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On the non-steady-state nucleation rate

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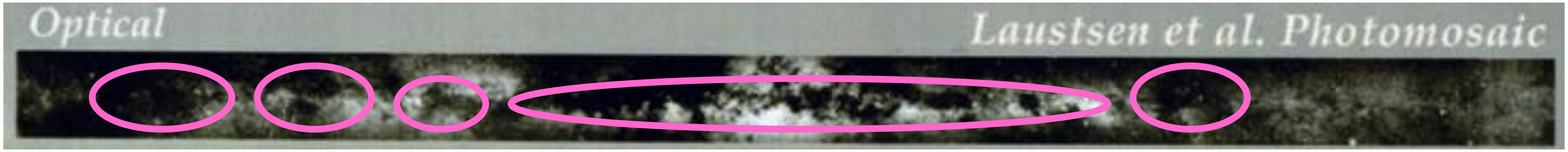
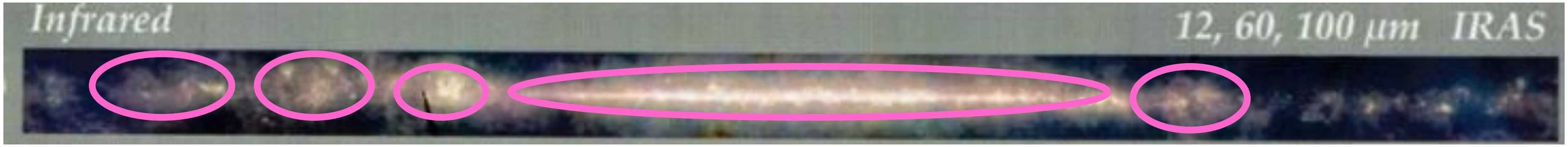
1-1. Introduction

cosmic dust universally exits in space

Milky Way (optical)



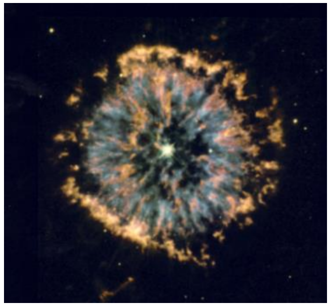
Milky Way (infrared)



Dust grains absorb UV/optical lights and reemit it by their thermal radiation at IR wavelengths!

where did these dust grains come from?

1-2. Formation sites of dust



- mass-loss winds from AGB stars

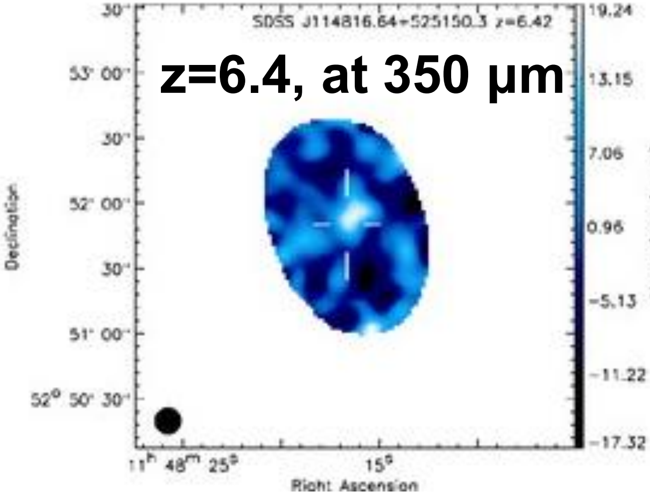
- expanding ejecta of supernovae (SNe)

— Huge amounts of dust grains ($>10^8$ Msun) are detected in quasars at redshift $z > 5$

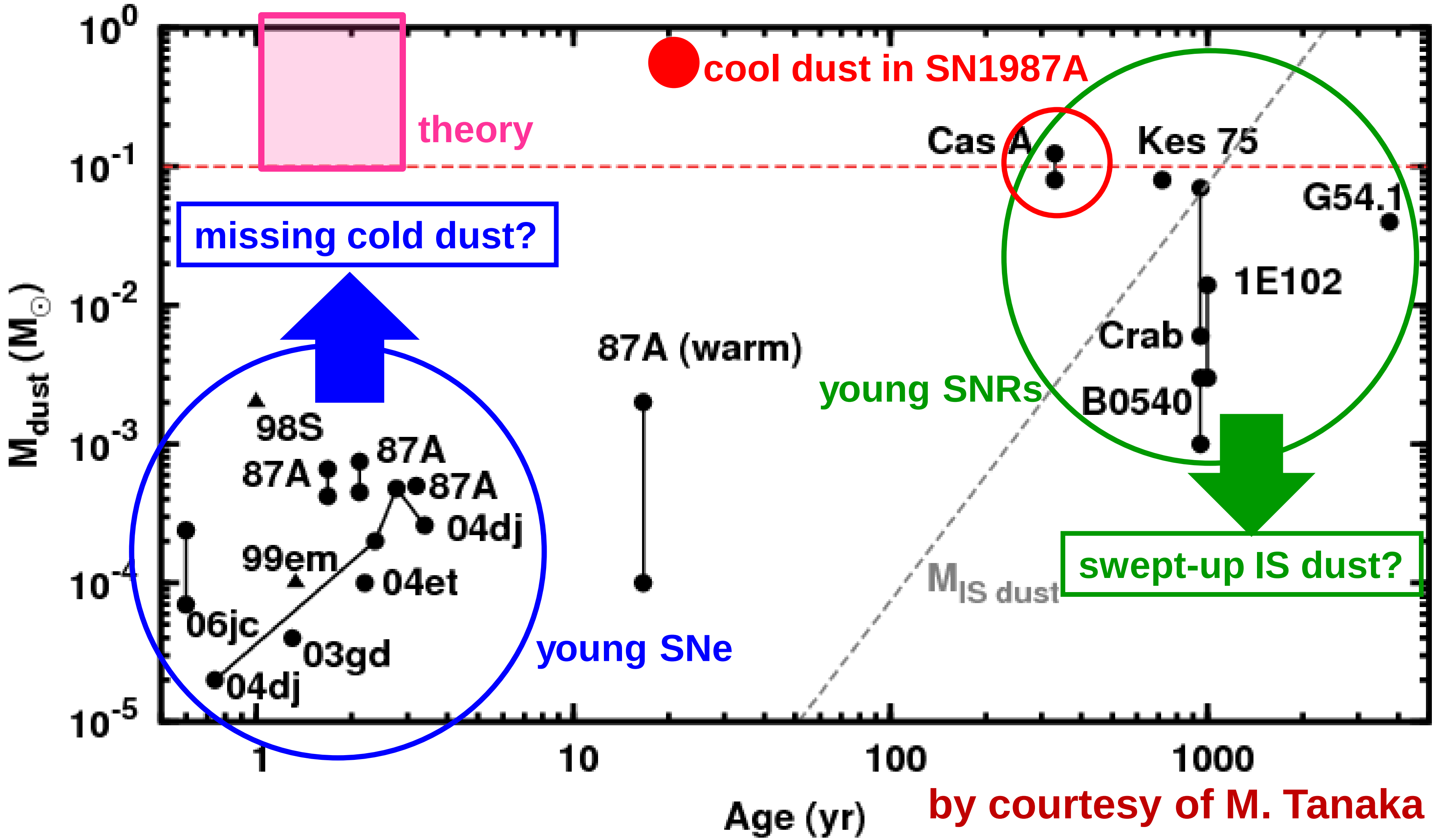
➔ 0.1 Msun of dust per SN is needed to explain such massive dust at high-z

theoretical works predict that 0.1-1.0 Msun of dust can form in SNe
(e.g., Nozawa et al. 2003)

N/MIR observations detect only $< 10^{-3}$ Msun of dust in nearby SNe
(e.g., Sakon et al. 2009)



1-3. Summary of dust mass in CC-SNe



by courtesy of M. Tanaka

Far-IR to sub-mm observations are essential for revealing the mass of dust grains produced in the ejecta of SNe

