

Dust formation theory in astronomical environments

Takaya Nozawa

(National Astronomical Observatory of Japan)

Contents:

1. Introduction
2. Classical (kinetic) nucleation
3. Chemical (molecular) nucleation
4. Implications

1-1. Origin of cosmic dust grains

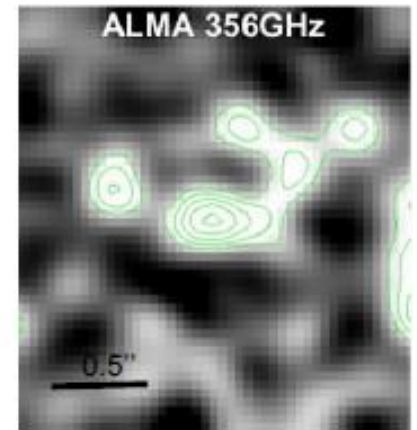
○ Cosmic dust grains

- universally exist in the universe
- cause various astrophysical phenomena
extinction, scattering, polarization, IR emission,
molecular formation on the surface, formation of planets ...

The origin of dust remains to be clarified !!

○ Massive dust at $z > 5$ discovered

in the early universe, CCSNe must play dominant roles in the enrichment of ISM with dust



$\sim 10^6 M_{\text{sun}}$ of dust in
a galaxy at $z = 8.4$

Laporte+2017

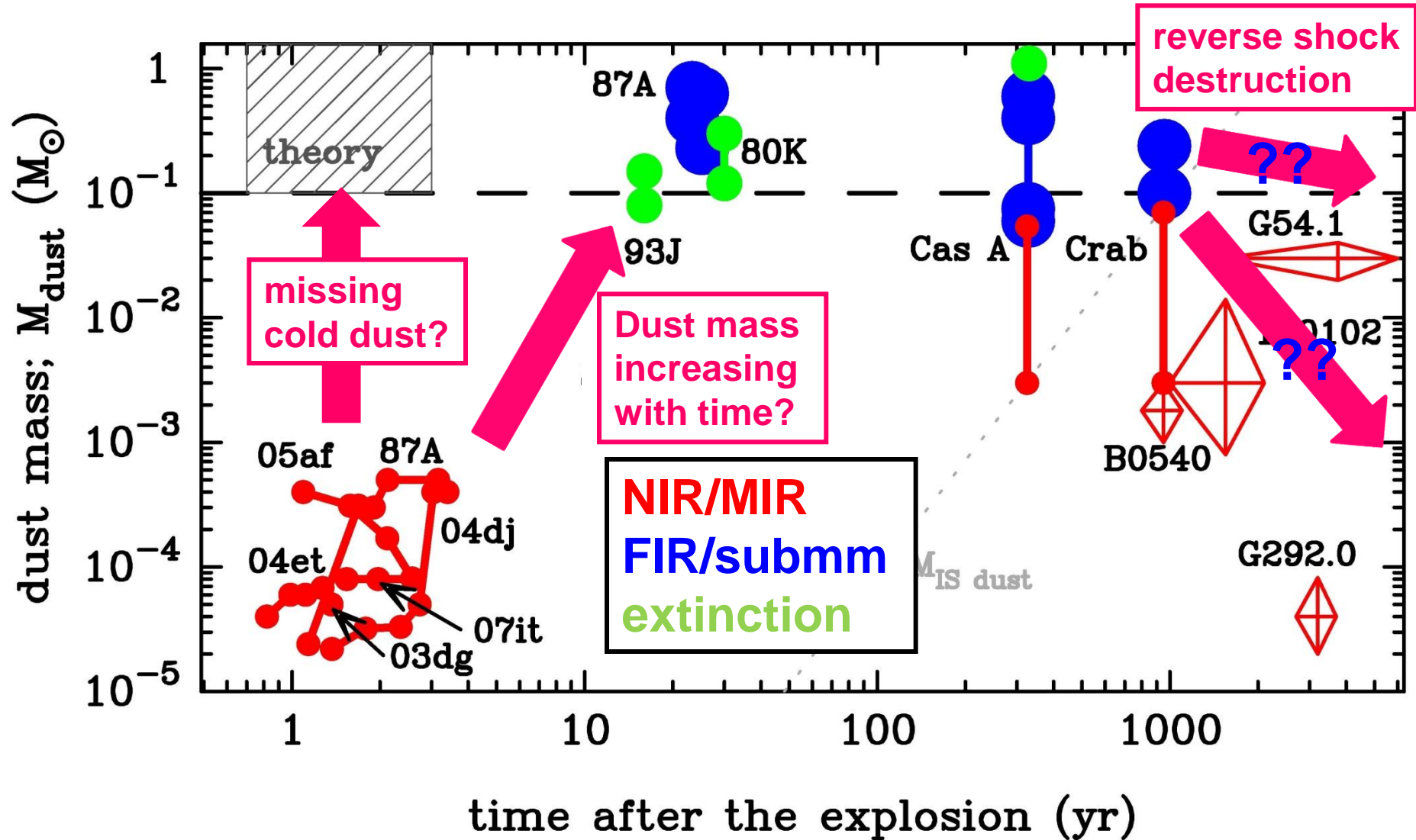
1-2. Key questions for dust formation

1. How much dust grains form?

2. What is the size distribution of dust?

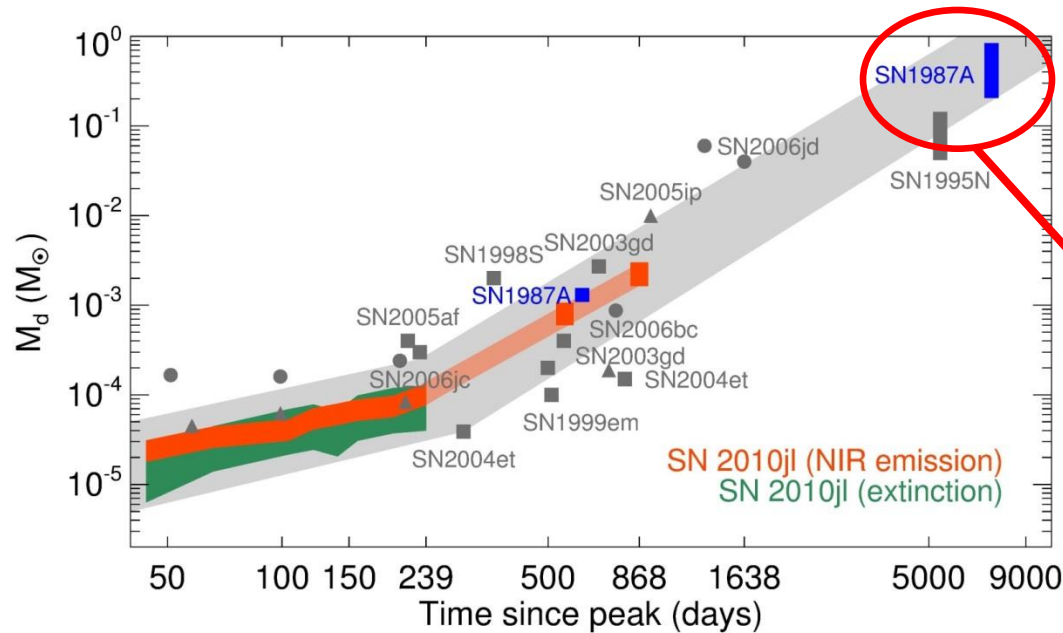
3. When do the majority of grains form?

