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Units of Time in Ancient China and Japan

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Abstract

The time systems employed in ancient China and Japan are discussed. It is well known that both in ancient China and Japan 1 day was divided into 12 double hours, and the first double hour began at 23 hr local time. However, it is confirmed in this paper that in the Chinese Song dynasty the first double hour began at 0 hr local time. One day was also divided into 100 equal parts, called ke, and ke was subdivided by a time unit called fen. The number of fen in 1 ke varied from dynasty to dynasty. These numbers were clarified by analyzing the tables of daytime duration given in the official Chinese chronicles. In ancient Japan, the time units ke and fen were also used, but the lengths of both of them varied depending on the era. It has been found that all of the daytime and nighttime, the times of sunrise and sunset, and the lengths of shadows given in the official Chinese chronicles refer to a particular latitude of about $34^{\circ}\!.5$, and that the Japanese system adopted this Chinese tradition. Symmetry of the data in tables with respect to certain dates was also investigated in detail in order to examine how the dates of 24 qis were determined.
Key words: ancient China — ancient Japan — history — time system — time unit

1. Introduction

Recently, studies on the historical records of astronomical phenomena by advanced techniques have been highlighted for investigations concerning the variation of the rate of the Earth's rotation, solar activities, geophysical phenomena, and so on (e.g. Stephenson, Morrison 1984; Han, Zhang 1996; Stephenson 1997, 2003; Tanikawa, Sôma 2004a,b; Kawabata et al. 2004). For studies using ancient astronomical records, such as timed solar eclipses, detailed knowledge on ancient units of time is required.

A detailed description on the units of time used in China can be found in official Chinese chronicles (the 24 Chronicles). On the other hand, Japanese units of time did not appear in official histories, like Nihongi, but were scattered in various private histories, diaries, temple records, and so on. Ancient Chinese units of time are described by Stephenson (1997) and Steele (2000). Ancient Japanese units of time are described by Hirayama (1913a,b) and Hashimoto (1966) in Japanese. However, their descriptions are insufficient to understand the accuracy of the records and relationship between Chinese and Japanese systems of units of time. Although the ancient Japanese astronomical system came from China, and used the same characters for time units, we can find some differences between the Chinese and Japanese systems of time units. The present article compiles and analyzes the Chinese official units of time until the Song ⟨Sung⟩ 宋 dynasty (13th century) and the Japanese official units of time until the end of the Muromachi shogunate (16th century) for the convenience of investigators of ancient and medieval Chinese and Japanese records using recorded times.

There are several ways to express Chinese terms in the Roman alphabet. In the past, the Wade–Giles system was commonly used, and it was this system that Stephenson (1997) and Steele (2000) used for Chinese terms. In 1979 a system called Pinyin was officially adopted in China, and is now widely used. Therefore, this Pinyin system is used in this article, although accents above individual letters indicating the tones of the vowels are omitted. However, for a comparison with past papers, some basic Chinese terms are also written in the Wade–Giles system using angle brackets in this article.

In China, the interval from one midnight to the next was divided into 100 equal parts from the legendary era, which were called ke ⟨k'o⟩ 刻. Stephenson (1997) and Steele (2000) use the term "mark" as the translation of ke in English.

For the representation of time, the twelve zhis ⟨chihs⟩ 十二支 were used. In this system, the interval from one midnight to the next was divided into 12 equal parts, and each double hour was named according to the twelve zhis (see Appendix). Each double hour was subdivided into ke. Because 1 ke is equivalent to 0.24 hours, a single double hour is equivalent to $[8 + (1/3)]$ ke. In spite of this inconvenience of not having an integral number of ke in a double hour, the same system of the units of time continued until AD 1628, except for very short exceptional periods, in China. In these exceptional periods, the number of ke in a day was 96, 108, or 120. After 1628, the number of ke in a day became 96. Furthermore, each ke was subdivided into fen 分. The length of fen depends on the dynasty and the system.