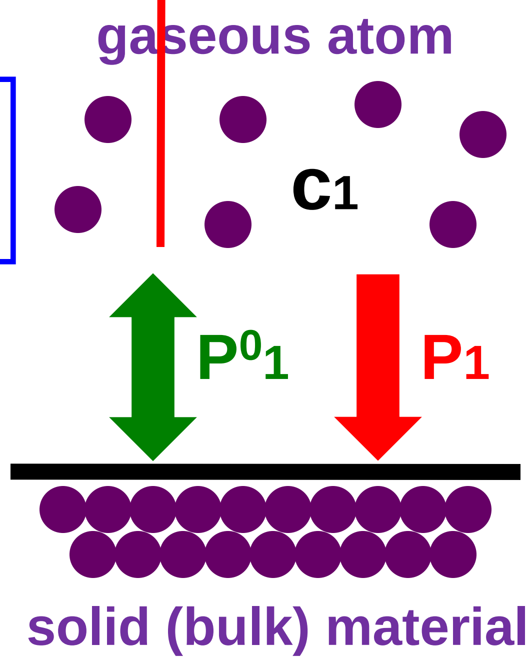
2-1. Supersaturation ratio

T_{dust} is assumed to equal to $T = T_{gas}$

· **supersaturation ratio, S**

➔ **ratio of partial pressure P_1 to equilibrium partial pressure P^0_1**

$$\ln S = \ln\left(\frac{p_1}{\mathring{p}_1}\right) = -\frac{1}{kT}\left(\mathring{g}_s - \mathring{g}_1\right) + \ln\left(\frac{p_1}{p_0}\right)$$



For condensation of dust,

$S = P_1 / P^0_1 > 1$

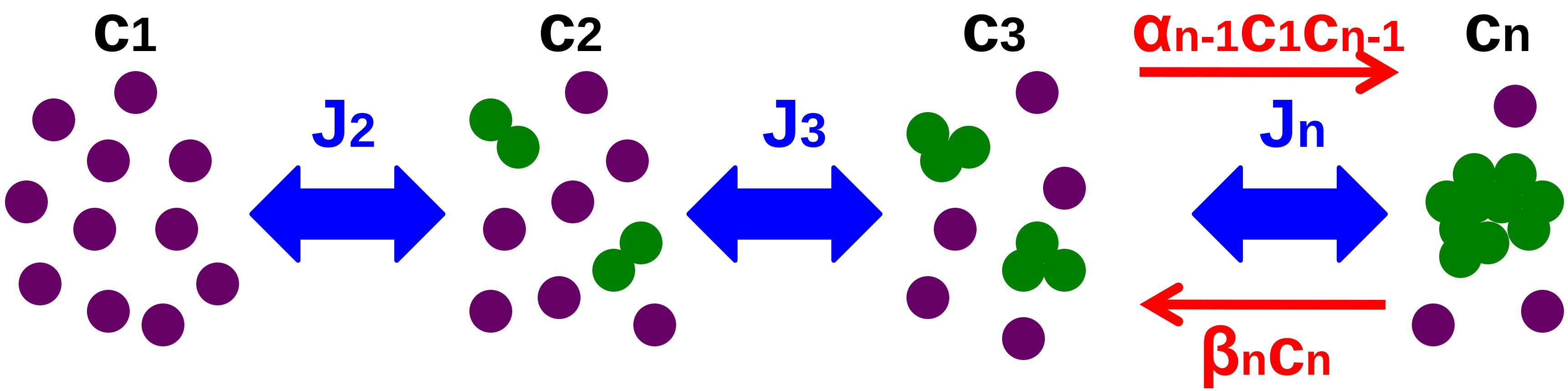
➔ **$\ln S = \ln(P_1 / P^0_1) > 0$**

$$\ln S = \frac{A}{T} - B + \ln\left(\frac{c_1 kT}{p_0}\right)$$

where $-\Delta G^0/kT = A/T - B$
($A = 8.64 \times 10^4$, $B = 19$ for C grain)

lnS is higher for lower T and higher c_1

2-2. Concept of nucleation theory



・ master equations

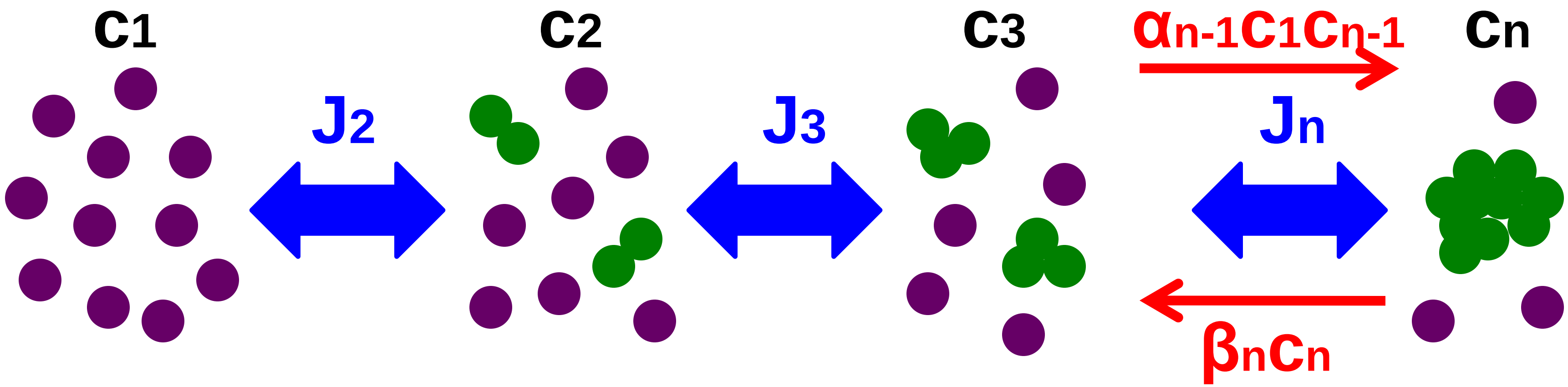
$$\frac{dc_n}{dt} = J_n(t) - J_{n+1}(t) \quad \text{for} \quad 2 \le n \le n_*,$$

$$J_n(t) = \alpha_{n-1} c_{n-1} c_1 - \beta_n c_n \quad \text{for} \quad 2 \le n \le n_*,$$

$$\alpha_n = \frac{s_n}{1 + \delta_{1n}} 4\pi a_0^2 \, n^{\frac{2}{3}} \left(\frac{kT}{2\pi m_n} \right)^{\frac{1}{2}},$$

$$\beta_n = \alpha_{n-1} \frac{\mathring{c}_{n-1}}{\mathring{c}_n} \mathring{c}_1,$$

2-3. Steady-state nucleation rate



· current density Jn

$$J_n(t) = \alpha_{n-1} c_1 \left(c_{n-1} - c_n \frac{\mathring{c}_{n-1}}{\mathring{c}_n} \frac{\mathring{c}_1}{c_1} \right).$$

· "steady" → $J_s = J_2 = J_3 = \cdots = J_\infty$

$$\frac{1}{J_s} = \frac{1}{\alpha_1 c_1^2} + \sum_{i=2}^{\infty} \frac{1}{\alpha_i c_1^2} \exp(\gamma_i').$$

2-4. Critical number of atom for nucleation

$$\frac{1}{J_s} = \frac{1}{\alpha_1 c_1^2} + \sum_{i=2}^{\infty} \frac{1}{\alpha_i c_1^2} \exp(\gamma_i').$$

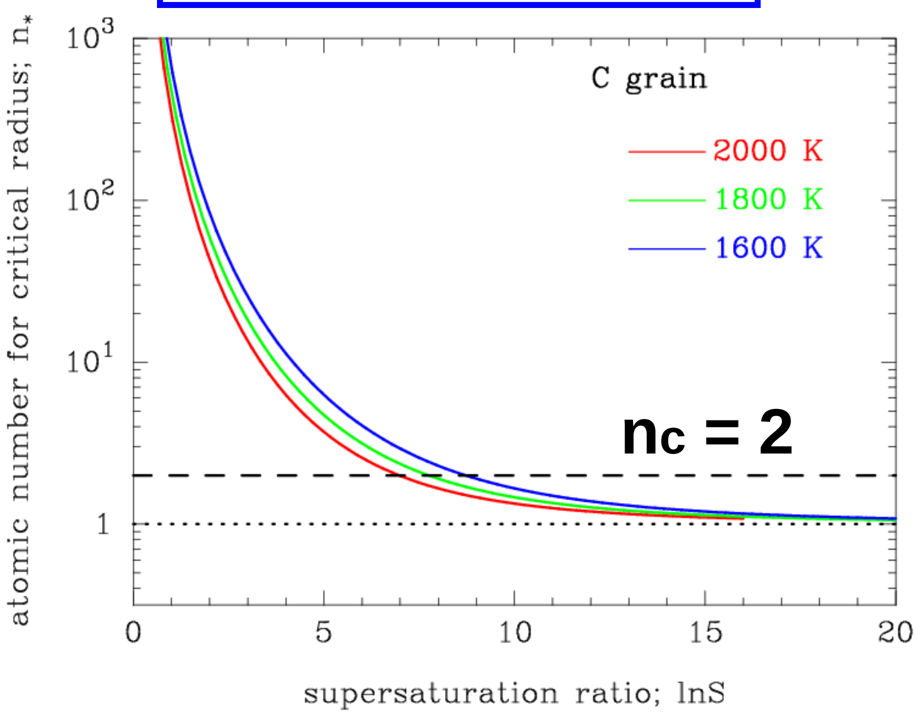
$$\gamma_n' = \mu\,(n-1)^{\frac{2}{3}} - (n-1)\ln S,$$

