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On the Totality of the Eclipse in AD 628 in the Nihongi

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Abstract

It is widely accepted that the solar eclipse on AD 628 April 10 (the reign of Empress Suiko, 36th year, 3rd month, 2nd day) recorded in the Nihongi (日本書紀) was not total, but partial at the site of observation, though it is written as an exhausted eclipse. A contemporary Japanese occultation observation on AD 681 November 3 is also suspected as being a missing of Mars in the glaring light of the Moon. We suggest in this paper that both records in the Nihongi may be true. Several reasonings are put forward. We then point out the possibility that the value of ΔT at around AD 600 is about 2000 s which is far less than 4500 s, the value adopted by Stephenson (1997, Historical Eclipses and Earth's Rotation). Lunar grazing occultation data are found to be very useful.

Key words: eclipses — histories and philosophy of astronomy — occultations

1. Introduction

> (推古天皇三十六年) 三月丁未朔戊申, 日有蝕盡之
> (The reign of Empress Suiko, 36th year) 3rd month starting on *ding-wei* (44),[1] 2nd day *wu-shen* (45), the Sun was exhausted by an eclipse.

We find this record in the Nihongi, the oldest history book in Japan. Though the expression is a little out of the Chinese rule of recording solar eclipses, this clearly asserts that there was a total eclipse on that day. Indeed, usually the expression total (既) is used. Here however, 'exhausted' (盡) is used. We find this expression in several history books of China, although it is used in a negative sense. Thus, for example, the Book of Houhan (後漢書) says 'Solar eclipse, nearly exhausted' (日有蝕之幾盡) in the record of (Houhanshu) Yuanchu, 6th year, 12th month, day *wu-wu* (55) (AD 120). As for the Suiko eclipse, there actually was a total eclipse on the Earth according to calculations (e.g., Watanabe 1979). This was on AD 628 April 10 in the Julian calendar. Our problem is whether or not the zone of totality passed the Japanese Islands.

Our aim in the present paper is to assert that the reliability of the record of the Suiko eclipse is not so low as has been believed by previous authors, and to direct general attention to seriously examine the ancient Japanese astronomical records.

A preliminary version of our work has been reported at the conference in Xingtai, China in 2001 October (Tanikawa, Sôma 2004).

The authors are concerned about the unfamiliarity of the topic of this paper to the readers. Thus, we give a rather long introduction before presenting our results. The authors hope that this paper will arouse young astronomers to have interest in the field described here.

1.1. Importance of the Eclipse

1.1.1. The first astronomical record in the Nihongi

This is the first formal record of solar eclipses in the Japanese history, although we have a famous record in the mythological age of the Nihongi, which is usually interpreted as implying the observation of a total solar eclipse: 'At this time, the Day Goddess … went into a rocky cave and shut herself up' (是時, 天照大神…, 入于天石窟, 閉磐戸而幽居焉).

1.1.2. An important epoch in Japanese history

We can see the rise and early development of Japan in Chinese chronicles. Japan was described in the Wei History (魏志) (Chen Shou 3rd century), in the Book of Song (宋書) (Chen Yue 488) in the fifth century, and in the Book of Sui (隋書) (Yan Shigu et al. 636) in the seventh century. We understand that Japan grew gradually and apparently during these centuries. Finally, Wo (倭国, i.e., the ancient Japan) declared independence in a sovereign message in the third year of Daye (大業三年, AD 607) (Nishijima 1994; Furuta 1988).

> 其王多利思比孤, 遣使朝貢. …
> 其国書曰, 日出処天子致書日没処天子無恙.
> 帝覧之不悦, …, 蛮夷書, 有無禮者. 勿復以聞.
> The King Tarishihiko sent messengers to greet the (Chinese) Emperor. …
> The sovereign message said that the son of heaven at the rising Sun sends a letter to the son of heaven at the sinking Sun with compliments.
> Emperor said "This barbarian's letter is impertinent.
> Never read this kind of letter in front of me."

China, from her really ancient past, recorded solar and lunar eclipses and various other astronomical phenomena in her history books. These records were important in view of standardizing the units of length and time, and making a calendar system. The first day of the month should be the new moon in a calendar system that adopted True Conjunction (定朔). If this was not the case, then the constants were adjusted and the calendar system had to be renewed. To renew

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[1] The number in the parentheses is the cyclic day number starting from jia-zi (甲子).

the system, we need a long-term accumulation of observational data. Independence requires this accumulation. Did Wo already have this in AD 607? We do not know the answer. Did Wo have an intention to have this? Yes. Wo started various kinds of observations sooner or later (Kanda 1935): comets (AD 634), bright meteors (AD 637), lunar eclipses (AD 643), approach to and occultation by the Moon of stars (AD 640), and other phenomena.

The Japanese original calendar system started as late as 1685 when Harumi SHIBUKAWA introduced the European idea. Until then, Japan constantly used Chinese calendars adapted to Japan (Nakayama 1969; Yabuuchi 1969).

1.1.3. Long-term variations of the Earth's spin and the Moon's orbital motion

At present, the Moon is known to recede from us at around $3.8\,\mathrm{cm\,yr^{-1}}$ (Dickey et al. 1994). We do not know whether or not this rate has been constant. Similarly, the rotation of the Earth has been known to slow down in the past several decades (Lambeck 1980). However, we do not know the rate of slow down during the past few thousand years.

We do not have an appropriate theory to answer the above questions. More precisely, to follow the rotational motion of the Earth for thousands of years, taking into account the existence of the sea, atmosphere, polar ice caps, mantle, liquid core under the influence of the Moon, Sun, and other planets, is a tremendous task. Even at present this is one of the most difficult scientific problems to solve. Ancient astronomical records give us a unique opportunity to determine the Earth's spin and the orbital motion of the Moon.

1.2. Conventional Views to the Eclipse

According to conventional views, the total eclipse record in the Nihongi of Empress Suiko is an exaggeration (Ogura 1916; Suzuki 1942; Watanabe 1979; Uchida 1979; Saito 1982a,b). Let us cite a comment of Uchida (1979) as a representative: "There is an assertion that the calendar system was already adopted at that epoch because of this record. I do not agree with this opinion. In this eclipse, the largest obscuration took place at Asuka just before nine thirty in the morning. The magnitude was more than 0.9, but less than one. Even if the eclipse is not total, the temperature goes down and the surrounding atmosphere becomes unfamiliar if the magnitude becomes greater than 0.9. This is a striking and notable phenomenon without any public prediction. So the record might have been the actual record of observation of amateurs. Then the record is not related to the adoption of a calendar system" (Uchida, 1979, p.525).

