CAN THE GROWTH OF DUST GRAINS IN LOW-METALLICITY STAR-FORMING CLOUDS AFFECT THE FORMATION OF METAL-POOR LOW-MASS STARS?

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ABSTRACT

The discovery of a low-mass star with such a low metallicity as $\leqslant 4.5 \times 10^{-5} Z_\odot$ reveals the critical role of dust in the formation of extremely metal-poor stars. In this Letter, we explore the effect of the growth of dust grains through accretion of gaseous refractory elements in very low metallicity pre-stellar cores on cloud fragmentation induced by dust emission cooling. Employing a simple model of grain growth in a gravitationally collapsing gas, we show that Fe and Si grains can grow efficiently at hydrogen densities of $\simeq 10^{10}$–10^{14} cm^{-3} in the clouds with metal abundances of $-5 \lesssim$ [Fe, Si/H] $\lesssim -3$. The critical metal number abundances, above which the grain growth could induce the fragmentation of the gas clouds, are estimated to be $A_{crit} \simeq 10^{-9}$–10^{-8}, unless the initial grain radius is too large ($\gtrsim 1 \mu m$) or the sticking probability is too small ($\lesssim 0.01$). We find that even if the initial dust-to-gas mass ratio is well below the minimum value required for the dust-induced fragmentation, the grain growth increases the dust mass high enough to cause the gas to fragment into sub-solar mass clumps. We suggest that as long as the critical metal abundance is satisfied, grain growth could play an important role in the formation of low-mass stars with metallicity as low as $10^{-5} Z_\odot$.

Key words: dust, extinction – ISM: clouds – stars: formation – stars: low-mass – stars: Population II – supernovae: general

Online-only material: color figures

1. INTRODUCTION

Dust grains in the early universe are considered to be important agents in triggering the formation of low-mass stars in metal-poor environments (Omukai 2000; Schneider et al. 2003); the cooling of the gas through the thermal emission of dust makes collapsing dense cores gravitationally unstable, leading to fragmentation into multiple sub-solar mass clumps at gas densities of 10^{12}–10^{14} cm^{-3} (Tsuribe & Omukai 2006; Dopcke et al. 2011). This scenario has been recently supported by the discovery of a Galactic low-mass star, SDSS J102915+172927 (Caffau et al. 2011), whose metal content is too low ($Z \leqslant 4.5 \times 10^{-5} Z_\odot$) to induce the fragmentation of star-forming clouds by metal-line cooling (see Klessen et al. 2012; Schneider et al. 2012b for details).

The condition that allows dust-induced fragmentation to be realized depends on the amount of dust grains as well as their size distribution in pre-stellar clouds (Omukai et al. 2005; Schneider et al. 2006, 2012a). Schneider et al. (2012a) found that the formation condition of the low-mass fragments obtained from numerical simulations is fully described in terms of the product of the dust-to-gas mass ratio $\mathcal{D}$ and geometrical cross section per unit dust mass $\mathcal{S}$ as follows:

$$\mathcal{S}\mathcal{D} > 1.4 \times 10^{-3} \text{ cm}^2 \text{ g}^{-1} \left(\frac{T_{gas}}{10^3 \text{ K}} \right)^{-\frac{1}{2}} \left(\frac{c_H}{10^{12} \text{ cm}^{-3}} \right)^{-\frac{1}{2}}, \quad (1)$$

where T_{gas} is the temperature of the gas and c_H is the hydrogen number density. Treating self-consistently the dust formation in the ejecta of supernovae (SNe) and the subsequent destruction of the dust by the reverse shocks, Schneider et al. (2012a) argued that the condition could not be satisfied in the collapsing star-forming clouds enriched with metals and dust from the first SNe

if the majority of grains formed in the SN ejecta are destroyed by the reverse shock (see also Schneider et al. 2012b). However, it could be possible that the accretion of gaseous refractory elements released by dust grains in the shocked gas onto the surfaces of the SN dust surviving in star-forming clouds changes the mass and size distribution of the dust, and thus affects the thermal evolution of the collapsing cores.[3]

In this Letter, we investigate the feasibility of grain growth in low-metallicity star-forming clouds to explore whether the grain growth can facilitate the formation of metal-poor low-mass stars. In Section 2, we describe the model of grain growth in collapsing dense clouds, and present the results of the calculations in Section 3. In Section 4, we estimate the critical metal abundances above which the grain growth could encourage the gas fragmentation into sub-solar mass clumps, and discuss the corresponding dust-to-gas mass ratio and total metallicity. Our conclusion is given in Section 5.