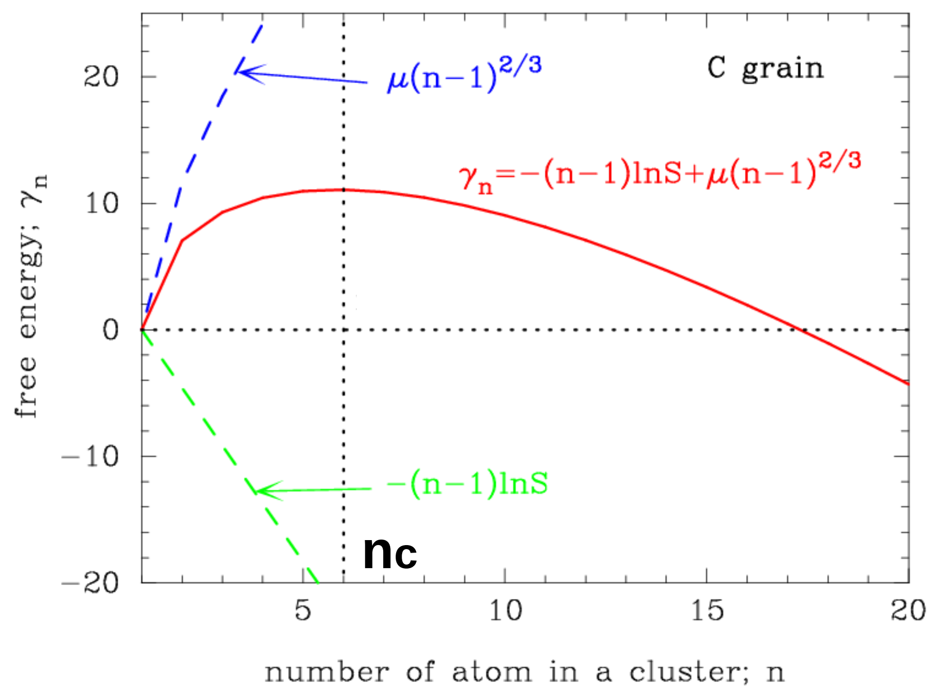
critical number: n_c

$$(n_c - 1)^{\frac{1}{3}} = \frac{2}{3}\frac{\mu}{\ln S}.$$

$\mu = 4\pi a_0^2 \sigma / kT$

(σ = 1400 erg cm^{-2} for bulk C materials)





2-5. Non-steady-state nucleation

・ steady-state nucleation rate: J_s

$$J_s = s\,\Omega_0 \left(\frac{2\sigma}{\pi m_1}\right)^{\frac{1}{2}} c_1^2 \,\exp\left[-\frac{4}{27}\frac{\mu^3}{(\ln S)^2}\right].$$

$$J_n(t) = \alpha_{n-1} c_1 \left(c_{n-1} - c_n \frac{\mathring{c}_{n-1}}{\mathring{c}_n}\frac{\mathring{c}_1}{c_1}\right).$$

→ **$J_s = J_2 = J_3 = \cdots = J_\infty$**

$$\gamma_n' = \mu\,(n-1)^{\frac{2}{3}} - (n-1)\ln S,$$

$$J_n(t) = \alpha_{n-1} c_1 \left[c_{n-1} - c_n \exp(\gamma_n)\right]$$

$$\gamma_n = \mu\left[(n-1)^{\frac{2}{3}} - (n-2)^{\frac{2}{3}}\right] - \ln S,$$

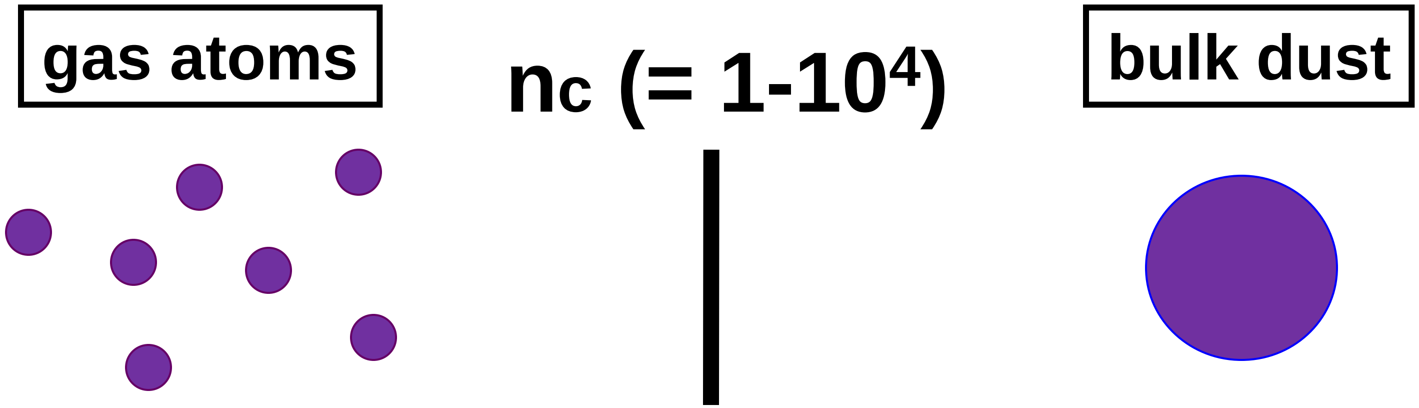
・ non-steady-state nucleation

$n_* = 100$

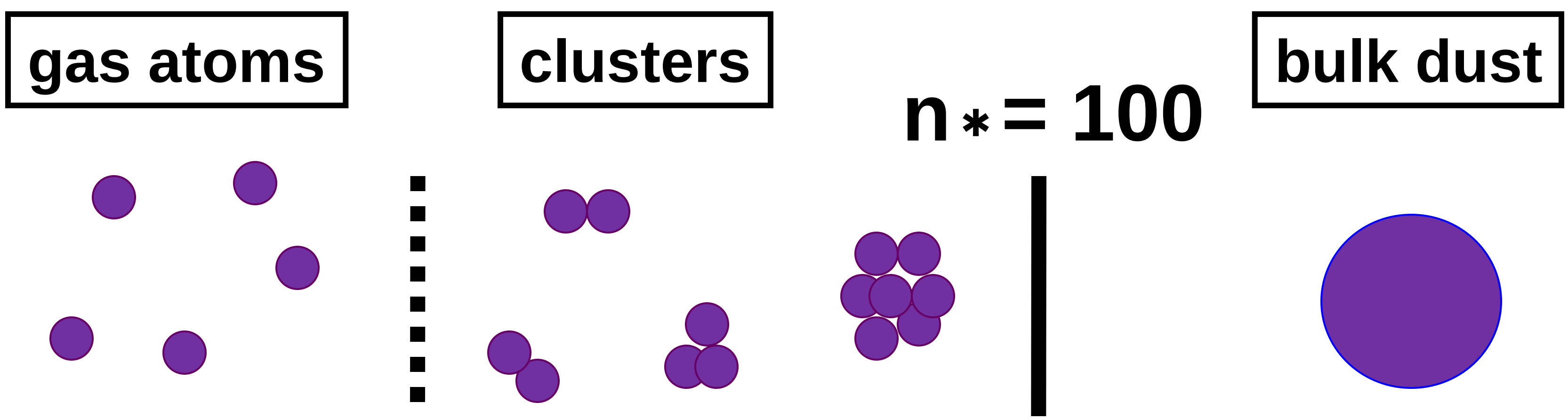
$$\frac{dc_n}{dt} = J_n(t) - J_{n+1}(t) \quad \text{for} \;\; 2 \le n \le n_*,$$