This view has strong support. The value of $\Delta T = \mathrm{TT} - \mathrm{UT}$ is widely accepted to be 4500 s or larger at around AD 600. Here, TT (Terrestrial Time) is the uniform time taking into account the general relativistic effect of gravitation, and UT (Universal Time) is the time obtained from the Earth's rotation. In this case, the zone of totality of the Suiko eclipse passes through the Pacific Ocean. Another support is the unreliability of the ancient eclipse records. Indeed, some of the records in Chinese chronicles and the Nihongi are not reliable. This is the case especially in the total eclipse records. We will later come back to this point.

There are comments from outside Japan. Stephenson (1997, p.267) says: "The recorded date proves to be in accurate accord with that of a computed solar eclipse. Although the obscuration of the Sun was said to be total, further description is lacking, so that I have included the observation in the lowest category. It is noteworthy that the event does not seem to be linked in any way with the Empress' death by the chronicler (unlike several similar occurrences in China)." Steele (2000) remarks that "In common with all of the other observations in the Nihongi, this record is not very detailed. No times are ever recorded for any of the eclipse records in this work."

1.3. Reliability of Eclipse Records in Ancient Chinese Books

Saito and Ozawa (1992) checked the reliability of Chinese astronomical records from Chunqiu (8th century BC) to Wudai (AD 10th century). The method of the check was to calculate the time and place (in the celestial sphere or on the Earth) of these phenomena using the parameter values coming from Neugebauer and Schock (1927), Neugebauer (1929), and Schoch (1927) to judge whether these actually occurred or not. Here, we take solar eclipse data from Saito and Ozawa (1992) and make some comments.

Saito and Ozawa (1992) compiled all the solar eclipse records in Chinese chronicles starting from Chunqiu to Wudai and expressed the reliability of the records as the percentage of the realized eclipses among predicted or recorded eclipses. They give table 1 (p.17). In their table, the fourth column shows the percentage.

The first impression when looking at this table is that the probability of success does not increase with time. There, of course, is a reason. Before explaining the reason, let us introduce notations following Saito and Ozawa. We have three different terms: NE (night eclipse), OB (out of eclipse zone), and OE (the eclipse zone is out of the Earth).

In China, from ancient times, astronomical phenomena were predicted using the calendar system. Predictions sometimes hit, but other times did not hit. Some predicted solar eclipses took place on the nightside of the Earth (NE); others were on the polar regions (OB), so that people at middle latitudes could not observe. Sometimes the eclipse shadow was out of the Earth (OE). The predictions seem to start in Houhan era (Ohashi 1999).

From the above observations, we can conclude that an examination of the reliability of individual eclipse records is a dangerous task. It is true that we can confirm the existence of a particular historical eclipse by calculation. Thus, at first glance, if the corresponding records are found in a history book, we are apt to believe that this was really observed. But we need to be careful. We do not know that the astronomers at that time actually observed the eclipse. It might have been cloudy. It may have been that the prediction hit. Moreover, we are not sure that our parameters in calculating the astronomical events are accurate enough to check the precise descriptions of ancient records.

The situation is more subtle for total or near total eclipses. For almost all cases, the corresponding eclipses are real. But did the astronomers in the capital of the empire see the totality? Saito and Ozawa (1992) answered to this question (see table 2). According to their calculations, no eclipses recorded as total in Chinese history books from 5th to 9th century were total.

Table 1. Eclipse records and reality in Chinese chronicles.*

Era	Number of records	Real	Probability	NE・OB	OE
Chunqiu (春秋)	37	35	95	1	1
Shiji (史記)	10	4	40	6	0
Hanshu (漢書)	61	42	69	1	18
Houhanshu (後漢書)	88	65	74	7	16
Sanguozhi (三国志)	12	11	92	0	1
Jinshu (晉書)	82	49	60	16	17
Songshu (宋書)	83	60	72	16	7
Weishu (魏書)	61	41	67	16	4
Nanqishu (南斉書)	5	4	(80)	1	0
Liangshu (梁書) Chenshu (陳書)	10	7	70	3	0
Nanshi (南史)	35	23	66	10	2
Beiqishu (北斉書)	2	2	(100)	0	0
Zhoushu (周書)	22	7	32	15	0
Beishi(北史)	82	46	56	34	2
Suishu (隋書) Liang (梁)	3	2	(67)	1	0
Suishu (隋書) Chen (陳)	1	0	0	1	0
Suishu (隋書) Qi (斉)	1	1	(100)	0	0
Suishu (隋書) Zhou (周)	10	2	20	8	0
Suishu (隋書) Sui (隋)	8	3	(37)	5	0
Jiutangshu (旧唐書)	99	69	70	25	5
Xintangshu (新唐書)	93	66	71	25	2
Jiuwudaishi (旧五代史)	20	15	75	5	0
Xinwudaishi (新五代史)	18	14	78	4	0

* Saito and Ozawa (1992, p.17). First column: the name of the history book; second column: the number of recorded eclipses; third column: real eclipses; fourth column: percentage of reality; fifth column: number of NE and OB eclipses; sixth column: number of OE eclipses (see text in subsection 1.3 for the meaning of NE, OB, and OE).

"We can interpret that the eclipse was judged to be total if the magnitude is greater than about 0.9. We cannot believe the ancient records of total eclipse as total" (Watanabe 1979).

The authors wonder: did people in Changan or Luoyang not see any total solar eclipse from the 5th to the 9th century? This wonder is one of the motivations of this work. We know an example in which the zone of totality runs parallel to the latitude lines. In this case, shifting the zone to the west or to the east does not bring the total eclipse zone to the capital. The event on AD 754 June 25 recorded as a total eclipse was this kind of record. The record surely is an exaggeration. When the zone of totality runs transverse to the latitude lines, there is a chance to have a total eclipse in a desired place. (See subsection 2.1 for the reasoning of the parallel shift of the eclipse zone.)

1.4. *Investigations by Stephenson*

Stephenson and Morrison (1995) and Stephenson (1997) compiled ancient and medieval solar and lunar eclipse records of China, Babylonia, Greece, Europe, and Arab, analysed these records one by one, gave weights to individual records, and fitted ΔT curves by spline curves with several nodes. Stephenson (1997, pp. 508 and 515) obtained $\Delta T = 4300\,\mathrm{s}$ for the period around AD 650 from lunar and solar eclipses. On the other hand, solar eclipses at around AD 650 give us $\Delta T = 2000\,\mathrm{s}$ as can be seen in Stephenson's graph. Solar eclipses are out of the fitted spline curve. This fitted line seems to be favorable to the lunar eclipses. The Japanese eclipse in AD 628, if it is a total eclipse, would lie far out of the line and stay with contemporary Chinese eclipses.