2. MODEL OF GRAIN GROWTH IN METAL-POOR STAR-FORMING CLOUDS

We consider the growth of dust grains in collapsing clouds that have been enriched with metals and dust grains produced by very early generation of SNe. Dust formation calculations by Nozawa et al. (2003) showed that various grain species condense in the unmixed ejecta of Population III SNe, and that Fe, Si, and C grains have relatively large average radii ($\gtrsim 0.01 \mu m$). Based on their dust models, Nozawa et al. (2007) investigated the evolution of dust in the hot gas swept up by the SN shocks and

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[3] Hirashita & Omukai (2009) examined the coagulation of dust in collapsing clouds with a variety of metallicity. They found that the dust coagulation can proceed even at metallicity as low as $10^{-6} Z_\odot$ but it does not have any impact on the thermal evolution of the star-forming clouds.

Table 1
Numerical Values Used in the Calculations

Numerical Values	Explanation of Symbols
$s = 1$	Sticking probability
$T_{gas} = 10^3$ K	Gas temperature
$\mu = 2.18$	Mean molecular weight
$a_{Fe,0} = 1.441$ Å	Radius of an Fe atom in the solid phase[a]
$a_{Si,0} = 1.684$ Å	Radius of an Si atom in the solid phase[a]
$m_{Fe} = 56 m_H$	Mass of an Fe atom
$m_{Si} = 28 m_H$	Mass of an Si atom
$A_{Fe,\odot} = 3.26 \times 10^{-5}$	Solar abundance of Fe relative to H[b]
$A_{Si,\odot} = 3.58 \times 10^{-5}$	Solar abundance of Si relative to H[b]
$\rho_{Fe} = 7.90$ g cm^{-3}	Bulk density of Fe[c]
$\rho_{Si} = 2.32$ g cm^{-3}	Bulk density of Si[c]

Notes.
[a] Nozawa et al. (2003).
[b] Anders & Grevesse (1989).
[c] Nozawa et al. (2006).

found that most of such large Fe, Si, and C grains can survive the destruction by the reverse shock to be predominantly injected into the early interstellar medium.

Being motivated by these studies, and to simplify the chemistry, we consider that Fe and Si grains composed of only one element grow through accretion of Fe and Si atoms in the gas phase, respectively.[4] We assume that dust grains are spheres, all of which have a single initial radius, although they might be expected to have the distribution of grain sizes. Suppose that the number density of a given refractory element i at a time t_0 is $c_{i,0} = c_i(t_0) = A_i c_{H,0}$, where A_i is the number abundance of the element i relative to hydrogen. In order to specify the number abundance of pre-existing seed grains, we introduce a parameter $f_{i,0}$ (>0), which is defined as the number fraction of the elements i originally locked in dust grains (i.e., condensation efficiency at t_0). Then, the number density of the dust whose initial radius is $r_{i,0}$ is described as $n_{i,0}^{dust} = f_{i,0} c_{i,0} (a_{i,0}/r_{i,0})^3$, where $a_{i,0}$ is the hypothetical radius of an atom in the dust phase.