1-3. Observed dust mass in CC-SNe/SNRs



Dust mass formed in the ejecta is dominated by cold dust

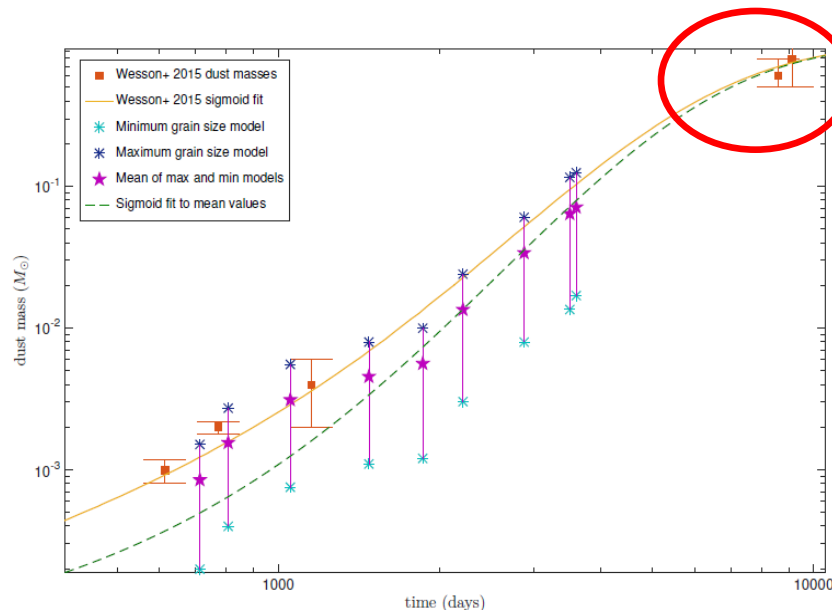
1-4. Dust mass increases with time?



Dust mass estimated from
IR emission

Gall et al. (2014, Nature)

Most of dust grains
($> 0.1 M_{\text{sun}}$) form at
~20 yr post-explosion



Evolution of dust mass in SN
1987A derived from extinction
of optical emission lines

Bevan & Barlow (2016)

1-5. Key questions for dust formation

1. How much dust grains form?

- FIR/submm obs. → 0.1-1 M_{sun}
- theoretical works → 0.1-1 M_{sun}

2. What is the size distribution of dust?

- what fraction of dust is destroyed by RS?
- smaller grains are destroyed more efficiently

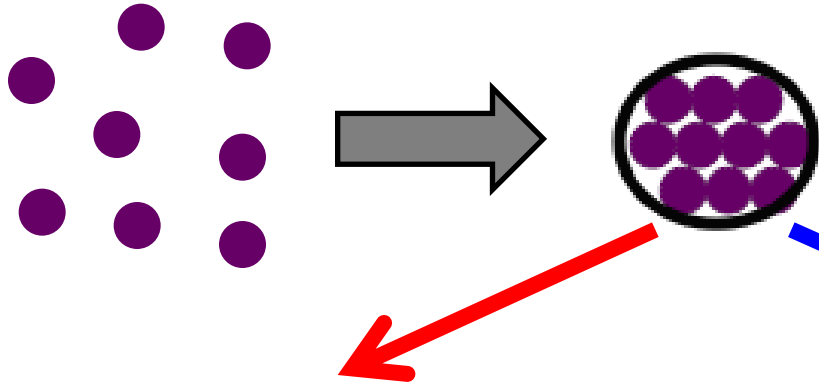
3. When do the majority of grains form?

- obs. → ~20 yr (dust mass gradually increases with time)
- theory → ~1-2 yr (within 5 yr)

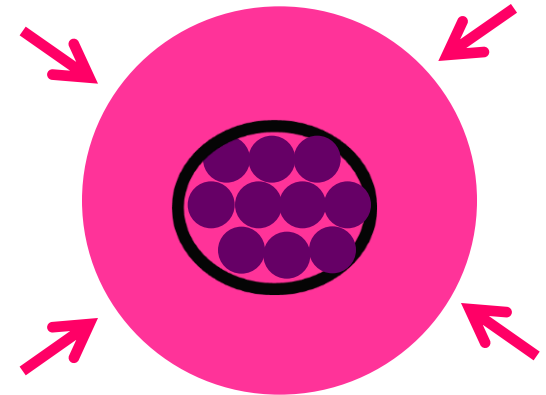
2-0. Two approaches for nucleation

1. Nucleation

(formation of stable clusters)



2. Grain growth



○ Classical nucleation (kinetic nucleation)

(Kozasa+1989, 1991, Yamamoto+2001, Todini & Ferrara 2001, Nozawa+2003, 2008, 2010, Schneider & Bianchi 2007, Falles+2011, Keith & Lazzati 2011, Schneider+2012, Nozawa & Kozasa 2013, Lazzati & Heger 2016, Marassi+2015, Bocchio+2016)

○ Chemical nucleation (molecular nucleation)

(Clayton+1999, 2001, Deneault+2003, Cherchneff & Dwek 2009, 2010, Sarangi & Cherchneff 2013, 2015, Biscaro & Cherchneff 2014, 2016, Sluder+2016)

2-1. Phase transition

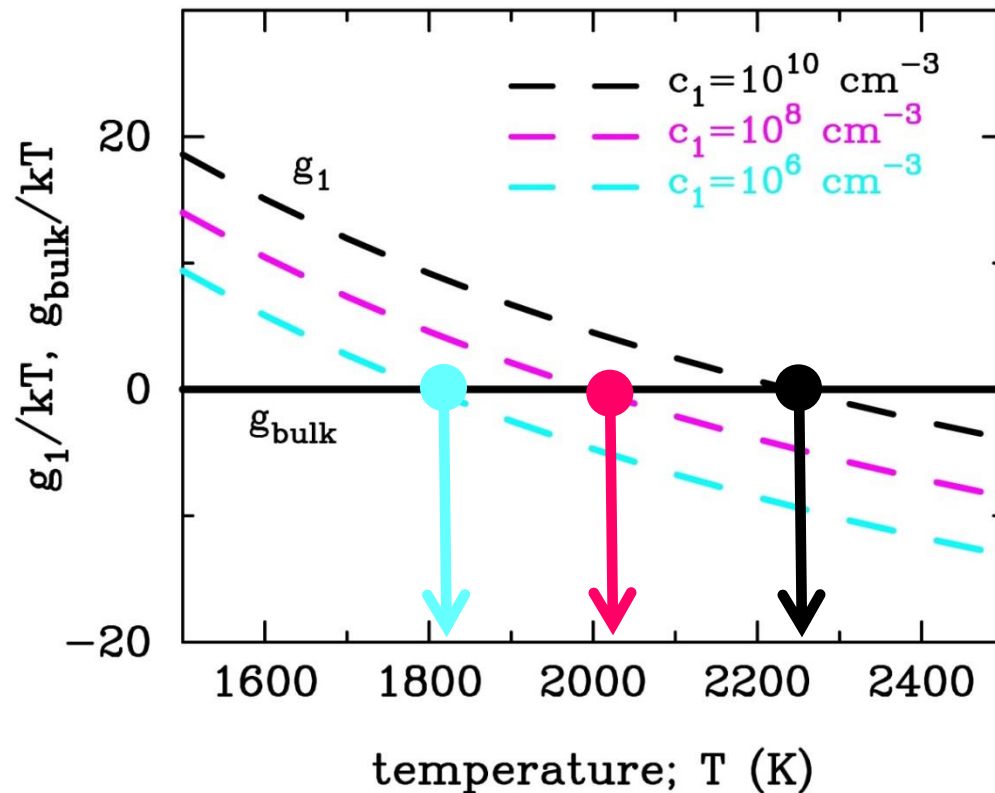
○ Gibbs free energy

gas

$$G_1 = n g_1(T) = n g_1^0(T) + nkT \ln \left(\frac{P_1}{P^0} \right)$$

solid

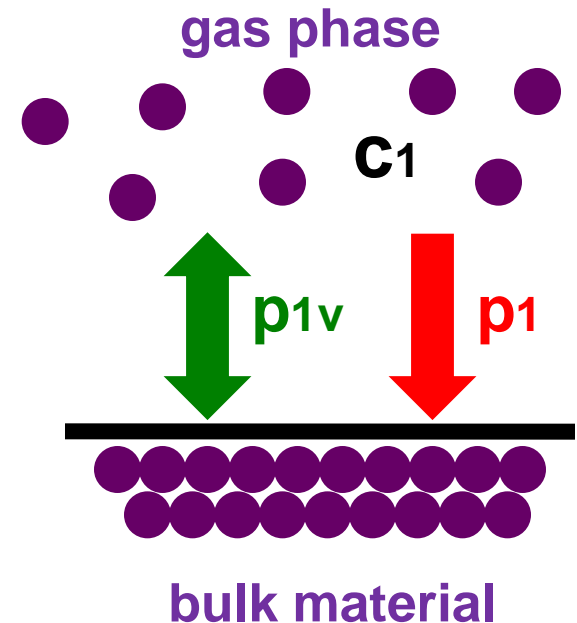
$$G_{\text{bulk}} = n g_{\text{bulk}}(T) = n g_{\text{bulk}}^0(T)$$