Our important question is: what is the reason for the inconsistency between the results from solar and lunar eclipses? Is it a simple coincidence? This is another motivation of our work.

2. Solar and Lunar Eclipses and the Secular Change of Earth's Spin

As is well-known, solar and lunar eclipses take place when the Sun, Earth, and Moon align. If the orbital motion of the Earth around the Sun, the orbital motion of the Moon around the Earth, and the revolution of the Earth around its spin axis are all steady, then these phenomena take place quasi-periodically. If one or more of the above motions change secularly, then the phenomena occur non-regularly.

In what follows, we consider the qualitative nature of the occurrence change of solar eclipses during thousands of years assuming the constancy of the orbital motion of the center of gravity of the Earth–Moon system around the Sun and allowing a secular change both of the orbital motion of the Moon with respect to the center of the gravity of the Earth–Moon system and the spin rate of the Earth.

It is to be noted here that our knowledge about the secular changes started less than 100 years ago for Earth's rotation and for the motion of the Moon, using modern techniques, like

Table 2. Total and near-total eclipses in China: 5th–9th centuries. *

Year(AD)	Chinese date	Magnitude	
(Empire)	Phenomena	$\Delta T \simeq 4000$	$\Delta T = 2000$
453.08.20 (Song)	Yuan-jia, 30th year, 7th month, 1st day *xin-chou*, Solar eclipse, total, stars appeared	OB	OB
454.08.10 (Song)	Xiao-jian, first year, 7th month, 1st day *bing-shen*, Solar eclipse, total	0.96	0.97
516.04.18 (Liang)	Tian-jian, 15th year, 3rd month, Solar eclipse, total	0.97	0.94
522.06.10 (Liang)	Pu-tong, 3rd year, 5th month, 1st day *ren-chen*, Solar eclipse, total	horizontal	0.97, non-horizontal
562.10.14 (Beizhou)	Bao-ding, 2nd year, 9th month, 1st day *wu-chen*, Solar eclipse, total (Zhoushu) Solar eclipse	0.35	0.225
616.05.21 (Sui)	Da-ye, 12th year, 5th month, 1st day *bing-xu*, solar eclipse, total	0.90, 'complete' is irrelevant	0.967
702.09.26 (Zezhou)	Chang-an, 2nd year, 9th month, 1st day *yi-chou*, Solar eclipse, nearly total At one degree of Jiao (Jiutangshu) Solar eclipse, like a hook, seen from the capital and its vicinity	0.99	1.016
754.06.25 (Tang)	Tian-bao, 13th year, 6th month, 1st day *yi-chou*, Solar eclipse, nearly total at 19 degrees of Dongjing, 京師分也 (Jiutangshu)Solar eclipse, like a hook	0.87	0.854
756.10.28 (Tang)	Zhi-de, first year, 10th month, 1st day *xin-si*, Solar eclipse, total, at 10 degrees of Di (Jiutangshu) total	0.96	0.886
761.08.05 (Tang)	Shang-yuan, 2nd year, 7th month, 1st day *gui-wei*, Solar eclipse, total, all the bright stars appeared, at 4 degrees of Zhang (Jiutangshu) total eclipse, all the bright stars appeared	0.99	1.011
846.12.22 (Tang)	Hui-chang, 6th year, 12th month, 1st day *wu-chen*, Solar eclipse, at 14 degrees of Nandou. (Jiutangshu) total	0.85	0.818
879.04.25 (Tang)	Gan-fu, 6th year, 4th month, 1st day *geng-shen*, Solar eclipse, total, at 8 degrees of Wei (Jiutangshu) No description	OE	OE

* The third column represents the comment of Saito and Ozawa (1992) and their calculation of the eclipse magnitude, adopting $\Delta T = 4000$ s. The fourth column represents ours, adopting $\Delta T = 2000$ s without any correction to the tidal term.

atomic clocks and lunar laser ranging. Modern values represent instantaneous ones compared with a long-term trend during two or three thousand years. Modern techniques are at present insufficient to make a definite conclusion concerning the long-term behavior of the lunar motion and Earth's spin.

2.1. *The Consequence of a Slowing Down of the Earth's Spin and Lunar Orbital Motion*

The contents of this section may be well-known. We add the section for completeness sake and also for expository purpose. It is well-known that both the spin rate of the Earth and the orbital motion of the Moon around the Earth slow down as time goes on due mainly to tidal effects. The slowing down of the spin rate is accumulated as the difference, ΔT, of the real and predicted rotation angles of the Earth, whereas the slowing down of the orbital motion of the Moon is expressed as coefficients of T^2 in the longitude of the Moon, where T is the time in centuries. The term due to the mutual tidal interaction between the Earth and Moon is called the tidal term.

2.1.1. *No secular change*

If there is no change in the orbital motion of the Moon nor the spin rate of the Earth, we can calculate the place, time, and duration of any eclipse at any time in the past. This situation is not a historical fact. If this is the case, historical eclipses are almost useless.

2.1.2. *The spin rate of the Earth slows down*

We assume that the spin rate of the Earth was larger in the past, and gradually slowed down. We assume, on the other hand, that the orbital motion of the Moon did not change. For simplicity of consideration, we assume that the spin axis of the

Earth is perpendicular to its orbital plane. We sit on the north pole of the earth and wind back the time and see what happens.

We fix the past time, say 1000 years ago, and suppose there was a total solar eclipse at some place. By assumption, the orbital motion of the Sun and Moon around the Earth can be traced back theoretically. This means we know exactly at which time of the eclipse day in the uniform time, the Sun, Moon, and the Earth came on a line. The remaining problem is which part of the Earth's surface was in the shadow of the Moon. This depends on the slowing down of the spin.

The spin was faster in the past. Thus we need to wind the Earth more to the west than the case of a constant spin. This implies that the place of the eclipse is shifted to the east compared with the predicted place under the assumption that the spin has been constant. Let TT be the uniform time and UT be the time measured by the rotation of the Earth as unit. Define

$$\Delta T = \mathrm{TT} - \mathrm{UT}. \tag{1}$$

We adjust the clock at some time in the modern era. Then, before this epoch, UT proceeds faster than TT; that is, the UT value is smaller in the past. Consequently, we have $\Delta T > 0$ in the past. The difference of two kinds of times can be transformed to the longitude difference of the eclipse place. The Earth rotates 15° per hour. Roughly speaking, $\Delta T = 3600\,\mathrm{s}$ is equivalent to a shift of 15° in longitude.