The time evolution of the number density $c_i(t)$ of an element i in the gas clouds collapsing with the timescale of free fall is given by

$$c_i(t) = c_{i,0} \left(1 - \frac{t}{2\tau_0^{ff}}\right)^{-2}, \quad (2)$$

where t is the elapsed time from t_0, $\tau_0^{ff} = (3\pi/32 G \mu m_H c_{H,0})^{1/2}$ is the free-fall time at the density $c_{H,0}$ with the gravitational constant G, the mean molecular weight μ, and the mass of a hydrogen atom m_H. Once grain growth is activated, the gaseous atoms are consumed, and the number density $c_i^{gas}(t)$ of an element i in the gas phase at the time t can be written as

$$c_i^{gas}(t) = c_i(t)\{1 - f_{i,0}[r_i(t)/r_{i,0}]^3\}, \quad (3)$$

with $r_i(t)$ being the radius of the ith grain species at t. Equation (3) is reduced to

$$f_i(t) = 1 - Y_i(t) = f_{i,0} X_i^3(t), \quad (4)$$

where $f_i(t)$ is the condensation efficiency at t, $Y_i(t) = c_i^{gas}(t)/c_i(t)$ represents the depletion of the gaseous atoms due to grain growth, and $X_i(t) = r_i(t)/r_{i,0}$.

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[4] The growth of C grains may not be expected in dense clouds. This is because at the high gas densities considered in this Letter ($c_H \gtrsim 10^8$ cm^{-3}), all C atoms can be bounded in CO molecules in gas clouds with the metallicity scaled by the solar abundance (Omukai et al. 2005).

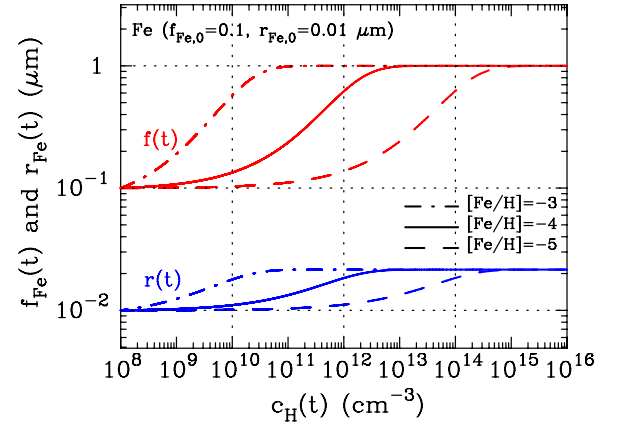**Figure 1.** Time evolutions of the condensation efficiency $f_{Fe}(t)$ (red) and grain radius $r_{Fe}(t)$ (blue) through the growth of Fe grains with the initial dust abundance $f_{Fe,0} = 0.1$ and the initial grain radius $r_{Fe,0} = 0.01$ μm as a function of hydrogen number density $c_H(t)$. Dot-dashed, solid, and dashed lines depict the results for the collapsing clouds with the Fe abundances of [Fe/H] = −3, −4, and −5, respectively.
(A color version of this figure is available in the online journal.)

In the dense clouds where almost all gaseous atoms are neutral, the growth rate of grain radius is given by

$$\frac{dr_i}{dt} = s_i \left(\frac{4\pi}{3} a_{i,0}^3\right)\left(\frac{kT_{gas}}{2\pi m_i}\right)^{\frac{1}{2}} c_i^{gas}(t)\left(1 - \frac{1}{S_i}\sqrt{\frac{T_{dust}}{T_{gas}}}\right), \quad (5)$$

where s_i is the sticking probability of the gaseous element i incident onto grains, k is the Boltzmann constant, and m_i is the mass of the element i. The gas temperature T_{gas} is assumed to be constant during the evolution of clouds in this study. The supersaturation ratio S_i is a function of the dust temperature T_{dust}. Since $T_{dust}/T_{gas} \ll 1$ and $S_i \gg 1$ under the condition considered here (Dopcke et al. 2011), Equation (5) is reduced to

$$\frac{dX_i}{dt} = \frac{Y_i(t)}{\tau_{i,0}^{gg}}\left(1 - \frac{t}{2\tau_0^{ff}}\right)^{-2}, \quad (6)$$

by introducing $(\tau_{i,0}^{gg})^{-1} = s_i 4\pi a_{i,0}^3 (kT_{gas}/2\pi m_i)^{1/2} A_i c_{H,0}/3r_{i,0}$. Then, integration of Equation (6) leads to the ratio of grain radius to the initial one $X_i(t)$:

$$X_i(t) = 1 + \frac{2\tau_0^{ff}}{\tau_{i,0}^{gg}} \int_0^u \frac{Y(u')}{(1-u')^2} du', \quad (7)$$

with $u = t/2\tau_0^{ff}$.