○ Supersaturation ratio, S

$$\begin{aligned} \ln S &= (g_1 - g_{\text{bulk}}) / kT \\ &= -\frac{1}{kT} (g_{\text{bulk}}^0 - g_1^0) + \ln \left(\frac{P_1}{P^0} \right) \\ &= \ln \left(\frac{P_1}{P_{1,v}} \right) \end{aligned}$$

solid stable for $\ln S > 0$ ($S > 1$)



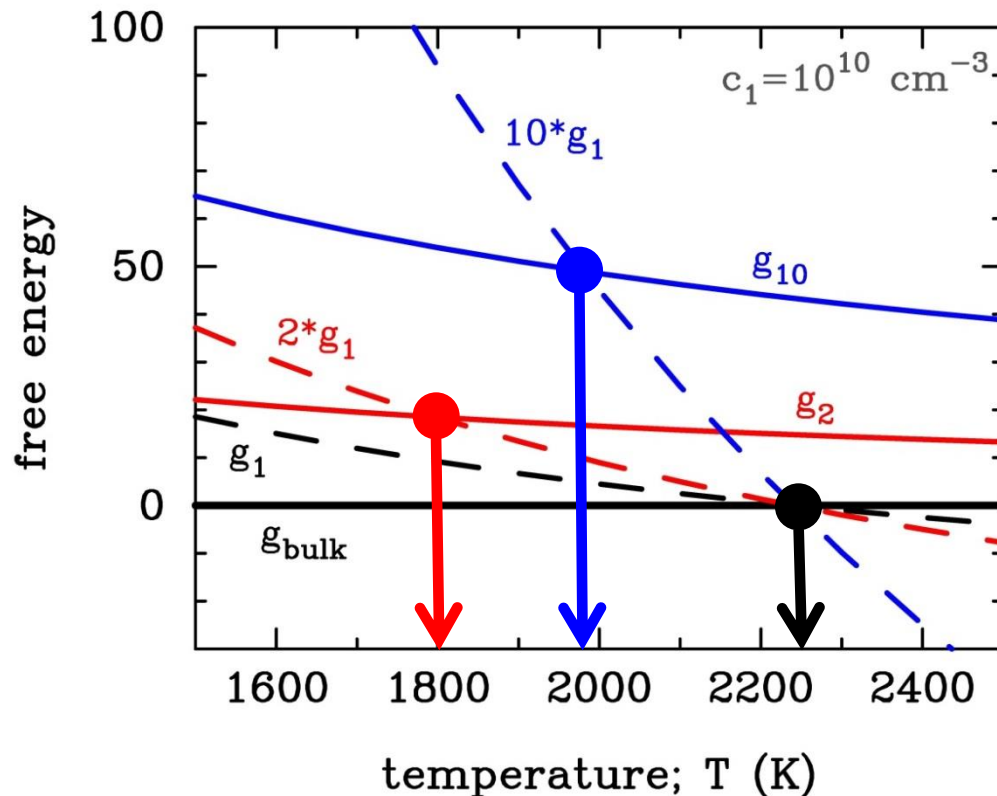
2-2. Formation of clusters

○ Free energy for clusters

$$G_n = n g_n(T) = n g_{\text{bulk}}^0(T) + E_{\text{bar}}$$

capillary approximation

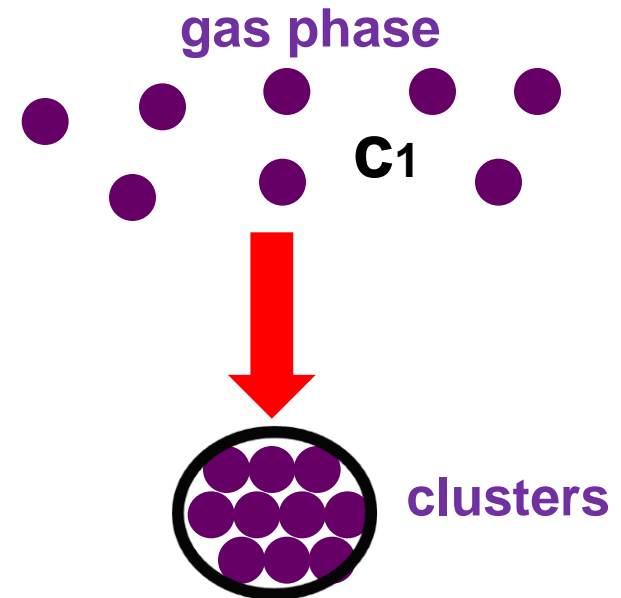
$$G_n = n g_{\text{bulk}}^0(T) + 4\pi a_0^2 n^{\frac{2}{3}} \sigma$$



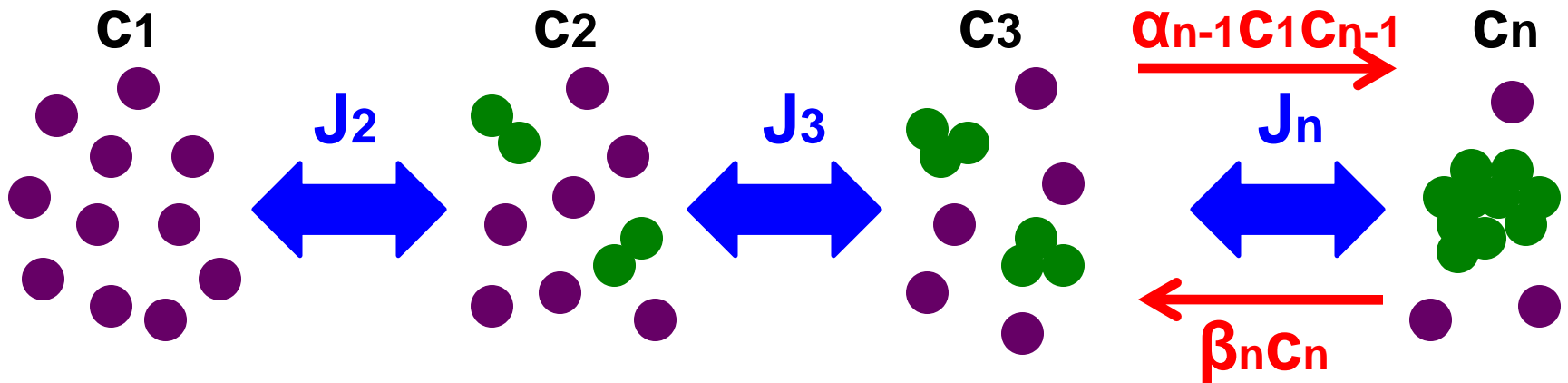
○ Energy involved in cluster formation

$$\begin{aligned} W_n &= (G_1 - G_n)/kT \\ &= n \ln S - E_{\text{surf},n}/kT \end{aligned}$$

where $E_{\text{surf},n} = 4\pi a_0^2 n^{\frac{2}{3}} \sigma$



2-3. Kinetic nucleation



○ Master equations

$$\frac{dc_n}{dt} = J_n(t) - J_{n+1}(t) \quad \text{for } 2 \leq n \leq n_*,$$

$$J_n(t) = \alpha_{n-1}c_1 [c_{n-1} - c_n \exp(\gamma_n)]$$

$$\alpha_n = \underbrace{\eta_s}_{\text{sticking coefficient}} 4\pi a_0^2 n^{\frac{2}{3}} \left(\frac{kT}{2\pi m_n} \right)^{\frac{1}{2}}$$

