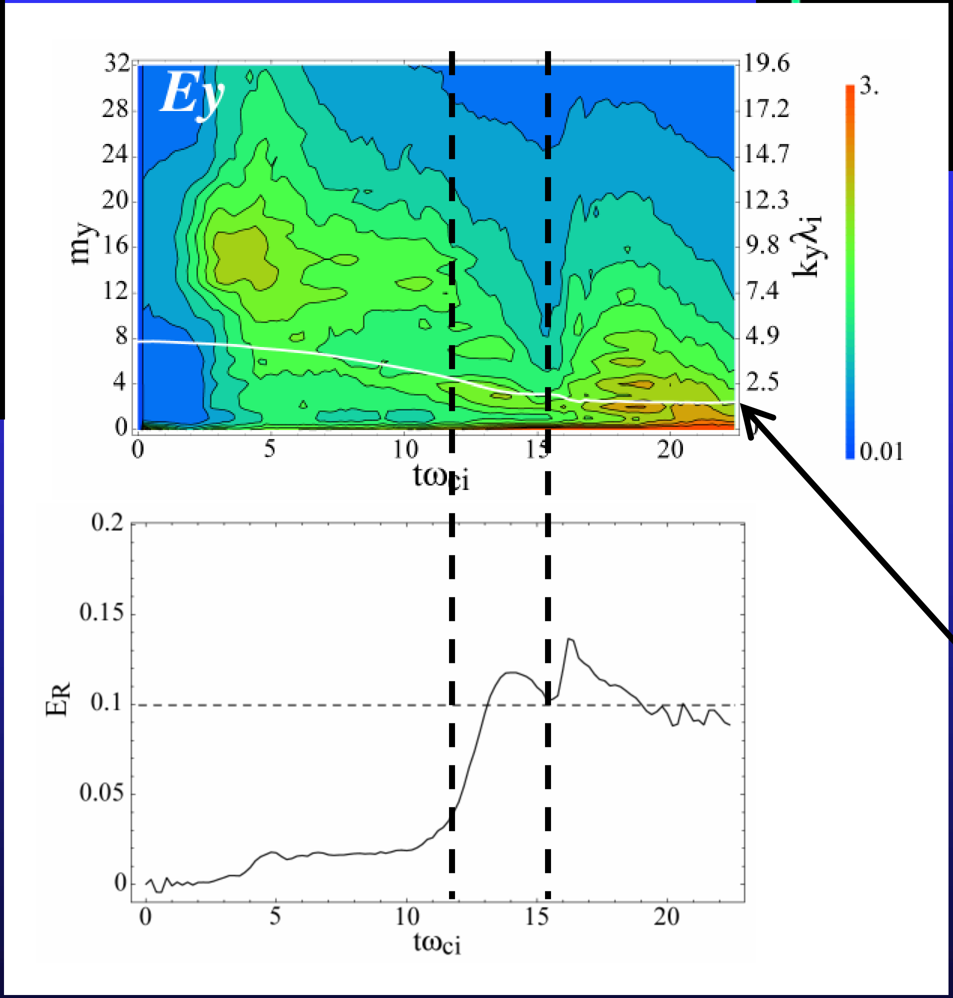


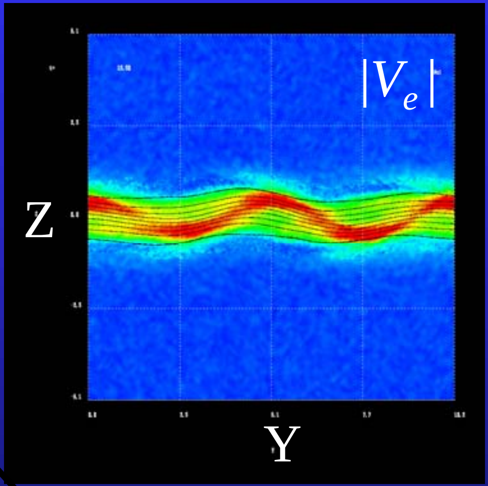


LHDI

$k_y \rho_e \sim 1$

$\gamma \sim \omega_{lh}$

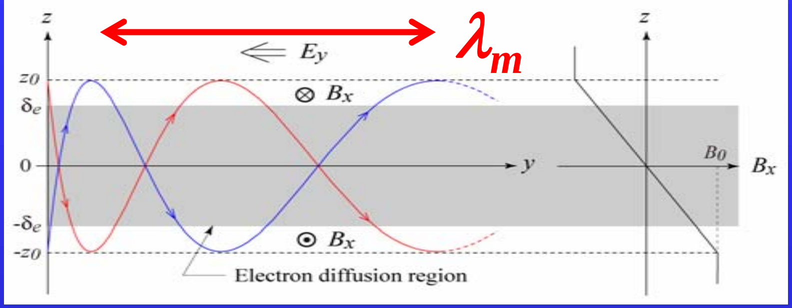




$$k_m = \frac{2\pi}{\lambda_m}$$

$$\lambda_m = 3\pi\lambda_e$$