In principle, by solving Equations (4) and (7) for a given set of $f_{i,0}$, $r_{i,0}$, and A_i, we can calculate the time evolution of $f_i(t)$ and $r_i(t) = r_{i,0} X_i(t)$. The values of the other parameters necessary for the calculations are summarized in Table 1. As shown later, for the metal abundances considered in this Letter, the grain growth operates at high gas densities of $c_H \gtrsim 10^{10}$ cm^{-3}, where the gas temperature is expected to be in the range of 500–2000 K (Dopcke et al. 2011). Thus, the calculations are started from $c_{H,0} = 10^8$ cm^{-3}, with $T_{gas} = 10^3$ K. Note that the results of the calculations are not sensitive to T_{gas} as long as the above range of T_{gas} is considered.

3. RESULTS OF CALCULATIONS OF GRAIN GROWTH

Figure 1 depicts the growth of Fe grains with $f_{Fe,0} = 0.1$ and $r_{Fe,0} = 0.01$ μm; the time evolutions of the condensation

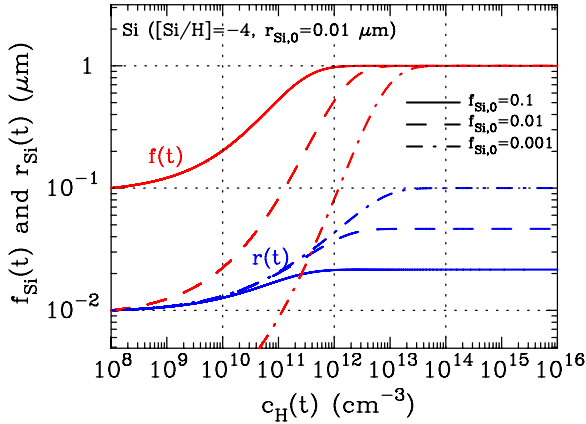**Figure 2.** Time evolutions of the condensation efficiency $f_{Si}(t)$ (red) and grain radius $r_{Si}(t)$ (blue) through the growth of Si grains of $r_{Si,0} = 0.01\,\mu m$ in the collapsing clouds with the Si abundance of [Si/H] = −4. Solid, dashed, and dot-dashed lines represent the results for the initial dust abundances of $f_{Si,0} = 0.1$, 0.01, and 0.001, respectively.

(A color version of this figure is available in the online journal.)

efficiency $f_{Fe}(t)$ and grain radius $r_{Fe}(t)$ versus hydrogen number density $c_H(t)$ for [Fe/H] = −5, −4, and −3, which correspond to $A_{Fe} = 3.26 \times 10^{-10}$, 3.26×10^{-9}, and 3.26×10^{-8}, respectively, with the solar abundance by Anders & Grevesse (1989). We can see that the grain growth is activated efficiently even in the gas clouds with [Fe/H] = −5, and the grain radius finally reaches a constant value $r_{Fe,0}(1/f_{Fe,0})^{1/3}$ by consuming all gaseous Fe atoms. However, the gas density at which a considerable fraction ($f_{Fe} \sim 0.5$) of Fe atoms is locked up in dust grains is higher for lower Fe abundance; $c_H \simeq 10^{10}$, 10^{12}, and 10^{14} cm⁻³ for [Fe/H] = −3, −4, and −5, respectively. Also, the gas density at which f_{Fe} reaches ∼0.5 is two orders of magnitude higher (lower) for $r_{Fe,0} = 0.1$ (0.001) μm than that for $r_{Fe,0} = 0.01\,\mu m$, though it is not presented in the figure. These behaviors of grain growth in the collapsing gas clouds can be seen from Equation (9) (see below); for given values of $f_{i,0}$ and $f_{i,*}$, $c_H(t) \propto (r_{i,0}/A_i)^2$ if $c_H(t)/c_{H,0} \gg 1$. Hence, for a fixed $f_{i,0}$, one order of magnitude higher $r_{i,0}$ or one order of magnitude lower A_i is compensated by two orders of magnitude higher c_H.