2-6. Steady and non-steady

・ steady-state nucleation rate: J_s



・ non-steady-state nucleation rate: J_*



3-1. Basic equations for dust formation

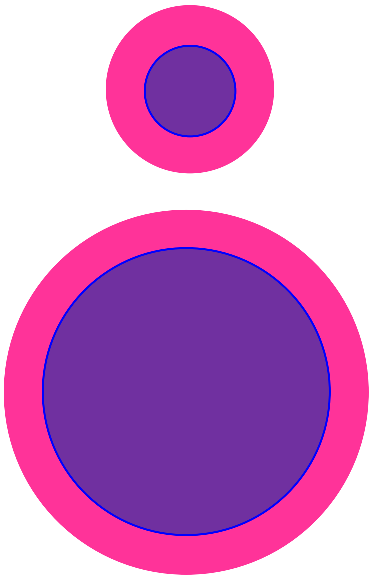
· Equation of mass conservation

$$c_{10} - c_1 = \underbrace{\sum_{n=2}^{n_*-1} n c_n}_{\text{(cluster)}} + \underbrace{\int_{t_0}^{t} J_{n_*}(t') \frac{a^3(t,t')}{a_0^3} dt'}_{\text{(bulk dust)}},$$

· Equation of grain growth

$$\frac{da}{dt} = s\Omega_0 \left(\frac{kT}{2\pi m_1}\right)^{\frac{1}{2}} c_1 \left(1 - \frac{1}{S}\right),$$

$$\frac{dV}{dt} = s\Omega_0 \, 4\pi a^2 \left(\frac{kT}{2\pi m_1}\right)^{\frac{1}{2}} c_1 \left(1 - \frac{1}{S}\right),$$



Growth rate is independent of grain radius

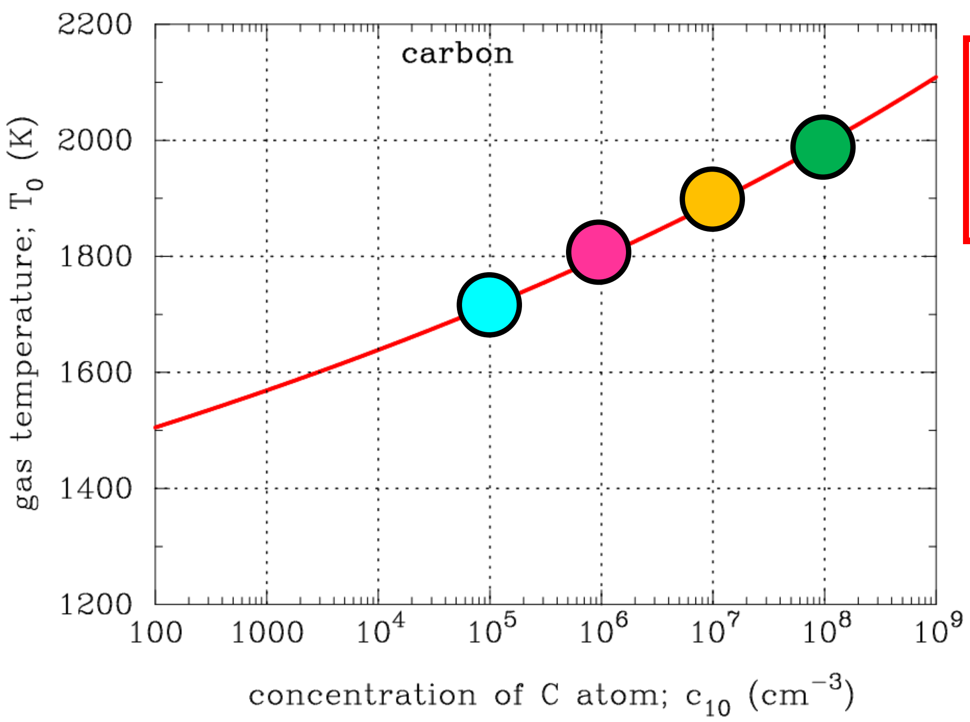
3-2. Evolution of gas temperature and density

・Time evolution of c_1 and T

$$c_1(t) = c_{10}\exp\left(-\frac{t-t_0}{\tau_{\exp}}\right)$$
$$T(t) = T_0\exp\left(-\frac{t-t_0}{\tau_{\rm cool}}\right),$$

$\tau_{\exp}$ = 100 day

$\tau_{\rm cool}$ = 100 day



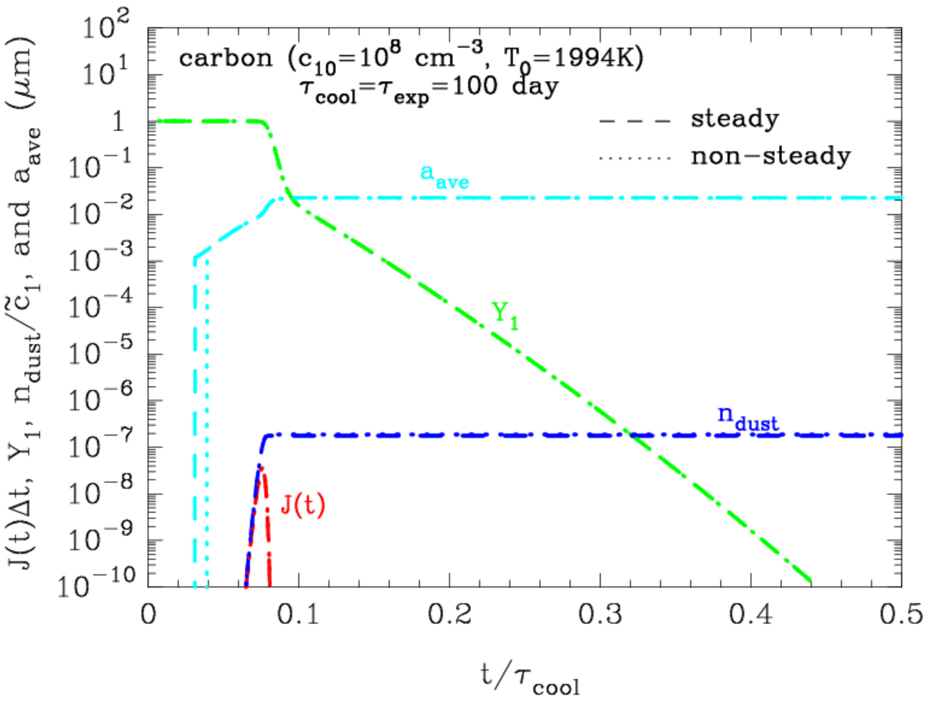
$$\ln S = \frac{A}{T_0} - B + \ln\left(\frac{c_{10}kT_0}{p_0}\right) = 0$$

c_{10} = 10^8, 10^7, 10^6, 10^5 cm^{-3}

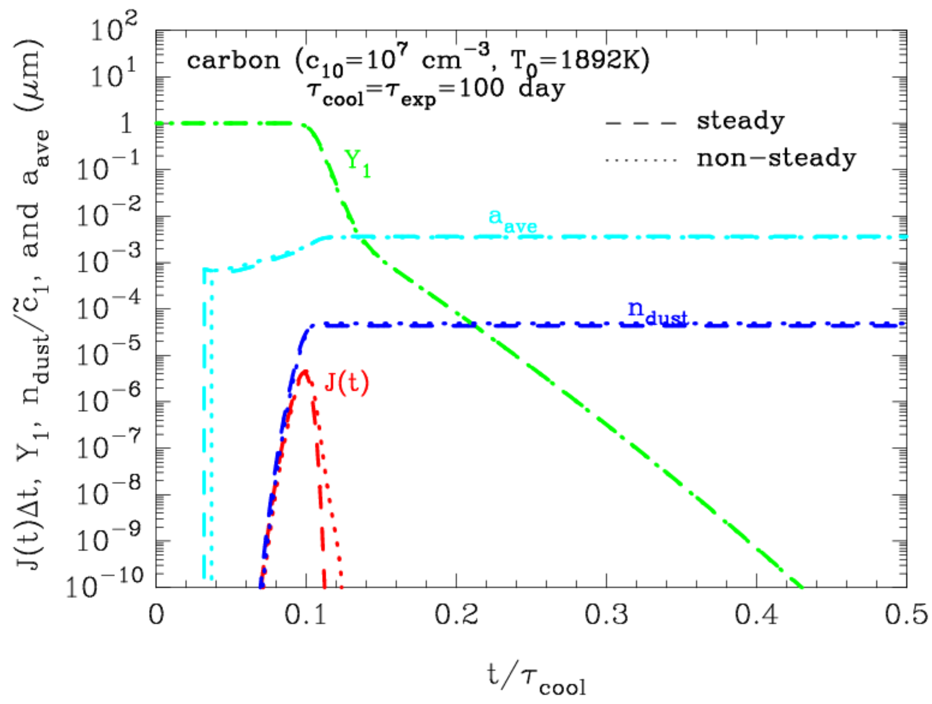
s = 1.0

4-1. Steady vs. Non-steady (1)