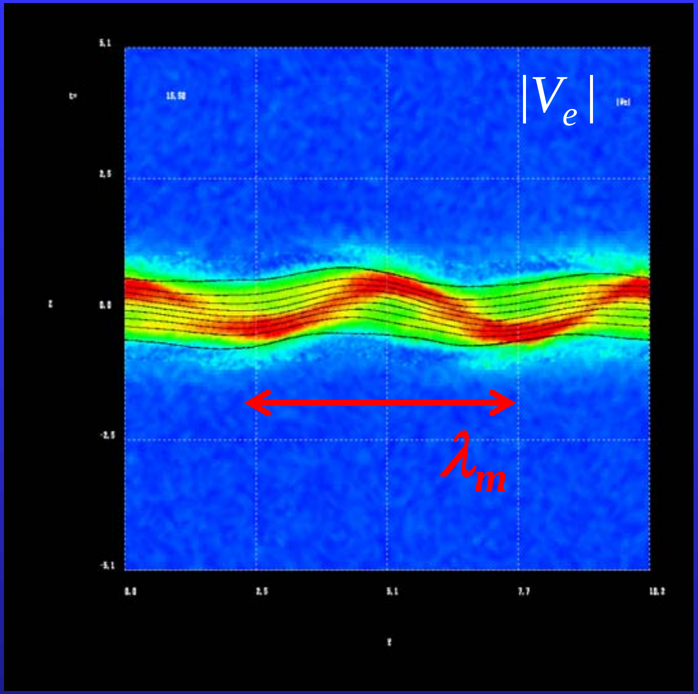
Scale of Electron Meandering Orbit



$$\omega_m \approx \frac{2}{3}\frac{V_{ey}}{c}\omega_{pe} \quad [\textit{Speiser}, 1965]$$

$$\lambda_m \approx V_{ey}\frac{2\pi}{\omega_m} = 3\pi\lambda_e$$

[*Fujimoto*, 2009]