sticking coefficient, $\eta_s = 1$

$$\gamma_n = \underbrace{\frac{1}{kT} (E_{\text{surf},n} - E_{\text{surf},n-1})}_{\text{energy barrier}} - \ln S$$

energy barrier : ~ 10

2-4. Classical nucleation

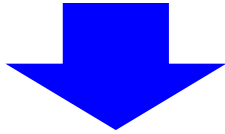
Steady-state nucleation rate

(assuming $J_s = J_2 = J_3 = \dots = J_n$)

$$J_s \propto \exp(-W_n)$$

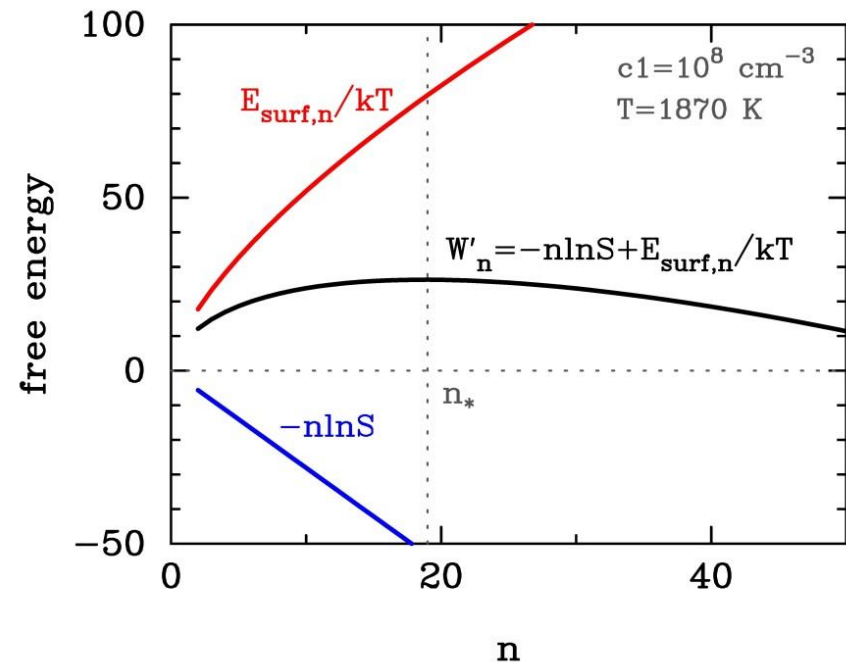
$$W'_n = -W_n = -n \ln S + \frac{4\pi a_0^2 \sigma}{kT} n^{\frac{2}{3}}$$

$$n_{\text{crit}} \simeq \left(\frac{8\pi a_0^2 \sigma}{3kT \ln S} \right)^3$$



$$J_s(t) = \eta_s \Omega_0 \left(\frac{2\sigma}{\pi m_1} \right)^{\frac{1}{2}} c_1^2(t) \exp \left[-\frac{4}{27(\ln S)^2} \left(\frac{4\pi a_0^2 \sigma}{kT} \right)^3 \right]$$

formation rate of stable clusters



2-5. Basic equations for dust formation

1. Nucleation

▪ Kinetic

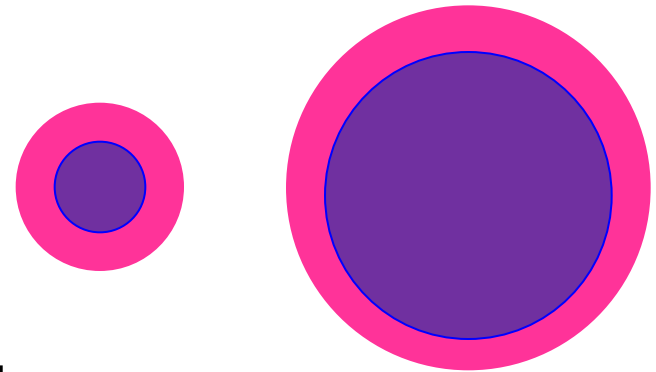
$$\frac{dc_n}{dt} = J_n(t) - J_{n+1}(t) \quad \text{for } 2 \leq n \leq n_*,$$

▪ Classical

$$J_s(t) = \eta_s \Omega_0 \left(\frac{2\sigma}{\pi m_1} \right)^{\frac{1}{2}} c_1^2(t) \exp \left[-\frac{4}{27(\ln S)^2} \left(\frac{4\pi a_0^2 \sigma}{kT} \right)^3 \right]$$

2. Grain growth

$$\frac{da}{dt} = \eta_g \Omega_0 \left(\frac{kT}{2\pi m_1} \right)^{\frac{1}{2}} c_1(t)$$



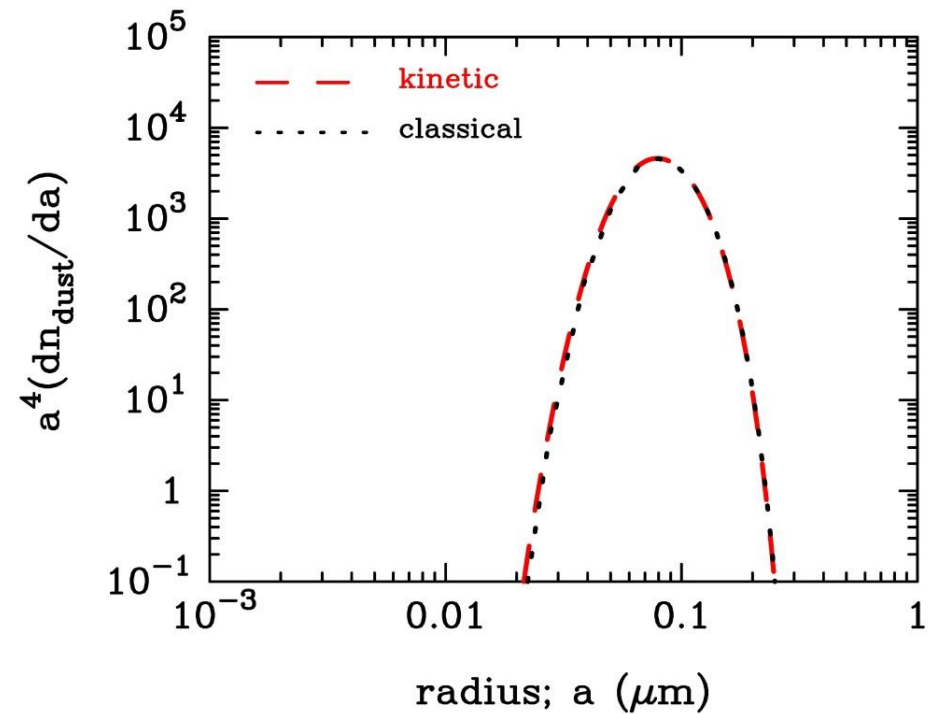
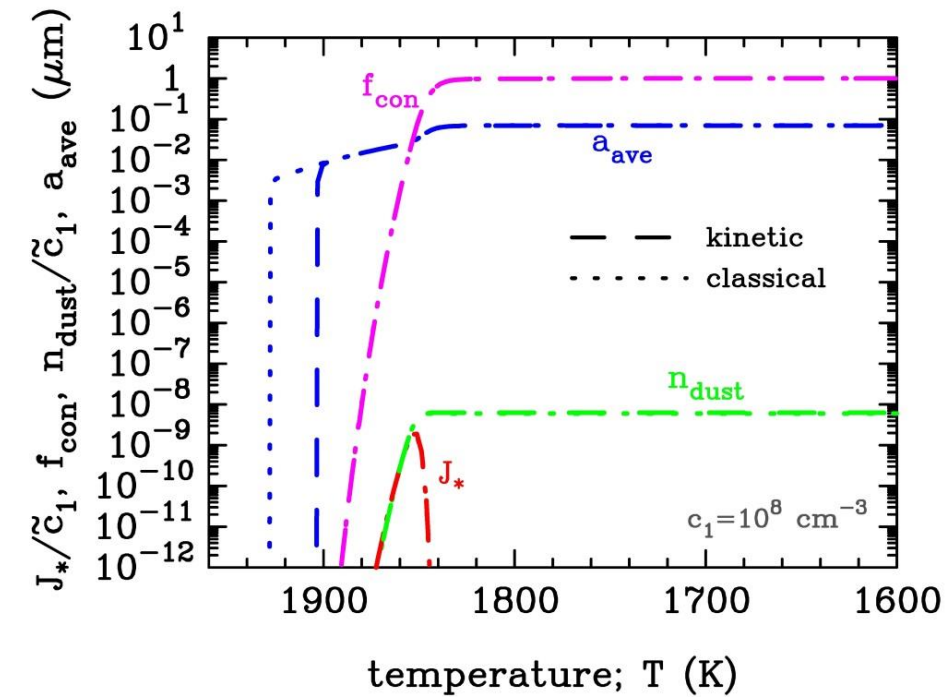
▪ free expansion