$c_{10} = 10^8$ cm^{-3}



$c_{10} = 10^7$ cm^{-3}

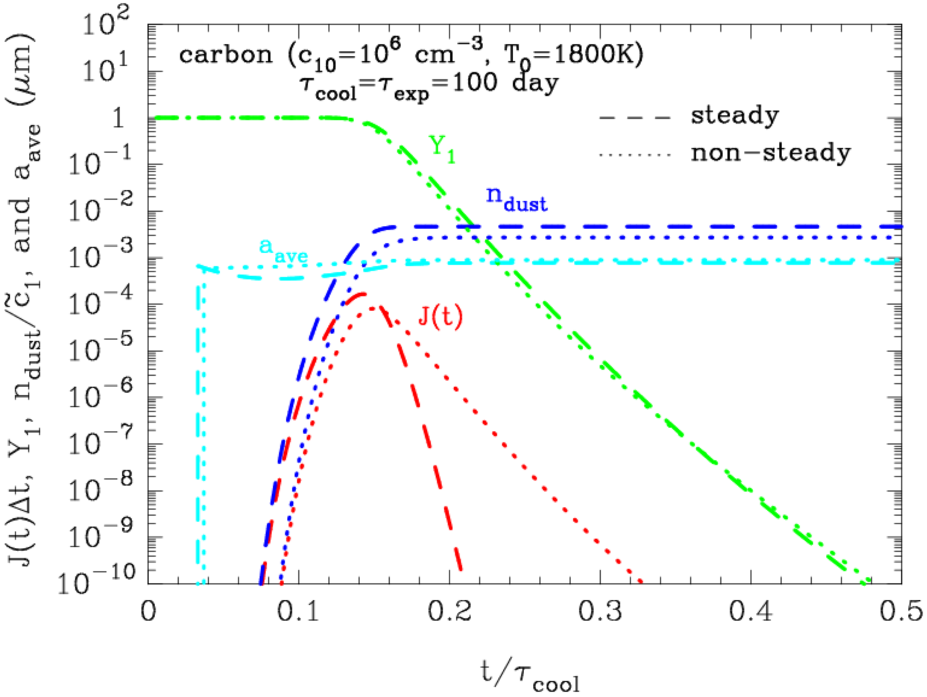


- dashed line : steady-state nucleation
- dotted line : non-steady-state nucleation

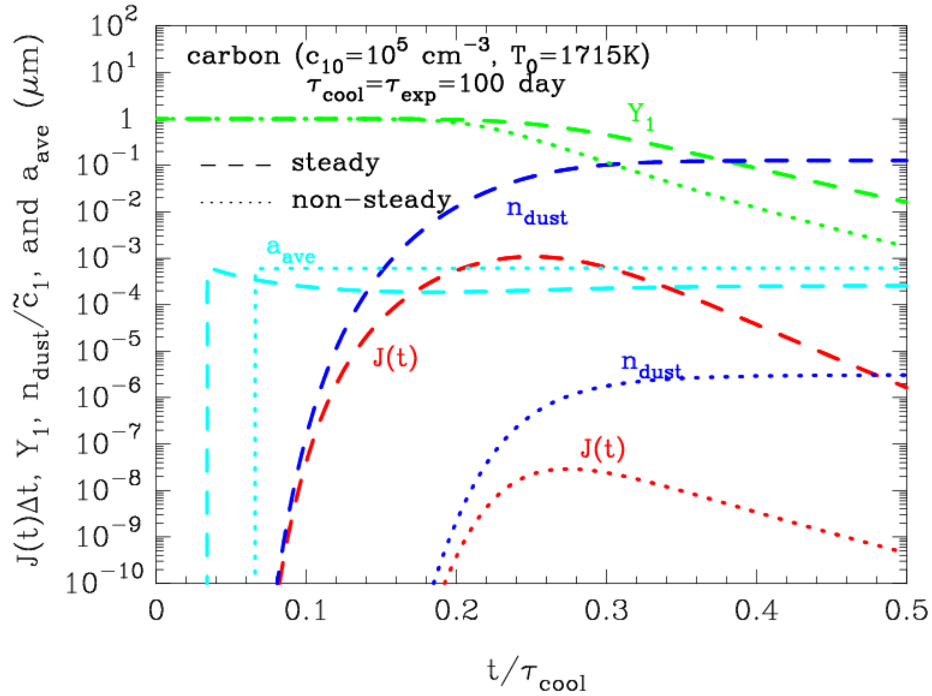
The difference between steady and non-steady nucleation is small for higher initial densities

4-2. Steady vs. Non-steady (2)

$c_{10} = 10^6$ cm^{-3}



$c_{10} = 10^5$ cm^{-3}



- **dashed line : steady-state nucleation**
- **dotted line : non-steady-state nucleation**

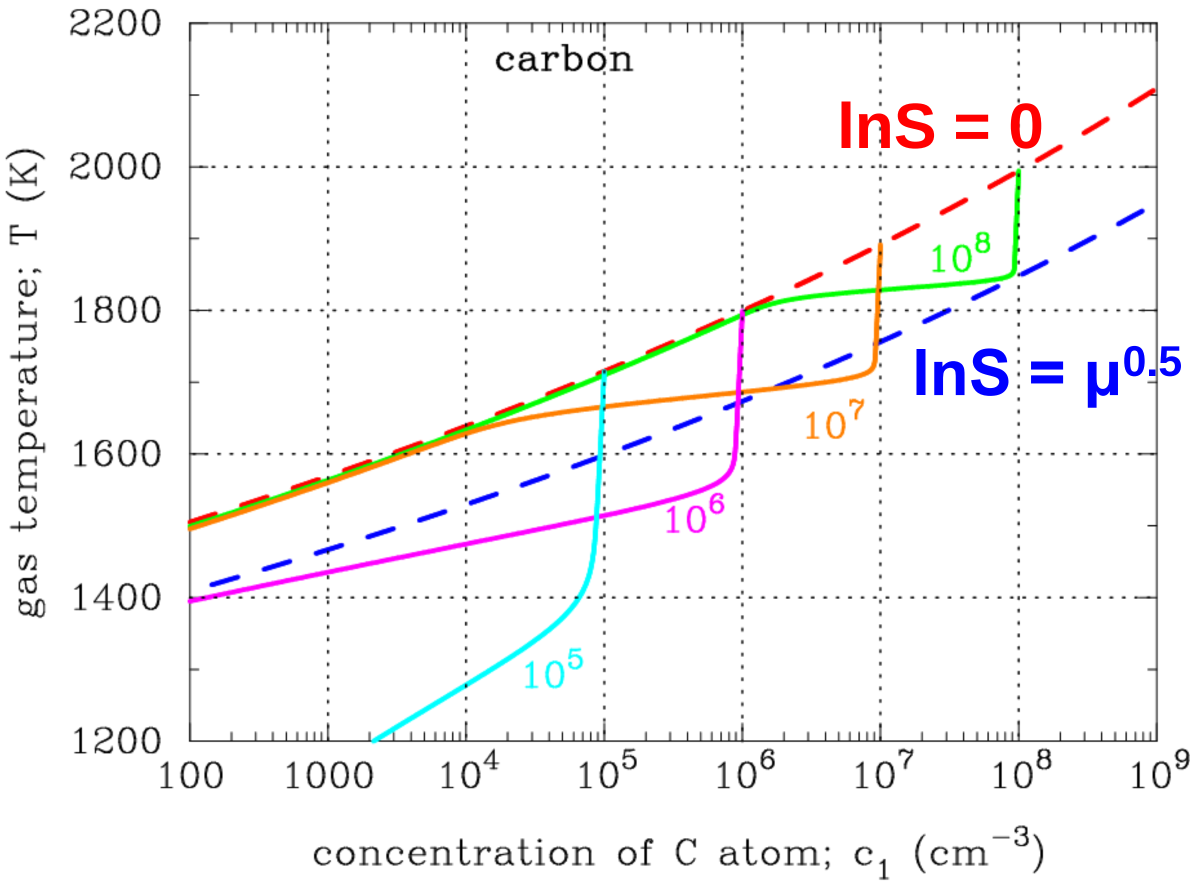
The difference between steady and non-steady seems significant for lower densities