Wavelength of EM mode ~ Electron meandering scale

→ Possibility of electron scattering and resultant anomalous resistivity

Comparisons With Observation

- Frequency

$$\lambda \approx 3\pi\lambda_e \qquad \sim \text{a few 100km}$$

$$V_{ph} = \frac{m_i V_{iy} + m_e V_{ey}}{m_i + m_e} \approx V_A$$

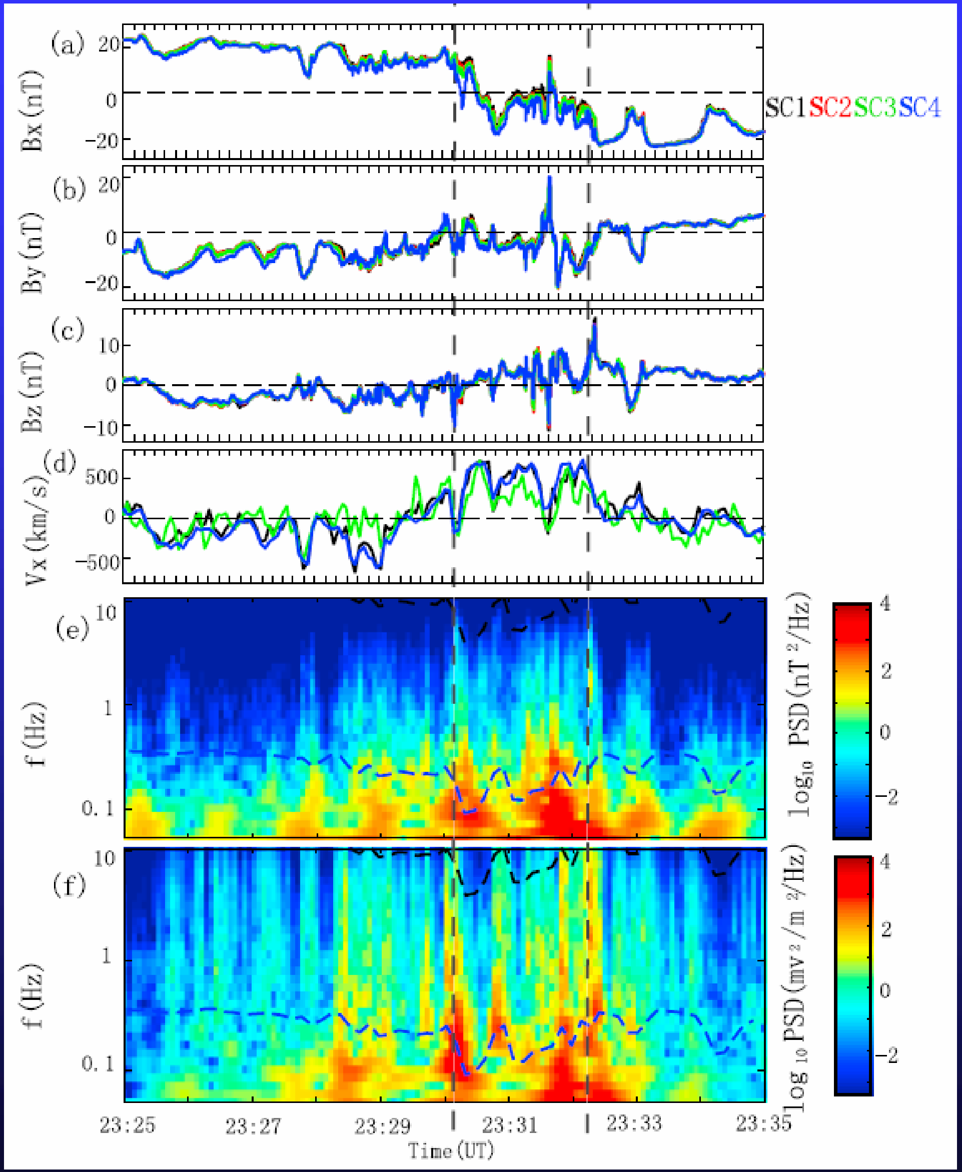
$$\omega = kV_{ph} \approx \frac{2}{3}\sqrt{\omega_{ci}\omega_{ce}} \sim \omega_{lh}$$

- Observation of Cluster

[*Zhou et al.*, JGR, 2009]

✓ ω_{lh}-range EM waves near the central current sheet.