The procedure to determine ΔT using the historical solar eclipse is as follows. Fixing the motion of the Moon to some model, one free parameter is ΔT. We adjust it so as to place the observed point (usually the capital) at the west and east boundaries of the zone of totality. Then, the range of ΔT is given. Additional information, such as the duration of the eclipse or the starting or ending time of the eclipse, yields a narrower range of ΔT.

2.1.3. *The case when the orbital motion of the Moon slows down*

Suppose that the orbital motion of the Moon was faster in the past, and it gradually slowed down. For simplicity, we assume that the spin rate of the Earth has not changed. We look at the Earth–Moon system as in the former subsection. The first prediction (to the past) is to assume the constancy of the Moon's orbital motion. Suppose that the Sun, Earth, and Moon are on a line in this case, and that the Moon is at point A on its orbit and its shadow on the Earth is at point A*. Now remember that the Moon's motion was faster in the past. Then, winding back the time, the Moon was to the west of point A. The shift is very small if we go back only a few thousand years. The Moon is in the parallel light rays of the Sun. As a result, the shadow of the Moon was to the west of point A* and the distance is almost exactly the difference of the positions of the Moon on its orbit. It turns out that the slow down of the orbital motion of the Moon has the opposite effect to the slow down of the Earth's spin on the position of the eclipse.

2.1.4. *Combined effects*

The position of the eclipse in the past was shifted to the east by the slow down of the Earth's spin and shifted to the west by the slow down of the orbital motion of the Moon both compared with predictions without these effects. To know the combined effects, we need to know the relation between the change rate of the two motions. Let us see qualitatively how the angular momentum is redistributed to the Earth's spin and lunar orbital motion.

Let us consider the conservation of the angular momentum of the Earth–Moon system. For simplicity, we neglect the existence of the Sun, i.e., the Earth–Moon system is moving with a constant velocity on a straight line. Let $M_\oplus$ and M_{moon} be the mass of the Earth and Moon. We write $M_{\mathrm{T}} = M_\oplus + M_{\mathrm{moon}}$. We denote by R the (average) distance of the Earth and Moon. O is the gravity center of the system. Let $L_{\mathrm{T}}^{\mathrm{orb}}$ be the total orbital angular momentum of the system with respect to O and let $L_\oplus^{\mathrm{spin}}$ and $L_{\mathrm{moon}}^{\mathrm{spin}}$ be the spin angular momenta of the Earth and Moon with respect to their own centers. Then, the total angular momentum L_{T} of the Earth–Moon system is

$$L_{\mathrm{T}} = L_{\mathrm{T}}^{\mathrm{orb}} + L_\oplus^{\mathrm{spin}} + L_{\mathrm{moon}}^{\mathrm{spin}}. \tag{2}$$

We are going to derive an expression of each term on the right-hand side of the above equation. We assume that the Earth and Moon move on a circular orbit with each other. Kepler's third law of motion is written as

$$GM_{\mathrm{T}} = n^2 R^3 \quad \text{or} \quad n = \sqrt{\frac{GM_{\mathrm{T}}}{R^3}}. \tag{3}$$

where n is the mean motion of the Earth's (or Moon's) orbit around O. Then, denoting $\mu = M_\oplus M_{\mathrm{moon}}/M_{\mathrm{T}}$, we have

$$\begin{aligned} L_\oplus^{\mathrm{orb}} &= M_\oplus \left(\frac{M_{\mathrm{moon}}}{M_{\mathrm{T}}} R\right)^2 n = \mu M_{\mathrm{moon}} \sqrt{\frac{GR}{M_{\mathrm{T}}}}, \\ L_{\mathrm{moon}}^{\mathrm{orb}} &= M_{\mathrm{moon}} \left(\frac{M_\oplus}{M_{\mathrm{T}}} R\right)^2 n = \mu M_\oplus \sqrt{\frac{GR}{M_{\mathrm{T}}}}. \end{aligned} \tag{4}$$

Adding two terms, we obtain

$$L_{\mathrm{T}}^{\mathrm{orb}} = L_\oplus^{\mathrm{orb}} + L_{\mathrm{moon}}^{\mathrm{orb}} = \mu\sqrt{GRM_{\mathrm{T}}}. \tag{5}$$

Let us next consider the spin angular momentum. Let $I_\oplus$ and I_{moon} be their momenta of inertia. Let $\omega_\oplus$ and ω_{moon} be the spin angular velocities of the Earth and Moon. We have

$$\begin{aligned} L_\oplus^{\mathrm{spin}} &= I_\oplus \omega_\oplus, \\ L_{\mathrm{moon}}^{\mathrm{spin}} &= I_{\mathrm{moon}} \omega_{\mathrm{moon}}. \end{aligned} \tag{6}$$

The basic constants are $G = 6.672 \times 10^{-11}$ MKS, $R = 3.844 \times 10^8\,\mathrm{m}$, $M_\oplus = 5.974 \times 10^{24}\,\mathrm{kg}$, and $M_{\mathrm{moon}} = 1.23 \times 10^{-2} M_\oplus$. We know that $\omega_\oplus = 2\pi\,\mathrm{d}^{-1}$, and $\omega_{\mathrm{moon}} = 2\pi/(27.3\,\mathrm{d})$. In MKS units,

$$\begin{aligned} L_{\mathrm{moon}}^{\mathrm{orb}} &= 2.82 \times 10^{34}, \quad L_\oplus^{\mathrm{orb}} = 3.47 \times 10^{32}, \\ L_\oplus^{\mathrm{spin}} &= 7.07 \times 10^{33}, \quad L_{\mathrm{moon}}^{\mathrm{spin}} = 2.37 \times 10^{29}, \end{aligned} \tag{7}$$

where we use the approximation $I_\oplus = (2/5) M_\oplus r_\oplus^2$ and $I_{\mathrm{moon}} = (2/5) M_{\mathrm{moon}} r_{\mathrm{moon}}^2$, that is, the Earth and Moon are assumed to be homogeneous spheres. $r_\oplus$ and r_{moon} are the radii of the Earth and Moon. Obviously,

$$L_{\mathrm{T}}^{\mathrm{orb}} > L_\oplus^{\mathrm{spin}} \gg L_{\mathrm{moon}}^{\mathrm{spin}}. \tag{8}$$

Let us achieve the objective of this section by estimating the redistribution of the total angular momentum of the Earth–Moon system to orbital and spin angular momenta according

Table 3. History of the Moon's tidal term.