$$\tilde{c}(t) = c_0 \left(\frac{t}{t_0} \right)^{-3},$$

▪ gas cooling

$$T(t) = T_0 \left(\frac{t}{t_0} \right)^{-3(\gamma-1)}, \quad (\gamma=1.25)$$

2-6. Kinetic vs. Classical



▪ Kinetic nucleation

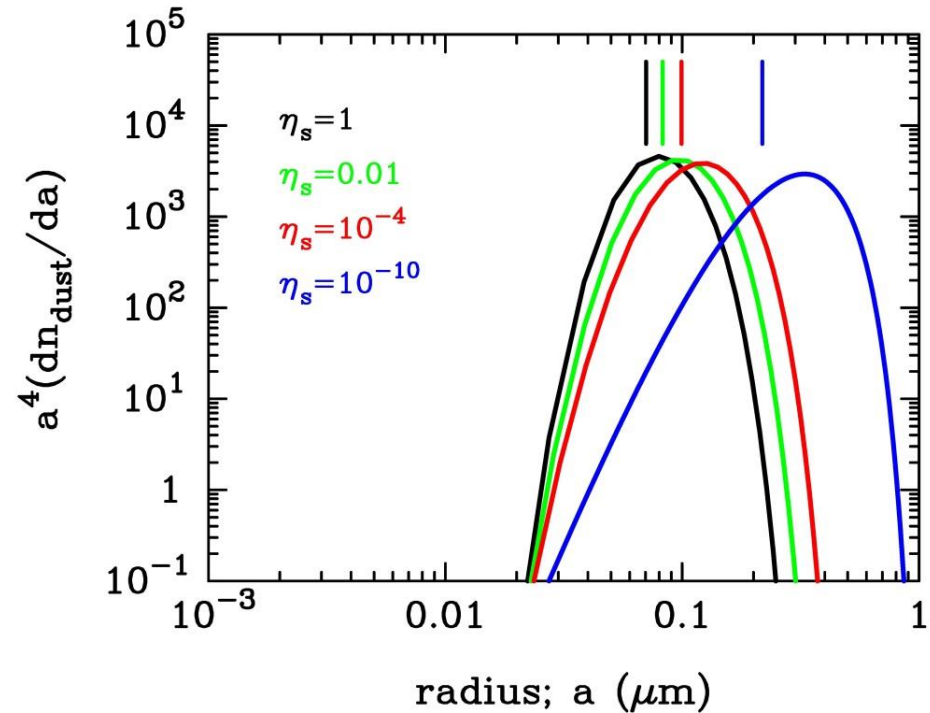
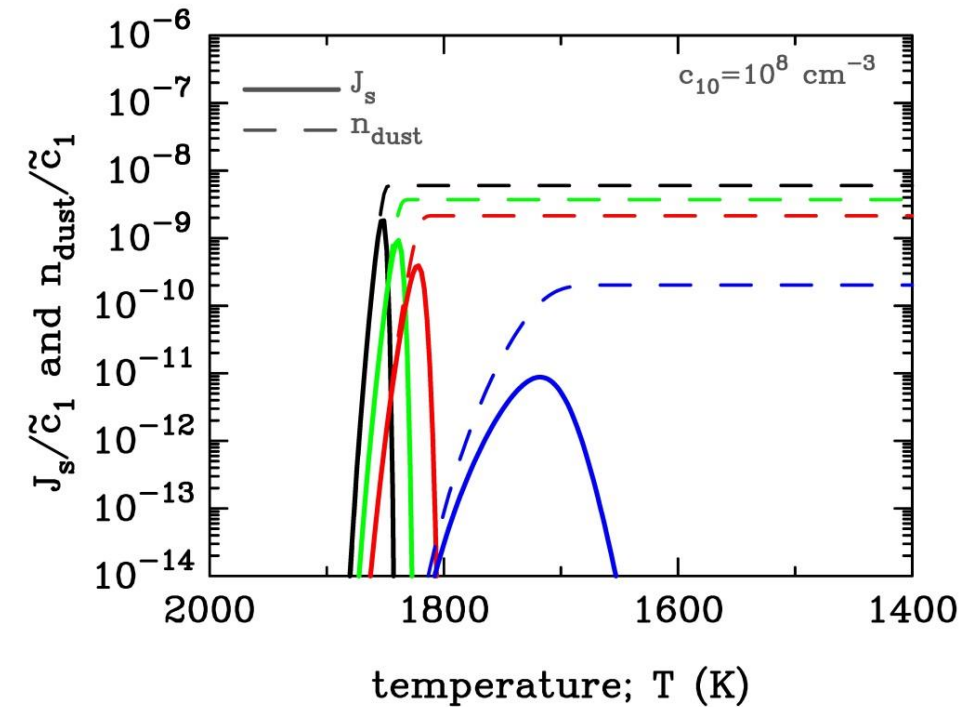
$$\frac{dc_n}{dt} = J_n(t) - J_{n+1}(t) \quad \text{for } 2 \leq n \leq n_*, \quad (n_*=100)$$

▪ Classical nucleation

$$\boxed{\eta_s = 1} \leftarrow J_s(t) = \eta_s \Omega_0 \left(\frac{2\sigma}{\pi m_1} \right)^{\frac{1}{2}} c_1^2(t) \exp \left[-\frac{4}{27(\ln S)^2} \left(\frac{4\pi a_0^2 \sigma}{kT} \right)^3 \right]$$

2-7. Dependence on sticking coefficient

$$J_s(t) = \eta_s \Omega_0 \left(\frac{2\sigma}{\pi m_1} \right)^{\frac{1}{2}} c_1^2(t) \exp \left[-\frac{4}{27(\ln S)^2} \left(\frac{4\pi a_0^2 \sigma}{kT} \right)^3 \right]$$