In Yuanshi ⟨Yuan-shih⟩ 元史, a modified system of units of time appeared. In this new system, each double hour was subdivided into two single hours. The first hour was called "initial" 初 and the second hour was called "central" 正. Each single hour was equivalent to $[4 + (1/6)]$ ke. According to

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Yuanshi, this modified system of units of time appeared to be used from the end of the Tang 唐 dynasty. Tables showing the relation between the times in both the double-hour system and the single-hour system and the local mean time are given by Stephenson (1997) and Steele (2000), although 1 hour should be added to their values for the times used in the Song dynasty, as is pointed out in subsection 3.6.

The Chinese system of time also employed geng ⟨keng⟩ 更 (night watches). This unit geng was a seasonal unit of time, and was defined as one fifth of the length of the time from dusk to dawn. Furthermore, each geng was subdivided into five equal parts, and each of them was called dian ⟨tien⟩ 点 (point). Because Stephenson (1997) describes geng in detail, we do not add any other description concerning geng in the Chinese system.

For the representation of the time, the twelve zhis and ke were also used in Japan, but the number of ke in a day was generally different from that used in China. In Japan, time intervals equally divided into 48 or 50 of the interval from one midnight to the next were also used as a unit of time, and they were also called ke. Although the same characters were used in Japan as in China, their pronunciations were different, in principle. However, the Chinese romanisation is also used for the Japanese units of time in this article for the convenience of comparing the Japanese and Chinese units of time.

Section 2 summarizes the Chinese calendar system and section 3 discusses the time systems appearing in the official Chinese history books in chronological order of the dynasties by comparing the data given in the official history books with calculations. Section 4 discusses the time systems appearing in various Japanese records, and section 5 summarizes the results found in this paper. Appendix gives the names of the 12 double hours employed in ancient China and Japan along with their corresponding time intervals.

2. Chinese System of Calendar

Although the contents given in this section may be well-known, we include it because they are needed to understand the rest of this article as background information.

The Chinese calendar system is luni–solar, and is thus based on the movement of the moon and the sun. The 1st day of the month is defined as the day on which the sun and moon are at conjunction. The mean period from a new moon to the next is 29.53… days, and a tropical year of 365.24… days is in excess of 12 months by 10.87… days. In order to reconcile the year and month, it was necessary to make a leap year with an intercalary month every two or three years. Initially, an intercalary month was inserted when it was judged to be necessary ad hoc.

After around 600 BC, the Chinese system used 24 qis ⟨ch’i⟩ 気 (table 1) starting from the winter solstice. The length of each qi was 15 days. The 1st day of each qi was defined as the day on which the solar ecliptic longitude was an integral multiple of 15°. The summation of the length of each qi was 360 days, and a tropical year was in excess of about 5.24 days over 24 qis. Thus, the Chinese system added an intercalary day, called mori 没日, once roughly every 70 days to make sure that the solar ecliptic longitude on the 1st day of qi would be an integral multiple of 15°.

Table 1. The 24 qis.*

L	Name of qi	Date
270°	冬至 dongzhi ⟨tung-chih⟩	Dec 22
285°	小寒 xiaohan ⟨hsiao-han⟩	Jan 6
300°	大寒 dahan ⟨ta-han⟩	Jan 21
315°	立春 lichun ⟨li-ch’un⟩	Feb 4
330°	雨水 yushui ⟨yu-shui⟩	Feb 19
345°	驚蟄 jingzhe ⟨ch’ing-che⟩	Mar 6
0°	春分 chunfen ⟨ch’un-fen⟩	Mar 21
15°	清明 qingming ⟨ch’ing-ming⟩	Apr 5
30°	穀雨 guyu ⟨ku-yu⟩	Apr 21
45°	立夏 lixia ⟨li-hsia⟩	May 6
60°	小満 xiaoman ⟨hsiao-man⟩	May 22
75°	芒種 mangzhong ⟨mang-chung⟩	Jun 6
90°	夏至 xiazhi ⟨hsia-chih⟩	Jun 22
105°	小暑 xiaoshu ⟨hsiao-shu⟩	Jul 8
120°	大暑 dashu ⟨ta-shu⟩	Jul 23
135°	立秋 liqiu ⟨li-ch’iu⟩	Aug 8
150°	處暑 chushu ⟨ch’u-shu⟩	Aug 24
165°	白露 bailu ⟨po-lu⟩	Sep 8
180°	秋分 qiufen ⟨ch’iu-fen⟩	Sep 23
195°	寒露 hanlu ⟨han-lu⟩	Oct 9
210°	霜降 shuangjiang ⟨shuang-hsiang⟩	Oct 24
225°	立冬 lidong ⟨li-tung⟩	Nov 8
240°	小雪 xiaoxue ⟨hsiao-hsueh⟩	Nov 23
255°	大雪 daxue ⟨ta-hsueh⟩	Dec 8