Authority	Adopted value	Method
Spencer Jones (1939)	$-11.^{\prime\prime}22\,T^2$	Lunar motion relative to planets
Van Flandern (1970)	$(-28^{\prime\prime} \pm 8^{\prime\prime})T^2$	Brown's lunar theory
Morrison (1973)	$(-21^{\prime\prime} \pm 3^{\prime\prime})T^2$	Brown's lunar theory
Van Flandern (1975)	$(-32.^{\prime\prime}5 \pm 9^{\prime\prime})T^2$	Numercal integration of the lunar orbit
Morrison & Ward (1975)	$(-13^{\prime\prime} \pm 1^{\prime\prime})T^2$	Lunar motion plus Mercury's transits
Calame & Mulholland (1978)	$(-12.^{\prime\prime}3 \pm 2.^{\prime\prime}5)T^2$	(LLR)*
Williams et al. (1978)	$(-11.^{\prime\prime}9 \pm 2.^{\prime\prime}0)T^2$	(LLR)
Ferrari et al. (1980)	$(-11.^{\prime\prime}9 \pm 1.^{\prime\prime}3)T^2$	(LLR)
Dickey et al. (1982)	$(-11.^{\prime\prime}9 \pm 0.^{\prime\prime}8)T^2$	(LLR)
Dickey et al. (1994)	$(-12.^{\prime\prime}94 \pm 0.^{\prime\prime}25)T^2$	(LLR)

* Lunar laser ranging.

as the secular change of the motion of the system. Let us take the derivative of

$$L_{\mathrm{T}} = \mu\sqrt{GRM_{\mathrm{T}}} + I_{\oplus}\omega_{\oplus} + I_{\mathrm{moon}}\omega_{\mathrm{moon}}, \tag{9}$$

with respect to time while keeping in mind the conservation law. Then,

$$0 = \frac{\mu}{2}\sqrt{\frac{GM_{\mathrm{T}}}{R}}\dot{R} + I_{\oplus}\dot{\omega}_{\oplus} + I_{\mathrm{moon}}\dot{\omega}_{\mathrm{moon}}. \tag{10}$$

The time derivative of Kepler's third law gives

$$2\dot{n}R = -3n\dot{R} \quad \text{or} \quad \dot{R} = -\frac{2}{3}R^2\sqrt{\frac{R}{GM_{\mathrm{T}}}}\dot{n}. \tag{11}$$

Substituting (11) in (10) we have

$$I_{\oplus}\dot{\omega}_{\oplus} = \frac{\mu}{3}R^2\dot{n} - I_{\mathrm{moon}}\dot{\omega}_{\mathrm{moon}}, \tag{12}$$

or neglecting the second term

$$\dot{\omega}_{\oplus} = \frac{\mu R^2}{3I_{\oplus}}\dot{n} \approx 45.0\dot{n}, \tag{13}$$

where as an inertial moment of the Earth we take $I_{\oplus} = 0.3307 \times M_{\oplus}r_{\oplus}^2$ of the 1999 IAG. The coefficient 45.0 is smaller than that (i.e., 49 ± 3) given in Christodoulidis et al. (1988). The difference may come from our neglecting the Sun as well as the non-sphericity, non-homogeneity, and non-rigidity of the Earth, or from neglecting the change of the moment of inertia of the Earth.

Let us expand the spin angular position, $\Lambda_{\oplus}$, of the Earth and the orbital angular position, $\lambda(t)$, of the Moon in powers of t as

$$\begin{aligned} \Lambda_{\oplus}(t) &= \widehat{A} + \widehat{B}t + \widehat{C}t^2 + \widehat{D}t^3 + \widehat{E}t^4 + \cdots, \\ \lambda(t) &= A + Bt + Ct^2 + Dt^3 + Et^4 + \cdots. \end{aligned} \tag{14}$$

Taking into account the relations

$$\dot{\Lambda}_{\oplus} = \omega_{\oplus}, \quad \dot{\lambda} = n, \tag{15}$$

and equating the terms of the same powers in t in (14), we obtain

$$\begin{aligned} &\widehat{C} = \frac{\gamma}{6}\Gamma, \quad \widehat{D} = \frac{1}{9}\left(-\frac{2}{3}\gamma^2 + \delta\right)\Gamma, \\ &\widehat{E} = \frac{1}{108}\left(\frac{14}{3}\gamma^3 - 12\gamma\delta + 9\epsilon\right)\Gamma, \end{aligned} \tag{16}$$

where

$$\begin{aligned} &\gamma = 2(C/B), \quad \delta = 3(D/B), \\ &\epsilon = 4(E/B), \quad \Gamma = \frac{\mu(GM_{\mathrm{T}})^{2/3}}{I_{\oplus}B^{1/3}}. \end{aligned} \tag{17}$$

We can determine the coefficients in (14) by appealing to ancient astronomical observations.

2.2. *Apparent Discrepancy of ΔT's from Solar and Lunar Eclipses*

The result of Stephenson (1997, figure 14.2) gives the present authors an impression that ΔT obtained from lunar eclipses between AD 300 and AD 600 is systematically larger than that obtained from three Chinese solar eclipses in the 7th century. Of course, ΔT might have changed very quickly during AD 600 and AD 650. The second interpretation may be that some of the data are erroneous. As a third interpretation, we here examine the possibility of changing the tidal term of the lunar motion. For reference, we give table 3 which shows the development of the determination of the tidal term. In the table, T is measured in units of 100 years. The adopted value at present is $-13^{\prime\prime}T^2$.

Before introducing the change in the tidal term, we need to confirm that it does not affect the ancient observations. As we can see in subsection 2.1, the slow down of the Earth's spin, and slow down of the orbital motion of the Moon have opposite effects on the place of eclipse zone. The Moon can be regarded approximately as moving in the parallel light from the Sun. Then, the distance that the Moon sweeps on its orbit is exactly the distance that its shadow sweeps on the Earth's surface, neglecting the curvature of the surface. One second of arc on the Moon's orbit corresponds to 1.86 km on the ground. The speed of the rotation of the Earth is $0.463\,\mathrm{km\,s^{-1}}$, that is, $\Delta T = 1$ s is equivalent with 0.463 km on the equator. $1^{\prime\prime}$ on the Moon's orbit is equivalent with $\Delta T = 4$s. So, roughly speaking, $\Delta T = 1000$s is equivalent with $250^{\prime\prime}$ on the Moon's orbit.

