(Formulation of Non-steady-state Dust Formation Process in Astrophysical Environments)

1-1. Core-collapse SNe as sources of dust

- **Discoveries of massive dust at high redshifts**
  - CCSNe must be main producers of dust grains

- **Dust formation in the ejecta of CCSNe**
  - Theoretical works predict that $0.1\text{-}1.0 \, M_{\odot}$ of dust can form in CCSNe (e.g., Nozawa+03; Nozawa+10)
  - FIR observations with Herschel reported $\sim0.1 \, M_{\odot}$ of cool dust in Cas A, SN 1987A, and Crab (Barlow+10; Matsuura+11; Gomez+12)

  ## Some of dust grains formed in the ejecta are destroyed by the reverse shock (e.g., Nozawa+07)

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Necessary to reveal the dust mass and size distribution!
1-2. Aim of this study

- **How do dust grains form?**
  
  atoms $\rightarrow$ molecules $\rightarrow$ clusters $\rightarrow$ bulk grains??

  reaction coefficients unknown!

- **Nucleation accompanied by chemical reactions**
  
  — kinetics of dust formation process is controlled by **key molecule**: gas species with the least collision frequency among reactants
    
    (Kozasa & Hasegawa 1987)

  — steady-state nucleation rate may not be applied in rarefied environments (e.g., Donn & Nuth 1985)

The aim of this study is to formulate a non-steady-state formation process of dust grains
2-1. Formulation of dust formation

- master equations

\[
\frac{dc_n}{dt} = J_n(t) - J_{n+1}(t) \quad \text{for} \quad 2 \leq n \leq n_*,
\]

\[
J_n(t) = \alpha_{n-1}c_{n-1}c_1 - \beta_n c_n \quad \text{for} \quad 2 \leq n \leq n_*,
\]

\[
\alpha_n = \frac{s_n}{1 + \delta_{1n}} \frac{4\pi a_0^2}{n^\frac{2}{3}} \left( \frac{kT}{2\pi m_n} \right)^{\frac{1}{2}},
\]

\[
\beta_n = \alpha_{n-1} \frac{\dot{c}_{n-1}}{\dot{c}_n} \frac{\dot{c}_1}{},
\]
2-2. Steady-state nucleation rate

**steady-state nucleation rate:** $J_s$

→ assuming $J_s = J_2 = J_3 = \cdots = J_\infty$

\[
(n_c - 1)^{\frac{1}{3}} = \frac{2}{3} \frac{\mu}{\ln S}.
\]

where

\[
\mu = 4\pi a_0^2 \sigma / kT
\]

$\sigma$: surface tension

$S$: supersaturation ratio

\[
( S = \frac{p_1}{p_{1v}} )
\]

\[
J_s = s_{\text{crit}} \Omega_0 \left( \frac{2\sigma}{\pi m_1} \right)^{\frac{1}{2}} c_1^2 \Pi \exp \left[-\frac{4}{27} \frac{\mu^3}{(\ln S)^2} \right],
\]

\[
\mu = 4\pi a_0^2 \sigma / kT
\]
2-3. Non-steady-state dust formation

- non-steady-state dust formation \[ n^* = 100 \]

\[
\frac{dc_n}{dt} = J_n(t) - J_{n+1}(t) \quad \text{for} \quad 2 \leq n \leq n^*,
\]

\[
J_n(t) = \alpha_{n-1}c_1\left[c_{n-1} - c_n \exp(\gamma_n)\right].
\]

\[
\gamma_n = \mu \left[\left(n - \frac{1}{\omega}\right)^{\frac{2}{3}} - \left(n - 1 - \frac{1}{\omega}\right)^{\frac{2}{3}}\right] - \ln S
\]

- Non-steady model: solving master equations
- Steady model: using a steady-state nucleation rate
2-4. Basic equations for dust formation

- Equation of mass conservation

\[ \tilde{c}_1 V - c_1 V = \sum_{n=2}^{n_*-1} n c_n V + \int_{t_0}^{t} V(t') J_{n_*}(t') \frac{a^3(t, t')}{a_0^3} dt', \]