✓ Wavelength λ ~ 352km



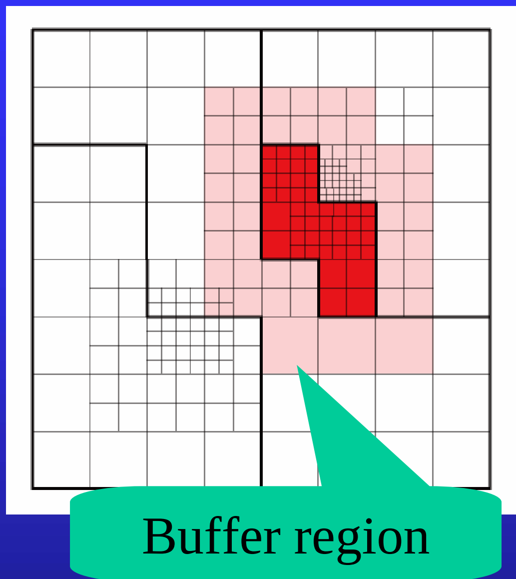
Summary

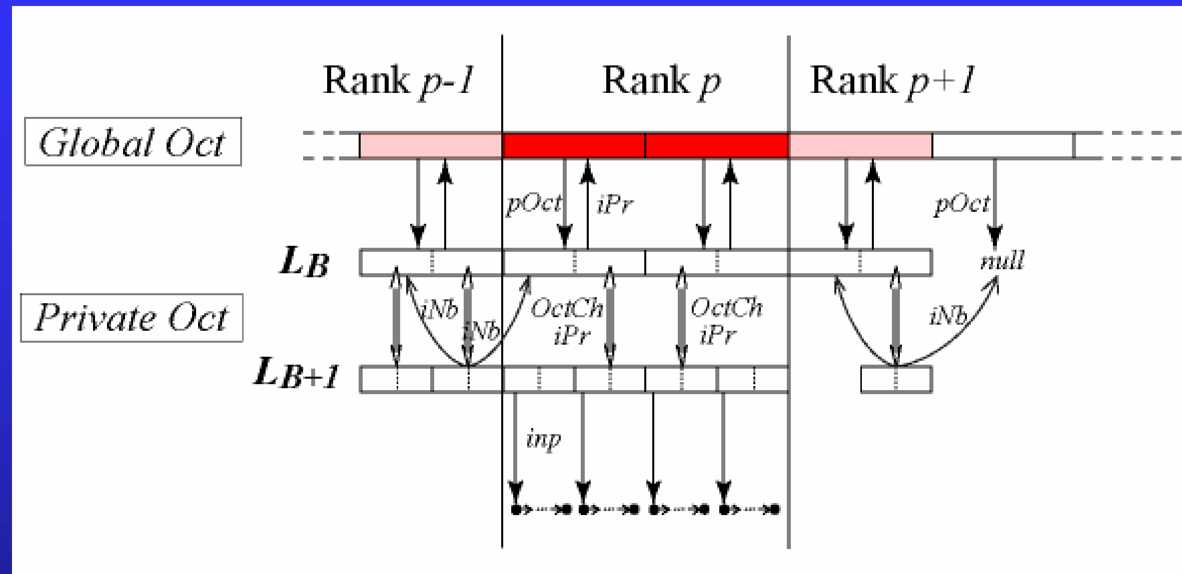
A new electromagnetic particle-in-cell (PIC) model with adaptive mesh refinement (AMR) has been developed for high-performance parallelization and applied to 3D magnetic reconnection.

- Staggering grid scheme + Charge conservation method + Smoothing
- Adaptive block technique
- More than 80% of parallel efficiency with 128 cores has been achieved for reconnection test simulations.
- Electromagnetic waves arise along the X-line and are enhanced associated with the splitting of the electron diffusion region.
- The wavelength is in the scale of the electron meandering orbit.
  ⇒ Electron scattering and resultant anomalous resistivity.

Adaptive Block Technique

Buffer region and private octs





Global oct

Shared by all the nodes

grank: Rank
gOctNb: Neighboring oct

pOct

iPr

Private oct

Allocated in each node

rank: Rank
iNb: Parent cell of neighboring oct
ipr: Parent cell
OctCh: Child oct
…
Physics data