4-3. Relaxation time

・ relaxation time, τ_{relax} (e.g., Gail et al. 1984)

$$\rightarrow \tau_{relax} = \tau_{coll} \left[(\ln S)^2 / \mu \right]$$

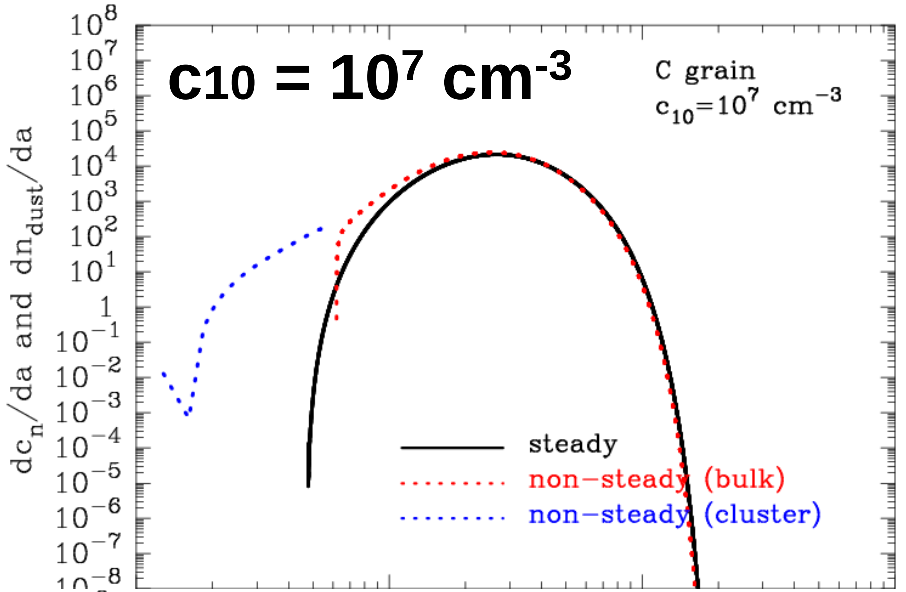
$$\tau_{coll} = \left[s_n \, 4\pi a_0^2 \, \langle v \rangle \, c_1 \right]^{-1}$$

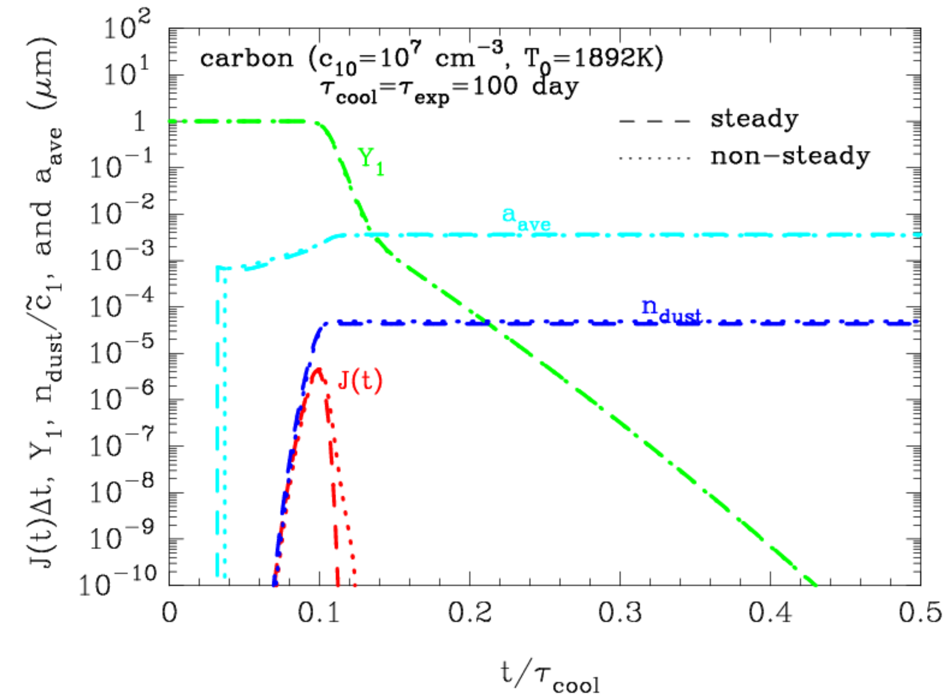


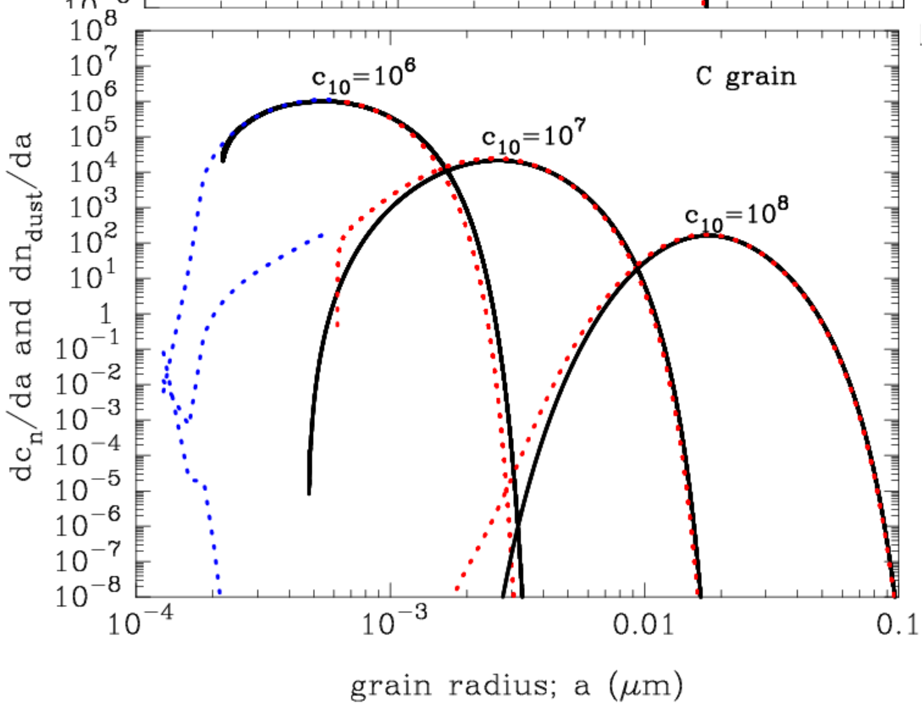
$\tau_{relax} / \tau_{coll} = \left[(\ln S)^2 / \mu \right] < 1$
➔ steady

$\tau_{relax} / \tau_{coll} = \left[(\ln S)^2 / \mu \right] > 1$
➔ non-steady

5-1. Steady vs. Non-steady: size distribution (1)