Figure 2 shows the time evolutions of the $f_{Si}(t)$ and $r_{Si}(t)$ of Si grains with $r_{Si,0} = 0.01\,\mu m$ for different initial dust abundances of $f_{Si,0} = 0.1$, 0.01, and 0.001. Here, the total abundance of Si atoms is set to be [Si/H] = −4 ($A_{Si} = 3.58 \times 10^{-9}$). For $f_{i,0} = 0.1$, the growth of Si grains proceeds somewhat earlier than that of Fe grains for [Fe/H] = −4, and the condensation efficiency increases to $f_{Si} = 0.5$ at $c_H \simeq 10^{11}$ cm⁻³. In cases of lower $f_{Si,0}$, higher gas densities are needed to achieve some level of the condensation efficiency, although the final grain radii are larger when all Si atoms are tied up in dust grains.

4. DISCUSSION: CRITICAL METAL ABUNDANCES

As shown in the last section, grain growth can be activated even in metal-poor star-forming clouds whose metallicity is only [Fe, Si/H] ≃ −5. However, in order that the grain growth affects the thermal evolution of collapsing cores, it must become effective before the cloud density increases to $c_H = 10^{12}$–10^{14} cm⁻³, where the optical depth becomes high enough to suppress dust emission cooling, and as a result gas fragmentation is expected to occur (e.g., Schneider et al. 2012a).

The metal abundance above which grain growth becomes important can be estimated by requiring that a certain fraction
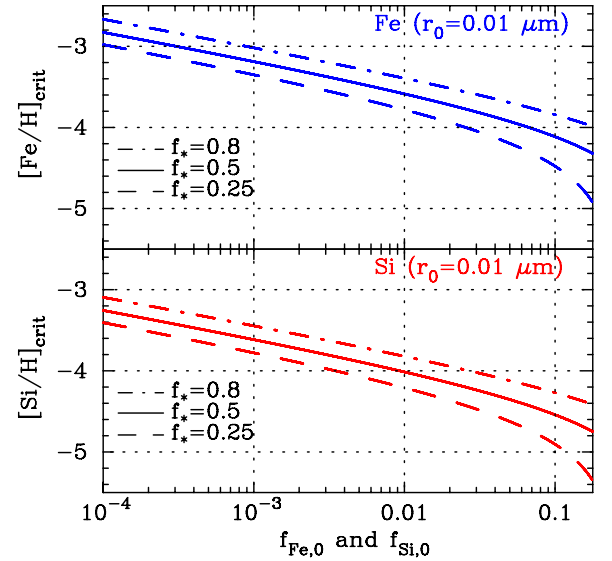

Figure 3. Critical abundances of Fe (upper panel) and Si (lower panel), for which the grain growth can lock up a fraction $f_{i,*} = 0.8$ (dot-dashed), 0.5 (solid), and 0.25 (dashed) of Fe and Si atoms in dust grains at the hydrogen number density $c_{H,*} = 10^{12}$ cm⁻³. The horizontal axis is the initial dust abundance $f_{i,0}$, and the initial grain radius is set to be $r_{i,0} = 0.01\,\mu m$.

(A color version of this figure is available in the online journal.)