Now if we adopt $-2.^{\prime\prime}0\,T^2$ for the correction to the tidal term in DE 406, the time of lunar eclipses changes by only 10 minutes from the nominal value in the 7th century. This value is negligible when we consider the timing of the ancient lunar eclipses. On the other hand, the difference in the position of the Moon on its orbit affects the position of the total solar eclipse. Thus, for example, we need to increase ΔT to 3000 s to have a total solar eclipse of AD 628 in Japan.

However, the correction $-2\overset{\prime\prime}{.}0\,T^2$ is incompatible with the LLR (Lunar Laser Ranging) data. Moreover, if the tidal term changed from the present value in the past, the correction should be even larger. It is very difficult to find the cause of such a large change of the tidal term during only a thousand years, since the tidal interaction may change on a geological time scale. Thus, the third interpretation may not be plausible.

3. Examination

If the total eclipse band is transversal to latitude parallels, it is always possible to find, for a fixed value of the tidal term of the lunar deceleration, an appropriate value of ΔT for which the eclipse is total at the desired site, which lies in the latitude range of the total eclipse band. Indeed, as we have seen in subsection 2.1, if the past ΔT is larger than the expected value, the eclipse band is to the east of the expected site, whereas if the past tidal term of the lunar deceleration is larger, the eclipse band is to the west of the expected site.

Thus, it is easy to get the total eclipse in the case of the Suiko eclipse. This does not assure that the record in the history book is a real observation, nor that the record reflects the truth. In the case of the Suiko eclipse, there is no further description, such as ‘bright stars become visible’ or ‘inky darkness.’ We need circumstantial evidence if there is no direct evidence.

In subsection 3.1, we look for parameter values that give the Suiko eclipse total. In subsection 3.2, we examine a contemporary lunar occultation of Mars, and show that the reliability of two records are related in the sense that they give similar parameters. In subsection 3.3, we check the number of recorded eclipses in the Nihongi, and argue for the reality of the observation of the Suiko eclipse.

As for the general reliability of the records on natural phenomena in the volume of Empress Suiko, see the note added in proof.

3.1. The Suiko Eclipse

If we take $\Delta T = 4300$ s at around AD 650 as Stephenson (1997) did, then the zone of totality passes through the Pacific Ocean on AD 628 April 10. This implies that the Suiko eclipse would have been partial. If we take $\Delta T = 2000$ s, then the eclipse becomes total in the Japanese Islands. However, in this latter case, there seems to be an inconsistency between the ΔT’s derived from solar and lunar eclipses. The smoothed curve of ΔT given by Stephenson (1997) seems to be favorable to lunar eclipses.

In order to resolve the inconsistency, as a trial, we add $-2\overset{\prime\prime}{.}0\,T^2$ to the tidal term of the lunar ephemerides. As we already pointed out, this correction does not affect the lunar eclipses. As for solar eclipses, the effect is large. Indeed, the eclipse of Suiko becomes total if we adopt $\Delta T = 3000$ s. This resolves the inconsistency, that is, ΔT obtained from the solar eclipse approaches ΔT determined from lunar eclipses. The result is shown in figure 1. However, as we pointed out in subsection 2.2, the correstion $-2\overset{\prime\prime}{.}0\,T^2$ is too large.

3.2. Grazing Occultation

Ancient astronomical records contain occultation observations. As in the case of partial solar eclipses, the site from which a particular stellar occultation is visible covers a wide range. This means that most of the occultation data are useless for determining our parameters. Only the grazing type of occultations is useful. Fortunately enough, we have this kind of observation in the 7th century. The occultation of Mars by the Moon on AD 681 November 3 is this.

The Nihongi says that the reign of Emperor Tenmu, 10th year, 9th month, day *gui-chou* (50), Mars (熒惑) went into the Moon. Saito (1982a, p.14) states that “The magnitude of this planet in the night was $-1\overset{\mathrm{m}}{.}3$, the Moon age was 17.3, the apparent radius of the Moon was $0\overset{\circ}{.}28$. According to the calculation, at 2 am, November 4 in the Julian calendar, Mars passed by the northern edge of the Moon $0\overset{\circ}{.}04$ apart. The Nihongi says that it entered the Moon. But the truth is that it was a passing-by (Fan, 犯). Supposedly, the observer lost Mars in the glaring Moon of age 17.3.” Our opinion is different. Even the Moon was bright, after two days from the full Moon, it is possible that the observer follows the motion of Mars of $-1\overset{\mathrm{m}}{.}3$ at $0\overset{\circ}{.}04 = 2\overset{\prime}{.}4$ from the lunar limb. The distance is more than twice the size of the resolving power of the eyes of an ordinary person. In particular, Mars entered the disk of the Moon from the bright side, but it came out from the dark side, though it is very thin. It is true that there were no occultation, except for the southern part of Kyushu Island if we adopt the present tidal value and $\Delta T = 4000$s. However, if we adopt $\Delta T = 2000$s, then the occultation could have been observed in the southern part of Kinki area and the whole of Kyushu. Further, if we add a correction $-2''$ century^{-2} to the tidal term and adopt $\Delta T = 3000$ s, then the occultation could be observed in Western Japan, including the Mid-Kinki area and the whole of Kyushu. Mars may have passed by the limb of the Moon with a distance smaller than, or comparable to, $30''$, the resolving size of human eyes. In this case, our conclusion is not affected very much.

In summary, this record of the Nihongi supports our parameter values (see figures 1 and 2).