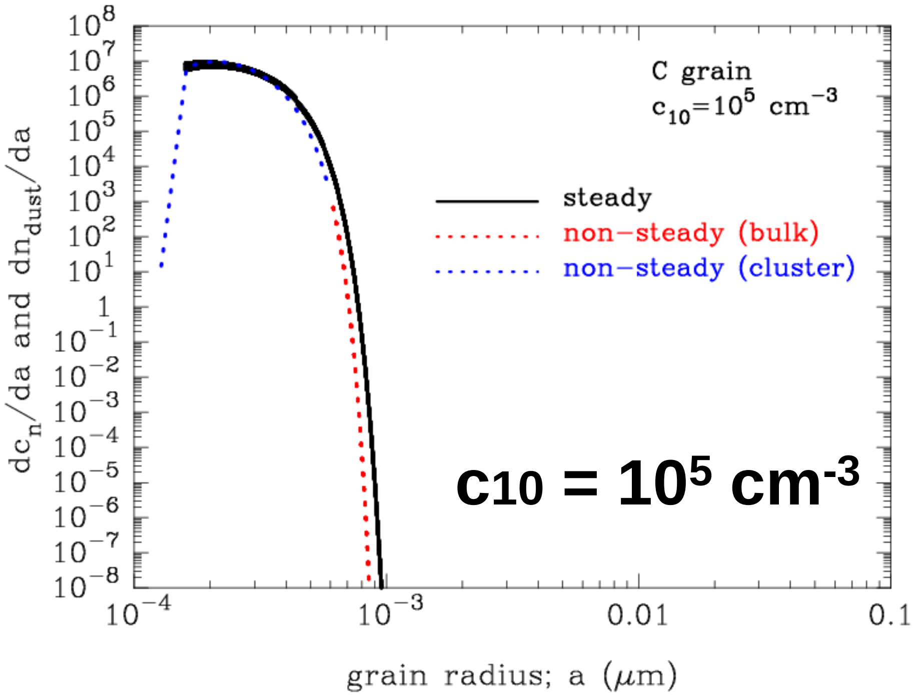


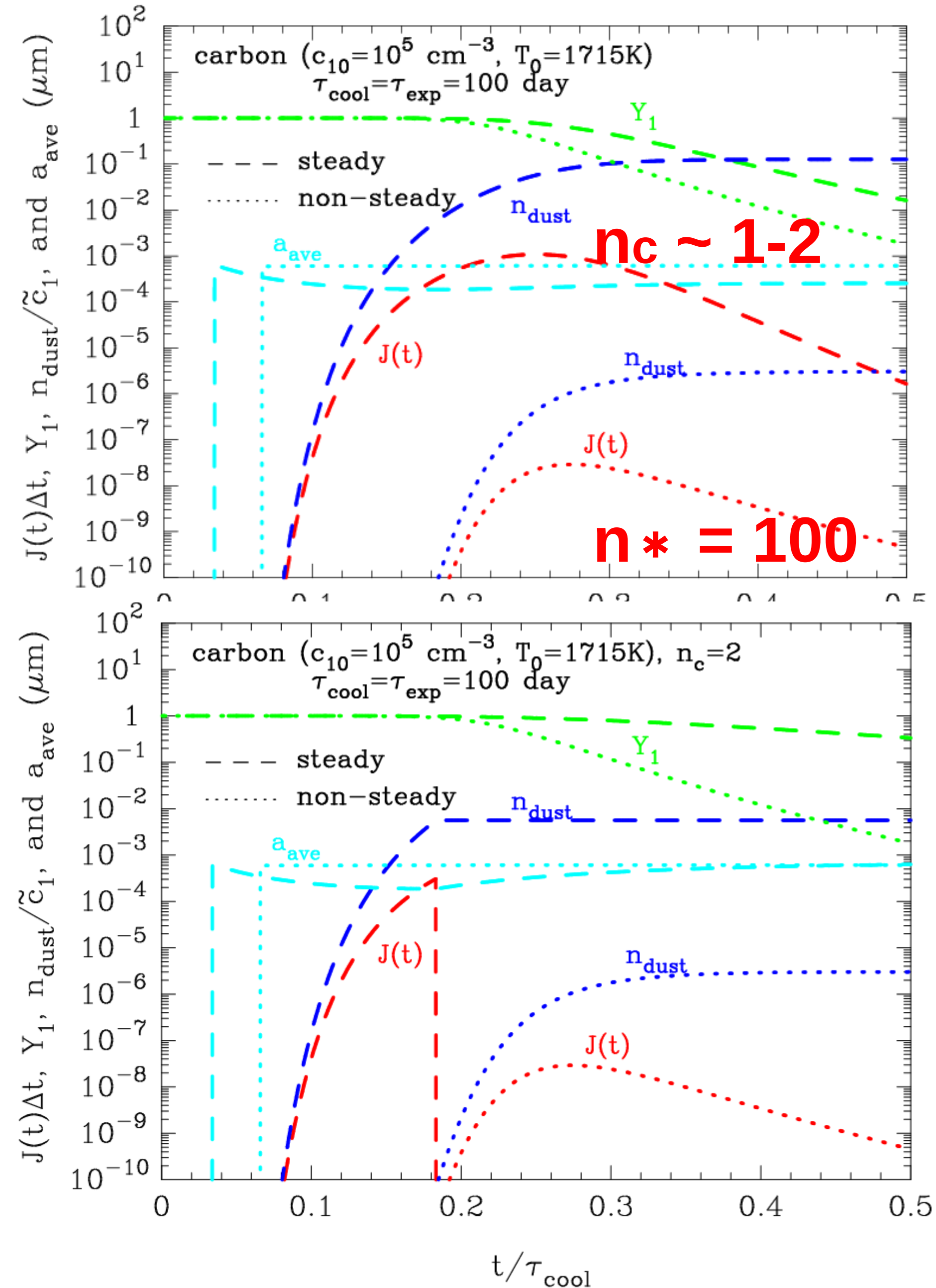




The size distribution of bulk grains from non-steady-state is consistent with that from steady-state

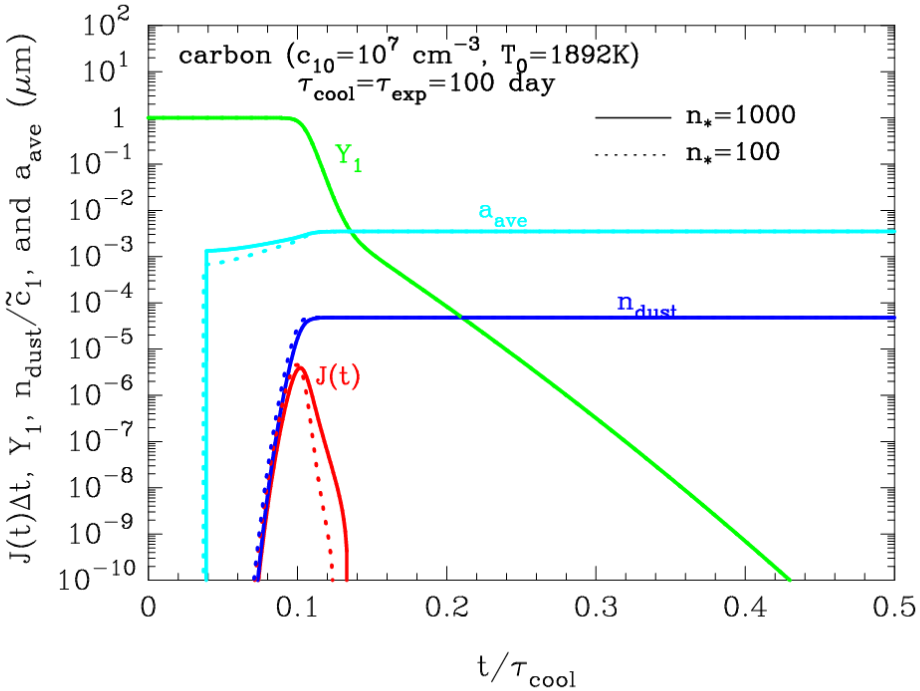
5-2. Steady vs. Non-steady: size distribution (2)