* L is the Sun’s apparent ecliptic longitude. Date is the approximate date in the Gregorian Calendar. 冬至 dongzhi, 春分 chunfen, 夏至 xiazhi, and 秋分 qiufen correspond to winter solstice, vernal equinox, summer solstice, and autumnal equinox, respectively. 驚蟄 was also written as 啓蟄 in Suishu 隋書 and Jiutangshu 舊唐書.

A day with the solar ecliptic longitude having an integral multiple of 30° was called zhong 中 and a day with the solar ecliptic longitude of an odd number multiple of 15° was called jie 節, to give a notch. Since the average length from a zhong to the next is about 30.4 days when we add intercalary days, and the average length from a new moon to the next is about 29.5 days, not all of lunar months include zhong. The month without zhong was inserted in the calendar as an intercalary month, and the year with an intercalary month was called a leap year.

There are two methods, called pingqi 平気 (or hengqi 恒気) and dingqi 定気, for determining the dates of 24 qis. In pingqi the variation of the speed of the Sun on the celestial sphere is ignored, and therefore the dates of 24 qis are determined by dividing the length of the tropical year into 24 equal parts, whereas in dingqi the dates of 24 qis are determined by the apparent longitude of the Sun by taking into account the variation of the Sun’s speed.

According to Yabuuchi (1969, p.88), in China the variation of the Sun’s speed was first recognized by Zhang Zixin 張子信 in c. AD 550, and knowledge of it was used to calculate eclipses in the Sui 隋 dynasty (581–619) onwards, but the dates by dingqi were first introduced in the calendar made in the Qing ⟨Ch’ing⟩ 清 dynasty (1616–1912).

Table 2. Comparison of the daytime duration given in Houhanshu with a calculation.