$f_{i,*}$ of the element i should be locked up in dust grains at a given hydrogen number density $c_{H,*}$. Noting that $Y_i(t) = 1 - f_{i,0}X_i^3(t) = 1 - f_i(t)$, Equation (7) can be rewritten as

$$K_i(f_{i,0}, f_{i,*}) = \int_1^{X_{i,*}} \frac{dX_i}{1 - f_{i,0}X_i^3}$$
$$= \frac{2\tau_0^{ff}}{\tau_{i,0}^{gg}} \int_0^{u_*} \frac{du'}{(1-u')^2} = \frac{2\tau_0^{ff}}{\tau_{i,0}^{gg}} \frac{u_*}{1-u_*}, \quad (8)$$

where $X_{i,*} = (f_{i,*}/f_{i,0})^{1/3}$ and $u_* = 1 - (c_{H,0}/c_{H,*})^{1/2}$. From this equation, we can derive the critical metal abundances above which the grain growth could facilitate the fragmentation of the clouds as follows:

$$A_{i,crit} = (1.0\text{--}2.5) \times 10^{-9} \, K_i \left(\frac{r_{i,0}}{0.01\,\mu m}\right) \left(\frac{10^{12}\,cm^{-3}}{c_{H,*}}\right)^{\frac{1}{2}},$$
$$(9)$$

using $c_{H,0}/c_{H,*} \ll 1$. The numerical factor 2.5 (1.0) corresponds to Fe (Si) grains. The function K_i increases with increasing $f_{i,*}$ and/or decreasing $f_{i,0}$; for $0.2 \leqslant f_{i,*} \leqslant 0.8$, $K_i = 0.3$–2.4 (5.2–15) at $f_{i,0} = 0.1$ ($f_{i,0} = 0.001$).

Figure 3 presents the critical abundances of Fe and Si in the form of [X/H] versus $f_{i,0}$ for $r_{i,0} = 0.01\,\mu m$; the dot-dashed, solid, and dashed lines give the abundances necessary for the condensation efficiency $f_{i,*}$ to reach 0.8, 0.5, and 0.25, respectively, at $c_{H,*} = 10^{12}$ cm⁻³. As expected, higher metal abundances are needed to attain higher $f_{i,*}$ and/or for lower $f_{i,0}$. For the case of $f_{i,*} = 0.5$, the critical abundances of Fe and Si spans the ranges of $-4.11 \leqslant$ [Fe/H] $\leqslant -3.19$ and $-4.54 \leqslant$ [Si/H] $\leqslant -3.62$, respectively, for the range of $0.1 \geqslant f_{i,0} \geqslant 0.001$. It would be interesting to mention that the above range of [Si/H] covers the abundance of Si observed for SDSS J102915+172927 ([Si/H] = −4.27; Caffau et al. 2011). This could suggest that the growth of Si grains might have worked in the parent cloud of this star.
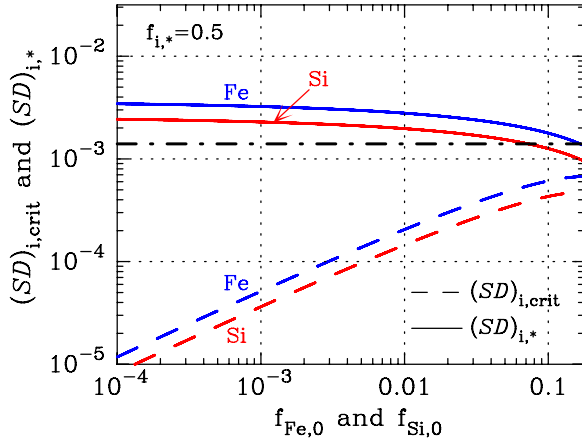
Figure 4. Products of dust-to-gas mass ratio $\mathcal{D}$ and geometrical cross section per unit dust mass $\mathcal{S}$ for Fe (blue) and Si (red) as a function of the initial dust abundance $f_{i,0}$. The dashed lines depict $(\mathcal{SD})_{i,\mathrm{crit}}$ corresponding to the critical abundances given by the solid lines in Figure 3, whereas the solid lines depict the resulting dust-to-gas mass ratio after the grain growth $(\mathcal{SD})_{i,*}$. The horizontal dot-dashed line indicates the minimum value above which the dust emission cooling causes the gas to fragment into low-mass clumps (Schneider et al. 2012a).