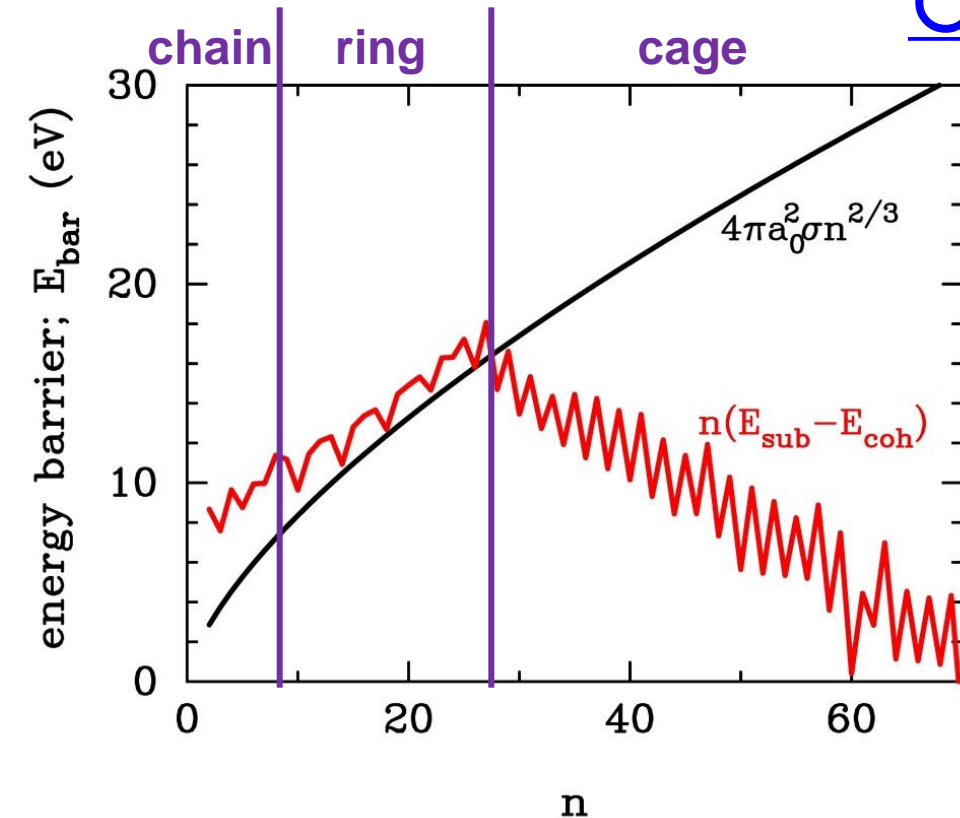


$$f_{\text{con}} M_C = n_{\text{dust}} \frac{4\pi}{3} \delta_C a^3$$

$$a \propto n_{\text{dust}}^{-\frac{1}{3}}$$

2-8. Energy barrier of clusters

○ Free energy for clusters



$$G_n = n g_{\text{bulk}}^0(T) + E_{\text{bar}}$$

▪ Classical nucleation

$$E_{\text{bar}} = E_{\text{surf},n} = 4\pi a_0^2 \sigma n^{2/3}$$

▪ Density fluctuation theory

$$E_{\text{bar}} = E_{\text{bind},n} = n(E_{\text{sub}} - E_{\text{coh}})$$

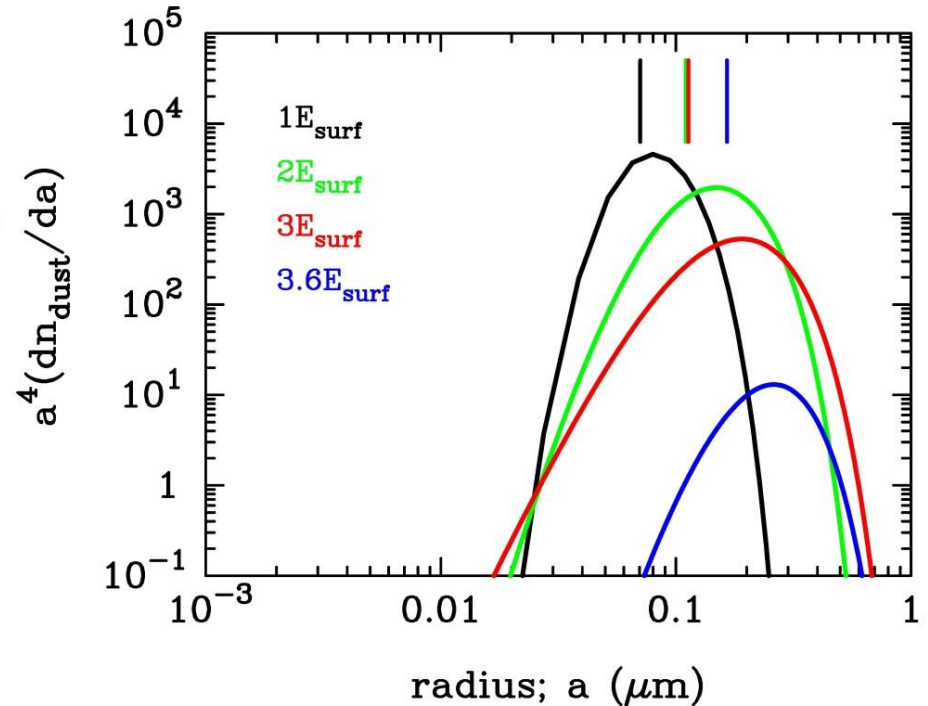
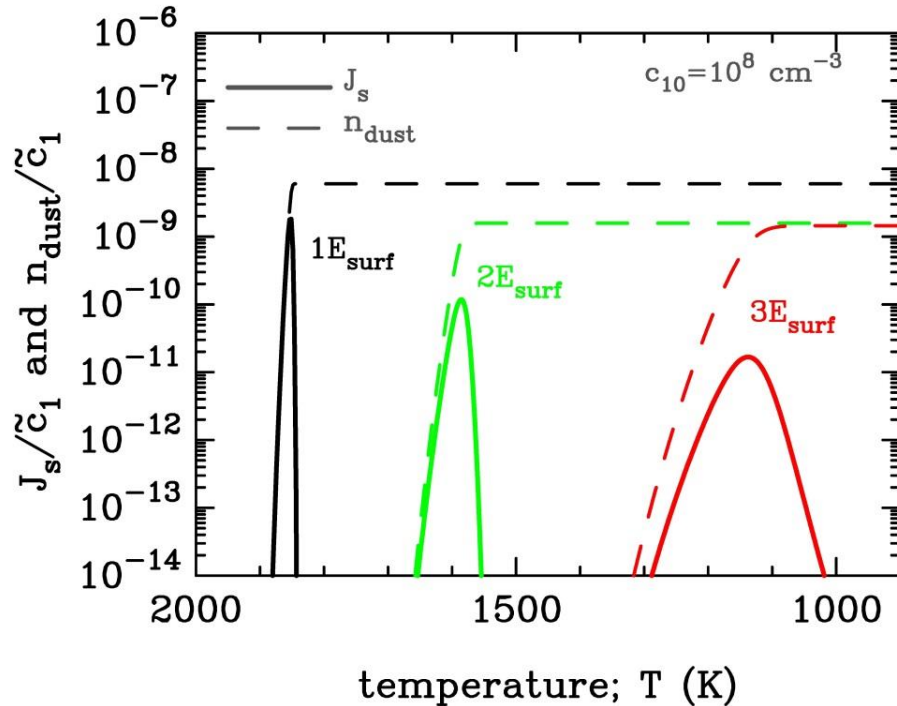
Mauney+2015

▪ Kinetic nucleation

$$\frac{dc_n}{dt} = J_n(t) - J_{n+1}(t) \quad \text{for } 2 \leq n \leq n_*,$$

2-9. Dependence on surface energy

300 day \longrightarrow 700 day



- Increasing the surface energy can lead to lower condensation temperature, retarding the formation of dust
- Dependence of sticking coefficient and energy barrier on the resulting size of dust is not strong

3-1. Chemical approach

○ Master equations

$$\frac{dc_n}{dt} = \sum_i \sum_j k_{i,j} c_i c_j - \sum_m k_{m,n} c_m c_n$$

○ Arrhenius expression

$$k_{i,j} = A_{i,j} \left(\frac{T}{300 \text{ K}} \right)^\nu \exp \left(-\frac{E_{\text{act}}}{kT} \right)$$

Empirical (fitting) formula to account for reaction rates obtained from experiments