(clusters) (grains)

- Equation of grain growth

\[ \frac{d a}{d t} = s \Omega_0 \left( \frac{kT}{2\pi m_1} \right)^{\frac{1}{2}} c_1 \left( 1 - \frac{1}{S} \right), \]

\[ \propto f_{\text{con}} \]

- Evolutions of gas density and temperature

\[ \tilde{c}(t) = c_0 \left( \frac{t}{t_0} \right)^{-3}, \quad T(t) = T_0 \left( \frac{t}{t_0} \right)^{-3(\gamma-1)} \]

(\( \gamma = 1.1-1.7 \))

Parameters: \( c_0, \gamma, t_0 \) (the time at which \( \ln S = 0 \))

Fiducial values: \( \gamma = 1.25, t_0 = 300 \text{ day} \)
3-1. Steady vs. Non-steady (1)

The results for steady and non-steady models are essentially the same for high gas densities.

decrease in temperature $\rightarrow$ increase in $S$ $\rightarrow$ increase in $I_\ast$ ($I_s$) $\rightarrow$ grain growth $\rightarrow$ consumption of gas $\rightarrow$ decrease in $I_\ast$ ($I_s$)
3-2. Steady vs. Non-steady (1): size distribution

The size distribution of grains for steady and non-steady models are identical

⇒ The steady-state nucleation rate is a good approximation for higher initial densities
3-3. Steady vs. Non-steady (2)

- \( I_* \): formation rate of clusters with \( n_* = 100 \)
- \( I_s \): formation rate of clusters with \( n = n_c (<100) \)

For \( \frac{t_{coll}}{t_0} \ll 1 \) → \( I_s = \ldots = I_n = I_{n+1} = \ldots = I_* \)

For \( \frac{t_{coll}}{t_0} \ll 1 \) → \( I_s > \ldots > I_n > I_{n+1} > \ldots > I_* \)
3-4. Steady vs. Non-steady (2): size distribution

For lower gas densities, the steady model overestimates the condensation efficiency and underestimates the average grain radius.
3-5. Scaling relation of average grain radius

- $\Lambda_{on} = \frac{\tau_{sat}(t_{on})}{\tau_{coll}(t_{on})}$: ratio of supersaturation timescale to gas collision timescale at the onset time ($t_{on}$) of dust formation

- $t_{on}$: the time at which $f_{con}$ reaches $10^{-10}$

- $f_{con,\infty}$ and $a_{ave,\infty}$ are uniquely determined by $\Lambda_{on}$
- steady-state nucleation rate is applicable for $\Lambda_{on} > 30$
3-6. Scaling relation of average grain radius

\[ \Lambda_{on} = \frac{\tau_{sat}(t_{on})}{\tau_{coll}(t_{on})} \]

- \( a \sim 0.1 \mu m \) in Type II-P SNe (Nozawa+03)
- \( a \sim 0.001 \mu m \) in Type IIb SNe (Nozawa+10)

\[ \#\# \Lambda_{on} = T_{sat}/T_{coll} \propto T_{cool} \eta_{gas} \]
5. Summary of this talk

We develop a new formulation describing nonsteady-state formation of small clusters and grains in a self-consistent manner, taking account of chemical reactions.

- **Steady-state nucleation rate is a good approximation if the gas density is high enough** \((T_{\text{sat}} / T_{\text{coll}} >> 1)\)
  
  - otherwise, non-steady effect becomes remarkable, leading to a lower \(f_{\text{con},\infty}\) and a larger \(a_{\text{ave},\infty}\)

- **Steady-state nucleation rate is applicable for** \(\Lambda_{\text{on}} > 30\)
  
  - \(f_{\text{con},\infty}\) and \(a_{\text{ave},\infty}\) are determined by \(\Lambda_{\text{on}} = T_{\text{sat}} / T_{\text{coll}}\) at the onset time \((t_{\text{on}})\) of dust formation

  - The approximation formulae for \(f_{\text{con},\infty}\) and \(a_{\text{ave},\infty}\) are given as a function of \(\Lambda_{\text{on}}\)