(A color version of this figure is available in the online journal.)

Here, we present how the fragmentation condition induced by grain growth depends on unknown parameters such as $r_{i,0}$ and s_i. The time duration Δt^{ff} for which the gas density increases from $c_{\mathrm{H},0}$ to $c_{\mathrm{H},*}$ by free fall is given by $\Delta t^{\mathrm{ff}} = 2\tau_0^{\mathrm{ff}}(1 - \sqrt{c_{\mathrm{H},0}/c_{\mathrm{H},*}})$. On the other hand, the time duration Δt^{gg} for which the condensation efficiency $f_{i,0}$ at $c_{\mathrm{H},0}$ increases up to $f_{i,*}$ at $c_{\mathrm{H},*}$ through grain growth is derived from Equation (8) as $(\Delta t^{\mathrm{gg}})^{-1} = (K_i \tau_0^{\mathrm{gg}})^{-1} + (2\tau_0^{\mathrm{ff}})^{-1}$. Since $\Delta t^{\mathrm{ff}}/\Delta t^{\mathrm{gg}} \geqslant 1$ is required for the fragmentation, we can obtain the condition

$$(1.0\text{--}2.5)\left(\frac{s_i}{1.0}\right)\left(\frac{A_i}{2.5 \times 10^{-9}}\right)\left(\frac{0.01\,\mu\mathrm{m}}{r_{i,0}}\right)$$
$$\times \left(\frac{c_{\mathrm{H},*}}{10^{12}\,\mathrm{cm}^{-3}}\right)^{\frac{1}{2}}\left(\frac{1.0}{K_i}\right) \geqslant 1.0, \qquad (10)$$

where the numerical factor 1.0 (2.5) is for Fe (Si) grains. This inequality demonstrates that the fragmentation condition is achieved more easily for initial grain radii smaller than $0.01\,\mu\mathrm{m}$. In contrast, if $r_{i,0} > 1.0\,\mu\mathrm{m}$ or $s_i < 0.01$, grain growth can no longer induce fragmentation for $A_i \lesssim 2.5 \times 10^{-7}$ ([Fe, Si/H] $\lesssim -2$). We also note that a smaller $f_{i,0}$ producing a larger K_i acts against gas fragmentation by grain growth.

Next we consider the dust-to-gas mass ratio to see whether the condition for the dust-induced fragmentation given in Equation (1) can be met. In the context of this Letter, we suppose that the product of $\mathcal{S}$ and $\mathcal{D}$ without grain growth is given as $(\mathcal{SD})_{i,\mathrm{crit}} = 3f_{i,0}A_{i,\mathrm{crit}}m_i/4\rho_i r_{i,0}\mu m_{\mathrm{H}}$, adopting the critical metal abundance evaluated in Equation (9), whereas the product resulting from the grain growth is given by $(\mathcal{SD})_{i,*} = (\mathcal{SD})_{i,\mathrm{crit}}(f_{i,*}/f_{i,0})^{2/3}$. Figure 4 shows the dependence of $(\mathcal{SD})_{i,\mathrm{crit}}$ and $(\mathcal{SD})_{i,*}$ on $f_{i,0}$, adopting $f_{i,*} = 0.5$. We can see that $(\mathcal{SD})_{i,\mathrm{crit}}$ is well below the minimum value required for the dust-induced fragmentation (dot-dashed line in Figure 4), whereas $(\mathcal{SD})_{i,*}$ exceeds this value. This indicates that even if the destruction by the SN reverse shock results in a lower $\mathcal{SD}$ than the criterion for the dust-induced fragmentation, the grain growth can enhance $\mathcal{SD}$ in the clouds and can enable the gas to fragment into sub-solar mass clumps. Note that the results

in Figure 4 are independent of the initial grain radius $r_{i,0}$ since $\mathcal{S}_i \propto r_{i,0}^{-1}$ and $\mathcal{D}_i \propto A_{i,\mathrm{crit}} \propto r_{i,0}$.