3.3. The Suiko Eclipse: Predicted or Observed?

There are arguments that the Suiko eclipse was predicted, but not observed. These arguments are rather baseless. The eclipse records of the Jito era are surely predictions because eclipses not observable in Japan were recorded, which reveals that the accuracy of the predictions was very low. On the other hand, in order to predict the totality of an eclipse, a high-precision calendar is necessary. As we pointed out above in subsubsection 1.1.2, the Japanese original calendar was first published only in the 17th century. In the Suiko era, Japanese astronomers must have used a Chinese calendar, i.e., the Yuanjia Calendar. They could not make an accurate prediction with calendars compiled at a different longitude. Actually, the Suiko eclipse was observed and recorded in China as a partial eclipse. If Japanese astronomers predicted the eclipse using the Chinese calendar, they might have predicted at the best a partial eclipse, since the longitude difference was not taken into account at that time. (In order to take into account the longitude difference, a long accumulation of observational

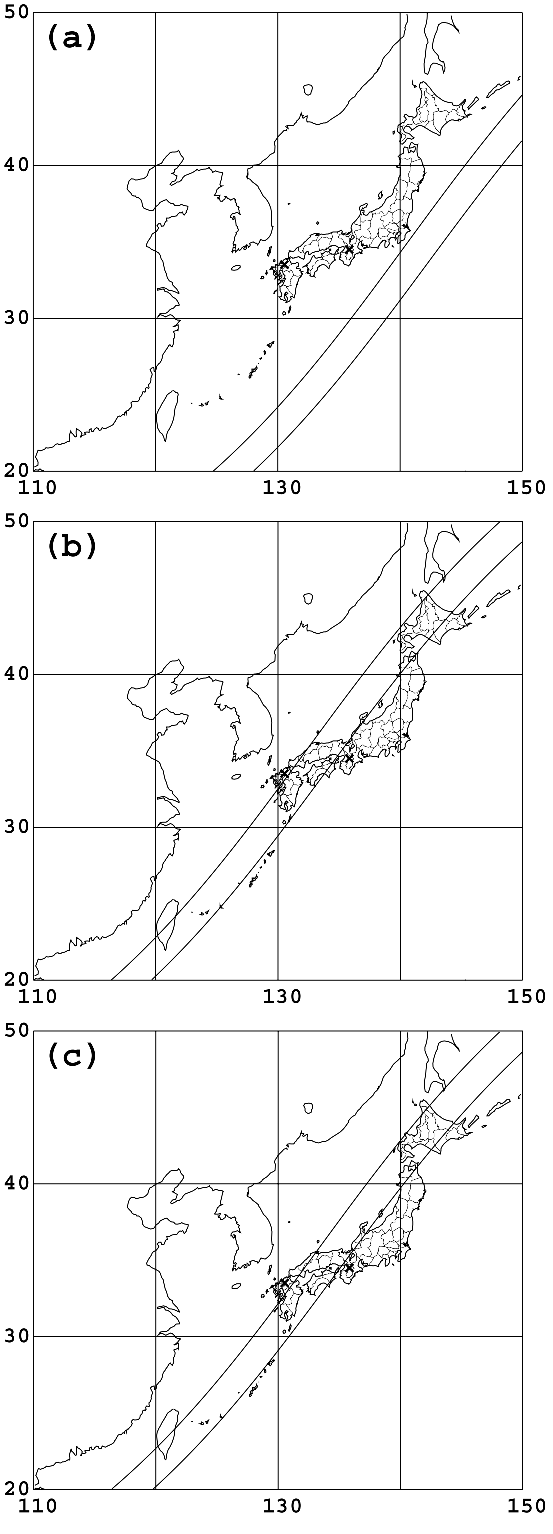

Fig. 1. Solar eclipse on AD 628 April 10 (thirty sixth year, third month, second day, of Empress Suiko). Shown are the zones of totality and possible observation sites, Asuka and Dazaifu, by crosses. Parameters are (a) $\Delta T = 4000\,\mathrm{s}$, correction to the tidal term = $0.''0\,T^2$, (b) $\Delta T = 2000$ s, correction to the tidal term = $0.''0\,T^2$, and (c) $\Delta T = 3000\,\mathrm{s}$, correction to the tidal term = $-2.''0T^2$.

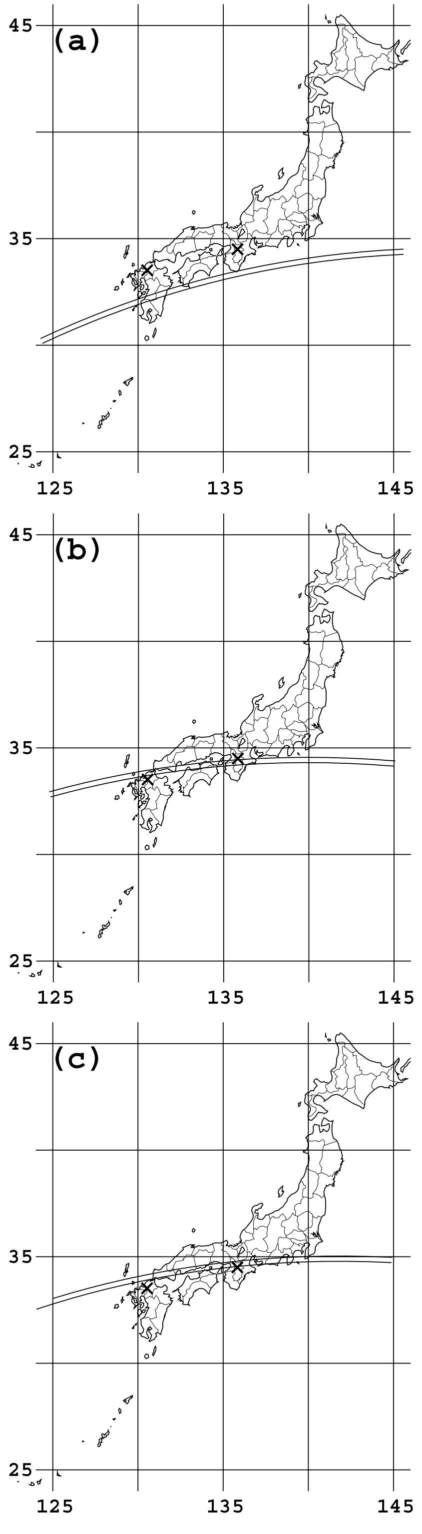

Fig. 2. Occultation of Mars by the Moon on AD 681 November 3. Shown are the bands of partial occultation. The total occultation took place to the south of the band. Parameters are (a) $\Delta T = 4000\,\mathrm{s}$, correction to the tidal term $t = 0.''0\,T^2$, (b) $\Delta T = 2000\,\mathrm{s}$, correction to the tidal term $t = 0.''0\,T^2$, and (c) $\Delta T = 3000\,\mathrm{s}$, correction to the tidal term $t = -2.''0T^2$.

data is necessary at the site where the calendar is to be edited.) In addition, we know that there were around twenty eclipses between AD 600 and AD 686 that could have been observed in western Japan. Only four eclipses other than one doubtful record of AD 637 are recorded in the Nihongi, whereas in the Jito era, AD 687–AD 696, six eclipses are recorded, though three eclipses could have been observed. Before the Jito era, only eclipses of large magnitude are recorded. This shows that there were no systematic predictions before the Jito era, whereas eclipses were predicted in the Jito era. Thus, the probability that the Suiko eclipse was predicted is very small.