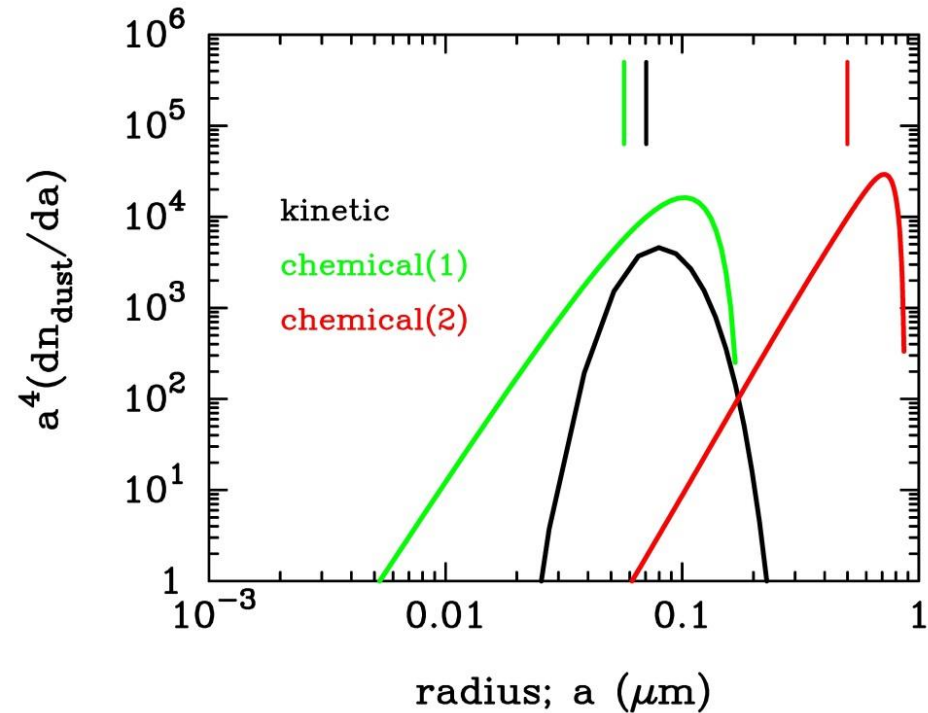
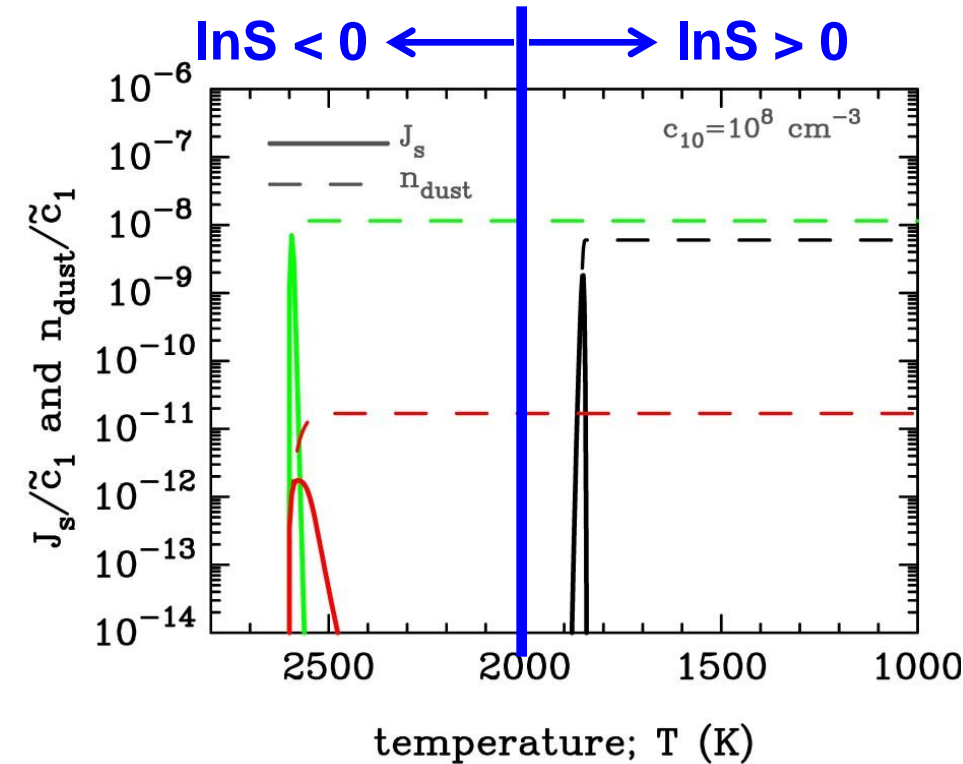
3-2. Reaction rates for carbon clusters

$$k_{i,j} = A_{i,j} \left(\frac{T}{300 \text{ K}} \right)^{\nu} \exp \left(-\frac{E_{\text{act}}}{kT} \right)$$

Carbon				C			
C1	C + C	→	C ₂ + hν	4.36 × 10 ⁻¹⁸	0.35	161.3	Andreazza & Singh 1997
C2	C + C ₂	→	C ₃ + hν	1.00 × 10 ⁻¹⁷	0	0	Clayton et al. 1999
C3	C + C ₃	→	C ₄ + hν	1.00 × 10 ⁻¹⁰	0	0	Clayton et al. 1999
C4	C + C ₄	→	C ₅ + hν	1.00 × 10 ⁻¹³	0	0	Clayton et al. 1999
C5	C + C ₄	→	C ₂ + C ₃	1.00 × 10 ⁻¹⁰	0	0	Clayton et al. 1999
C6	C + C ₅	→	C ₆ + hν	1.00 × 10 ⁻¹⁰	0	0	Clayton et al. 1999
C7	C + C ₆	→	C ₇ + hν	1.00 × 10 ⁻¹³	0	0	Clayton et al. 1999
C8	C + C ₆	→	C ₂ + C ₅	1.00 × 10 ⁻¹⁰	0	0	Clayton et al. 1999
C9	C + C ₆	→	C ₃ + C ₄	1.00 × 10 ⁻¹⁰	0	0	Clayton et al. 1999
C10	C + C ₇	→	C ₈ + hν	1.00 × 10 ⁻¹⁰	0	0	Clayton et al. 1999
C11	C + C ₈	→	C ₉ + hν	1.00 × 10 ⁻¹³	0	0	Clayton et al. 1999
C12	C + C ₈	→	C ₂ + C ₇	1.00 × 10 ⁻¹⁰	0	0	Clayton et al. 1999
C13	C + C ₈	→	C ₃ + C ₆	1.00 × 10 ⁻¹⁰	0	0	Clayton et al. 1999
C14	C + C ₈	→	C ₄ + C ₅	1.00 × 10 ⁻¹⁰	0	0	Clayton et al. 1999
C15	C + C ₉	→	C ₁₀ + hν	1.00 × 10 ⁻¹⁰	0	0	Clayton et al. 1999

Cherchneff & Dwek (2010)

3-3. Chemical vs. Kinetic (Classical)



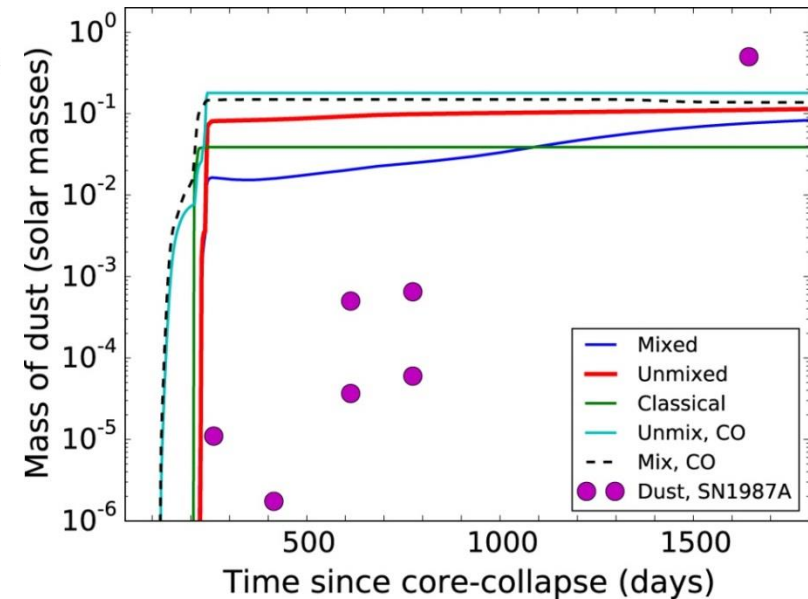
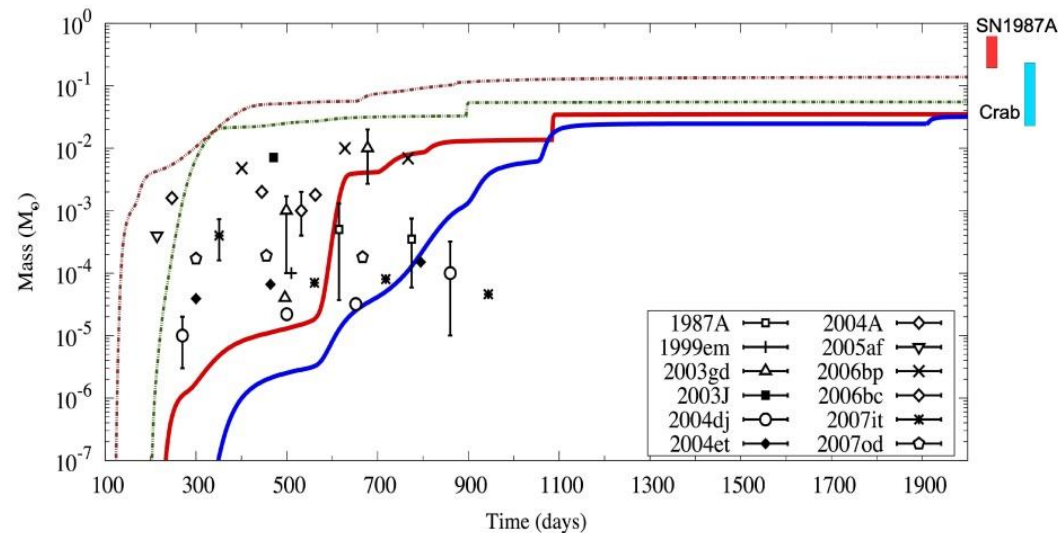
green line: only for forward reactions