Finally, we relate the critical abundance to the total metallicity Z. Equation (9) suggests that the critical metal abundances are generally in the range of $A_{\mathrm{crit}} \simeq 10^{-9}\text{--}10^{-8}$, depending on $f_{i,0}$, $f_{i,*}$, $r_{i,0}$, and grain species. By representing the mass ratio of refractory elements condensible into dust grains to the total heavy elements as $\mathcal{R}$, the metallicity Z can be related to A_{crit} as $\mathcal{R}Z \simeq A_{\mathrm{crit}}(\mu_{\mathrm{ref}}/\mu)$ with μ_{ref} being the mean atomic mass of refractory elements. Then, we have

$$Z \simeq (5\text{--}50) \times 10^{-6}\left(\frac{0.2}{\mathcal{R}}\right)Z_{\odot}, \qquad (11)$$

where we use $\mu_{\mathrm{ref}}/\mu = 20$ and $Z_{\odot} = 0.02$. Equation (11) implies that, if the grain growth does work efficiently, it can drive the gas fragmentation of low-mass clumps in the star-forming clouds enriched with metallicity $\sim 10^{-5}\,Z_{\odot}$. In other words, as long as the abundance of a given refractory element satisfies the critical abundance in Equation (9), the formation of hyper-metal-poor low-mass stars with metallicity lower than $Z \simeq 4.5 \times 10^{-5}\,Z_{\odot}$ observed in SDSS J102915+172927 could be possible.

5. CONCLUDING REMARKS

We have investigated the growth of dust grains in metal-poor protostellar clouds. Our simple model shows that grain growth can operate efficiently even in collapsing dense cores with metal abundances as low as [Fe, Si/H] $\simeq -5$. We also present the critical metal abundances above which grain growth could affect the fragmentation process of collapsing gas clouds. This abundance is estimated to be $A_{\mathrm{crit}} \simeq 10^{-9}\text{--}10^{-8}$, which suggests that the formation of low-mass stars with metallicity of $\sim 10^{-5}\,Z_{\odot}$ can be possible. We conclude that even if the initial dust-to-gas mass ratio does not satisfy the condition required for the dust-induced fragmentation, the grain growth can increase the dust-to-gas mass ratio high enough to facilitate the formation of metal-poor low-mass stars.

Our results suggest that if grain growth is considered, the formation of low-mass protostars can occur not only at very low metallicity but also at higher metallicity. The final mass of a newly born star is determined by the accretion of the surrounding gas onto the protostars (McKee & Ostriker 2007 and references therein), and its mass accretion rate would be regulated by the mass of the central protostar induced by the grain growth. Thus, the grain growth in collapsing clouds might be a fundamental physical process to control the stellar initial mass function in the present universe.

It should be mentioned that we have considered only the growth of single-component Fe and Si grains with a single initial radius. However, it might be possible that Si and Fe atoms condense as silicates or oxides in an oxygen-rich gas. Since the growth of such compound grains with no monomer molecule has been usually treated by considering Si or Fe element as a key element (e.g., Zhukovska et al. 2008), the mass and radius of dust given in this Letter are considered to be lower limits. On the other hand, the timescale of grain growth is sensitive to the initial grain radius (Hirashita & Kuo 2011). Thus, the effect of the initial size distribution as well as the growth of compound grains should be explored. Furthermore, we have assumed the sticking probability of $s_i = 1$ and a constant gas temperature $T_{\mathrm{gas}} = 10^3$ K during the collapse of the clouds. In particular, too low sticking probabilities ($s_i \lesssim 0.01$) may prevent

grain growth from becoming efficient for metal abundances of $A_i \lesssim 2.5 \times 10^{-7}$. We note that our conclusions obtained with the simple model should be confirmed by more sophisticated simulations of the thermal evolution of star-forming clouds involving grain growth.

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