4. Discussions and Summary

There may be a doubt that the interpretation of the term rendered "exhausted" really did not imply totality in early Japanese history. Nominally, 'exhausted' is equivalent with 'total'. No one disagrees with this interpretation. It is true that usage of 'total' and that of 'exhausted' were slightly different in ancient Chinese history. In China, 'exhausted' was used in cases where the eclipse was nearly complete — 'nearly exhausted', and the eclipse was not total but remained like a hook — 'not exhausted, but remained like a hook'. In this sense, the usage of the term 'exhausted' in the Suiko eclipse is exceptional. We cannot state a definite conclusion only from the usage of the term. One possibility is that astronomers in ancient Japan were not used to the difference in 'total' and 'exhausted', and they used 'exhausted' instead of 'total'. Another possibility is that Japanese astronomers expressed independence by using a different word. Yet another possibility is that a small portion of the sun remained and the portion was extremely small.

Previous Japanese authors believed that the ΔT, or its equivalent, at around AD 600 was 4000 s or larger. From this they judged that the record of the Suiko eclipse in the Nihongi was an exaggeration. This judgment has been in accordance with the idea of historians that the Nihongi contains much forgery. The present authors consider that the previous authors' arguments are incorrect because they take for granted that their supposed ΔT value or its equivalent is real.

As for the occultation of Mars by the Moon, the situation is similar. Previous authors argued based on $\Delta T \geq 4000$ s that Japanese observers must have missed the image of Mars in the glaring light of the Moon of the 17th day. Experiences in occultation observations of one of the authors (M. Sôma) says that if a star of magnitude -1 is apart from the lunar limb by more than twice the resolving size of human eyes, the star as a bright spot can be identified. Our tentative conclusion is that Mars at the worst passed by the lunar limb within a distance of $30''$, which is the resolving size of human eyes. Of course this should be examined further in real situations. In addition, occultations of planets could not be predicted in the seventh century because the motion of planets in the celestial sphere is complicated. This means that the record of AD 681 November 3/4 is not a prediction, but an observation. If the occultation on AD 681 November 3/4 is real, or at least a close passing-by of Mars near the lunar limb, then the eclipse on AD 628 April 10 was total at the capital of Japan and vice versa, provided that ΔT changes little in fifty years.

Finally, the authors would like to stress that though each of the two records may seem to be a datum of doubtful reliability, their reliability increases drastically if they are looked at together and together with contemporary Chinese observations of solar eclipses. The records of Suiko eclipse on AD 628 April 10 and the occultation of Mars on AD 681 November 3/4 are independent in the sense that ancient astronomers did not know that the totality of the eclipse on AD 628 April 10 is related to the occurrence of the occultation of Mars by the Moon. As we pointed out above, both are observations in high probability. These two observations are independent from Chinese eclipse data of the seventh century. Stephenson (1997, figure 14.2 and table A4) shows that the Chinese three solar eclipses give us an average ΔT value 2000 s smaller than the value on the fitted spline curve. These values are in accordance with the value derived from the reality of the Suiko eclipse and the occultation of Mars by the Moon.

Let us summarize our assertion:

i) The Suiko eclipse in high probability was actually observed. Observations should have been made by professionals. In fact, Wo (Old Japanese Empire) sent a letter of declaration of independence in AD 607, which implies that official astronomers may have existed in AD 628 (K. Kawabata 2001, private communication). Evidence is that the records of other astronomical phenomena started at nearly the same epoch: comets (AD 634), bright meteors (AD 637), approaches of stars and the Moon (AD 640), lunar eclipses (AD 643). These cannot be the work of amateurs. We know that Wo might have used the Chinese calendar issued in the fifth century (Ogawa 1946).

ii) If we take $\Delta T = \mathrm{TT} - \mathrm{UT} = 2000$ s, the eclipse was total in western Japan. The contemporary Chinese total eclipse data also are favorable to this value of ΔT, as is shown by Stephenson (1997). The lunar occultation of Mars on AD 681 November 3 is also explainable as in the record. At worst, the phenomenon was a very close passing-by of Mars along the lunar limb.

iii) If we take $\Delta T = 3000$ s and introduce a correction $-2\overset{''}{.}0$ to the adopted value of the coefficient of T^2 of the lunar longitude, both records are real observations. However, correction $-2\overset{''}{.}0$ may be too large, since the tidal interaction between the Earth and Moon may change only on a geological timescale.

There remains a problem. A correction of $-2''$ century^{-2} to the coefficient of the tidal term is too large. An alternative explanation may be necessary for the discrepancy between the deceleration of the lunar orbital motion and the rate of spin down of the Earth.

Our next target will be to obtain ΔT and the motion of the Moon for various epochs using ancient Chinese and other astronomical data, including occultation data. We have started to compile these data in a machine readable form from Siku-Quanshu (四庫全書). In this direction, Stephenson and Morrison (1995) and Stephenson (1997) obtained long-term variations of the Earth's rotation using mainly solar and lunar eclipses, and fixing the motion of the Moon. Interestingly, their results have the largest inconsistency at around the seventh and eighth centuries, in which epoch we are most interested. Our ideas are to include grazing occultation observations. The simplest approximation will be to determine the coefficients of equation (14). As another effort, we will model the change of the inertial moment of the Earth due to the variation in the amount of polar ice.

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Note added in proof (2002 May 1):

We have strong support that the eclipse in AD 628 was really observed. Kawabata et al. (2002, 2004) examined the reliability of astronomical records in the Nihongi. Their work is based on the classification of various volumes of the Nihongi by historian Mori (2000). According to Mori, all volumes of the Nihongi are divided into three groups: α, β, and the remaining volume 30 (Jito; Kawabata et al. called this γ). Mori's conclusion is that the authors are different for different groups of the volume. Kawabata, Tanikawa, and Sôma (2002) argue that astronomical data in volumes of β group are real, i.e., are based on observations. Volumes 22 (Suiko), 23 (Jomei), 28, 29 (Tenmu) belong to β. The reality is also true for geophysical observations, such as volcano activities and earthquakes. In volumes of group α, there are no astronomical data, except for one lunar eclipse, which could not have been observed from Japan. In volume 30 (Jito), group γ, the astronomical data are all predictions, as is well known. The argument of Kawabata et al. (2002) is convincing because the classifications of the volumes of the Nihongi were independently made based on quite different reasonings.

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