$$\frac{dc_n}{dt} = k_{i,j} c_i c_j$$

red line: destruction reaction $\text{C}_2 + \text{O} \implies \text{CO} + \text{C}$

If chemical reaction rates and paths are not accurate, the chemical approach may produce erroneous results

4-1. Mass evolution of newly formed dust

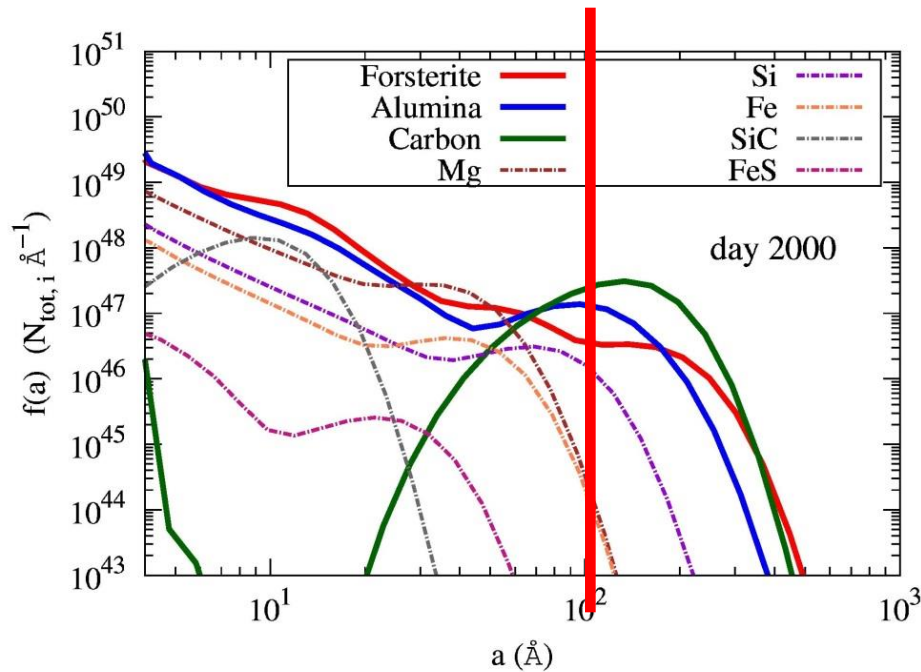


Molecules: chemical
Clusters: chemical
Grains: coagulation
Sarangi & Cherchneff (2015)

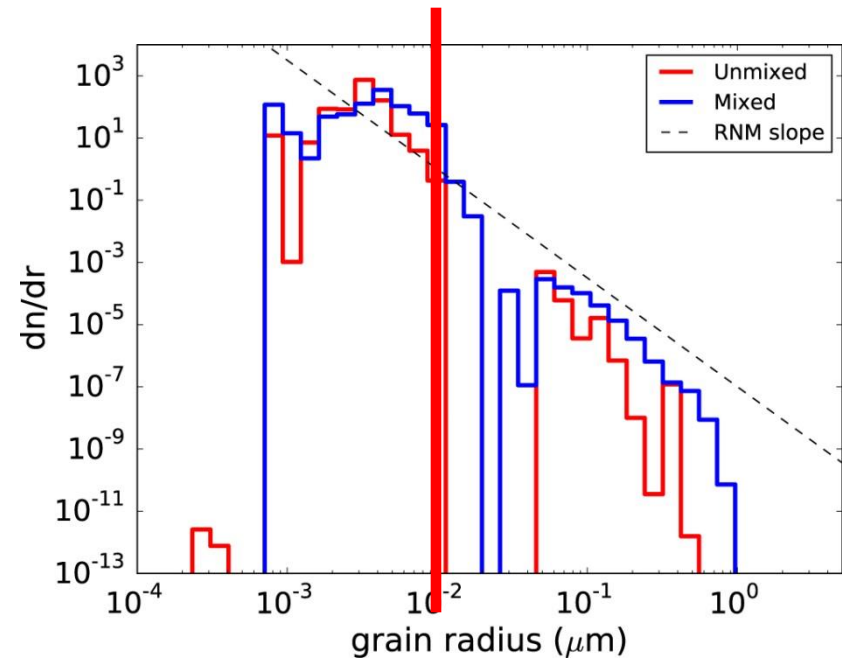
Molecules: chemical
Clusters: kinetic
Grains: accretion
Lazzati & Heger (2016)

Both nucleation approaches predict that $\sim 0.1 M_{\text{sun}}$ of dust condenses a few years after the explosion

4-2. Size distribution of newly formed dust



Molecules: chemical
Clusters: chemical
Grains: coagulation
Sarangi & Cherchneff (2015)



Molecules: chemical
Clusters: kinetic
Grains: accretion
Lazzati & Heger (2016)

The radii of newly formed dust ranges from $\sim 0.001 \mu\text{m}$ to $\sim 1 \mu\text{m}$ though there are some subtle differences

4-3. Timescale of grain growth

$$\tau_{\text{grow}}^{-1} = \frac{1}{a} \left(\frac{da}{dt} \right) = \left(\frac{1}{a} \right) \eta_g \Omega_0 c_1 \left(\frac{kT}{2\pi m_1} \right)^{\frac{1}{2}}$$



$$\tau_{\text{grow}} \simeq 50 \left(\frac{\eta_g}{1.0} \right)^{-1} \left(\frac{a}{0.01 \mu\text{m}} \right) \left(\frac{T}{50 \text{ K}} \right)^{-\frac{1}{2}} \left(\frac{M_C}{0.01 M_{\odot}} \right)^{-1} \\ \times \left(\frac{V_{\text{core}}}{10^3 \text{ km s}^{-1}} \right)^3 \left(\frac{t}{20 \text{ yr}} \right)^3 \left(\frac{f_{\text{density}}}{10} \right)^{-1} \text{ yr}$$

At 20 yr, the gas density is too low to form dust grains in the freely expanding ejecta

5. Summary of this talk

1) Classical (kinetic) nucleation

- well established on the basis of thermodynamics
- still one of the good tools to describe dust formation

2) Chemical (molecular) nucleation

- consistently follows the formation of a variety of molecules and small clusters, given that reaction paths and rates are well known

3) Both nucleation approaches predict

- the formation of dust in the ejecta within 5 yr
(seems hard to form massive dust at ~20 yr)
- dust size range of 0.001-1 μm