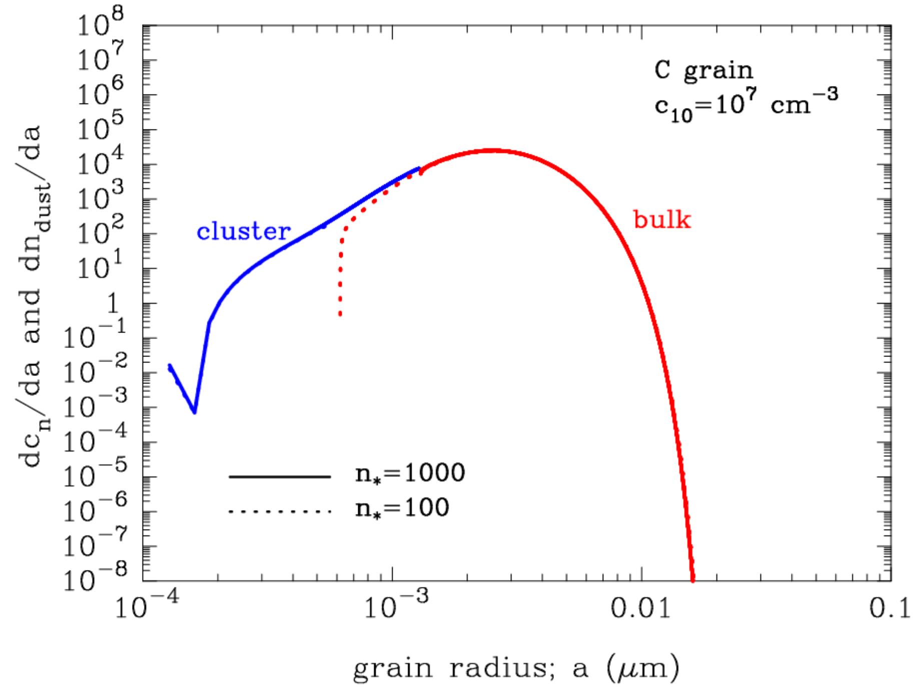




Even for $c_{10} = 10^5$ cm^{-3}, the size distribution is quite similar between steady-state and non-steady-state

5-3. Dependence on cluster maximum size



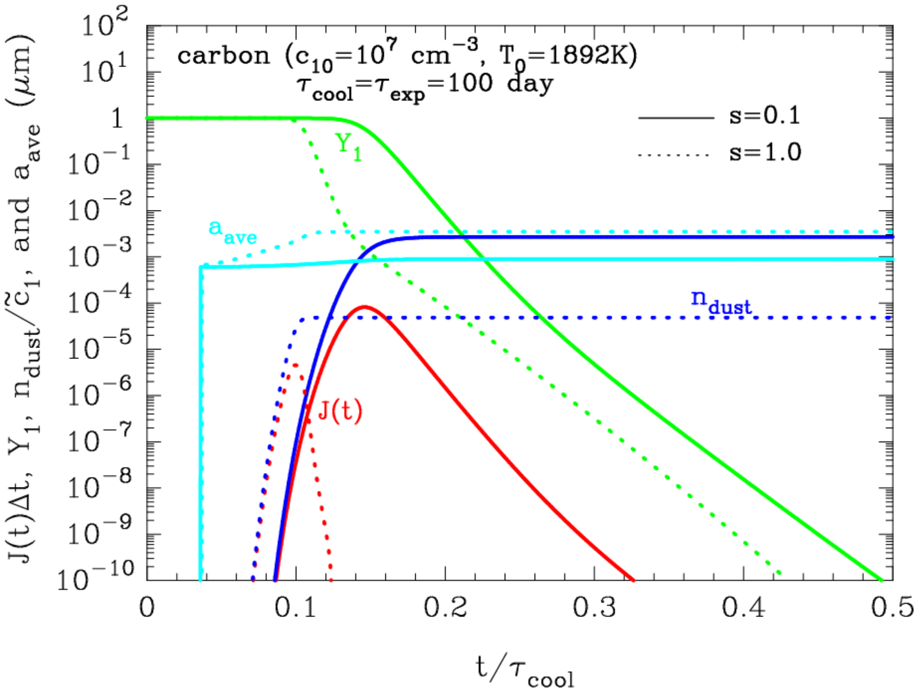


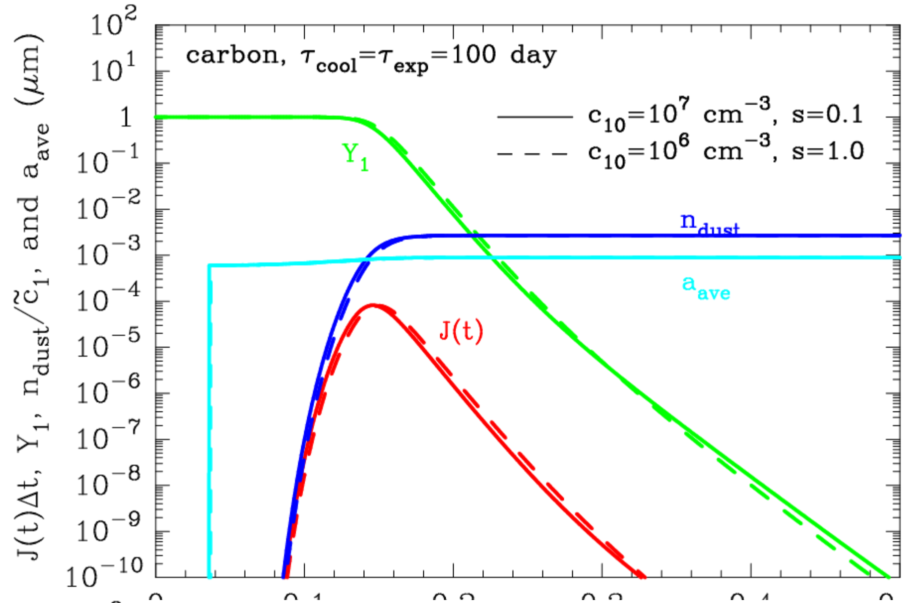
- solid line : non-steady-state, n_* = 1000
- dotted line : non-steady-state, n_* = 100

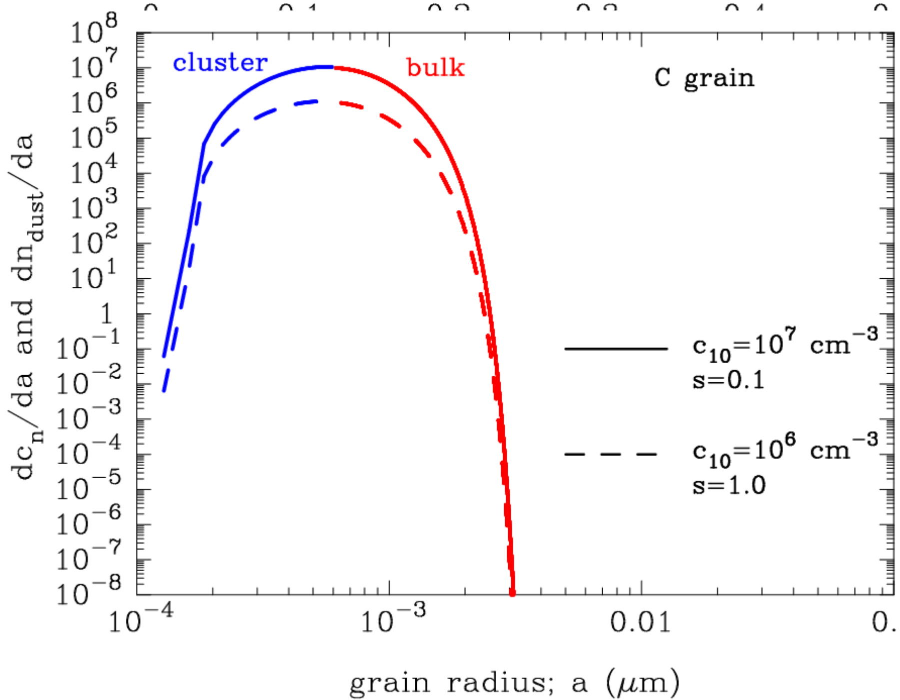
The size distribution does not depend on n_*

How many atoms are needed to be defined as bulk dust grains?

5-4. Dependence on sticking probability







The result for c_{10} = 10^7 cm^{-3} and s = 0.1 is similar to that for c_{10} = 10^6 cm^{-3} and s = 1, but not completely same

6. Summary

- **The difference between steady and non-steady state nucleation is not significant**
  - steady-state nucleation rate is a good approximate
  - what is the definition of bulk dust grains?

- **In the future work**
  - to extend to multiple-element grains like silicate
  - to calculate the temperature of dust (clusters)
    - ➔ Dust (cluster) temperature is assumed to be equivalent to gas temperature
  - unknown quantities: sticking probability, shape and surface energy of small clusters