氣 Qi	晝漏刻 Daytime	夜漏刻 Nighttime	Duration from sunrise to sunset (ke)	Rec	Cal	Rec − Cal (min) without refraction 平気 pingqi	定気 dingqi	with refraction 平気 pingqi	定気 dingqi
冬至	45 刻	55 刻	40.0	9:36.0	9:40.1	−4.1	−4.0	−10.6	−10.5
小寒	45 刻 8 分	54 刻 2 分	40.8	9:47.5	9:47.3	+0.2	+1.7	−6.2	−4.8
大寒	46 刻 8 分	53 刻 2 分*	41.8	10:01.9	10:05.3	−3.4	−0.2	−9.6	−6.5
立春	48 刻 6 分	51 刻 4 分	43.6	10:27.8	10:30.9	−3.1	+1.5	−9.1	−4.5
雨水	50 刻 8 分	49 刻 2 分	45.8	10:59.5	11:00.9	−1.4	+4.1	−7.2	−1.7
驚蟄	53 刻 3 分	46 刻 7 分	48.3	11:35.5	11:32.9	+2.6	+8.4	−3.1	+2.6
春分	55 刻 8 分	44 刻 2 分	50.8	12:11.5	12:05.6	+5.9	+11.5	+0.2	+5.8
清明	58 刻 3 分	41 刻 7 分	53.3	12:47.5	12:37.8	+9.7	+14.6	+3.9	+8.9
穀雨	60 刻 5 分	39 刻 5 分	55.5	13:19.2	13:08.6	+10.6	+14.6	+4.7	+8.7
立夏	62 刻 4 分	37 刻 6 分	57.4	13:46.6	13:36.4	+10.2	+12.9	+4.1	+6.8
小満	63 刻 9 分	36 刻 1 分	58.9	14:08.2	13:59.2	+9.0	+10.3	+2.7	+4.1
芒種	64 刻 9 分	35 刻 1 分	59.9	14:22.6	14:14.5	+8.1	+8.4	+1.6	+2.0
夏至	65 刻	35 刻	60.0	14:24.0	14:20.0	+4.0	+4.0	−2.5	−2.5
小暑	64 刻 7 分	35 刻 3 分	59.7	14:19.7	14:14.5	+5.2	+5.5	−1.3	−0.9
大暑	63 刻 8 分	36 刻 2 分	58.8	14:06.7	13:59.0	+7.7	+8.8	+1.4	+2.6
立秋	62 刻 3 分	37 刻 7 分	57.3	13:45.1	13:35.7	+9.4	+11.4	+3.3	+5.3
處暑	60 刻 2 分	39 刻 8 分	55.2	13:14.9	13:07.2	+7.7	+10.3	+1.8	+4.4
白露	57 刻 8 分	42 刻 2 分	52.8	12:40.3	12:35.7	+4.6	+7.4	−1.2	+1.7
秋分	55 刻 2 分	44 刻 8 分	50.2	12:02.9	12:02.7	+0.2	+2.9	−5.4	−2.8
寒露	52 刻 6 分	47 刻 4 分	47.6	11:25.4	11:29.3	−3.9	−1.7	−9.6	−7.5
霜降	50 刻 3 分	49 刻 7 分	45.3	10:52.3	10:56.7	−4.4	−3.1	−10.2	−8.9
立冬	48 刻 2 分	51 刻 8 分	43.2	10:22.1	10:26.6	−4.5	−4.2	−10.5	−10.2
小雪	46 刻 7 分	53 刻 3 分	41.7	10:00.5	10:01.7	−1.2	−1.6	−7.4	−7.9
大雪	45 刻 5 分	54 刻 5 分	40.5	9:43.2	9:45.2	−2.0	−2.6	−8.4	−9.1
冬至	45 刻	55 刻	40.0	9:36.0	9:40.1	−4.1	−4.0	−10.6	−10.5

* The nighttime duration of dahan was given as 53 刻 8 分, but it is inconsistent with the daytime duration, and therefore the value 53 刻 2 分 is adopted.

3. Chinese Units of Time

In this section, we describe the time system appearing in each official history book in chronological order of the dynasties.

The word ke as a unit of time first appeared in official Chinese histories in Chapters 11, 26, and 75 of Hanshu ⟨Han-shu⟩ 漢書 edited by Ban Gu ⟨Pan Ku⟩ 班固 (compiled in c. AD 82) in the Houhan dynasty; in Chapter 26 it is written that the number of ke was temporarily changed from 100 to 120, but any further detailed description on the units of time can not be found in Hanshu.

The first detailed description on units of time can be found in Houhanshu ⟨Hou-han-shu⟩ 後漢書. According to Songshu ⟨Sung-shu⟩ 宋書, astronomical records of Houhan (Later Han) were in the hand of Wu 呉 when Wei 魏 usurped the throne of Houhan. These materials were finally in the hand of Song 宋 of Nanchao 南朝. In the next official history, Sanguozhi ⟨San-kao-shih⟩ 三国志, edited by Chen Shou 陳壽 in AD 285–297, none of the descriptions on units of time can be found, although Chapter 53 of Yuanshi says that the double hour system employing the twelve zhis was already used in the Sanguo ⟨San-kao⟩ 三国 era.

Descriptions of time units next to Houhanshu can be found in Jinshu ⟨Chin-shu⟩ 晉書, Songshu, Suishu ⟨Sui-shu⟩ 隋書, Jiutangshu ⟨Chiu-t'ang-shu⟩ 舊唐書, Songshi ⟨Sung-shih⟩ 宋史, and so on in chronological order of the dynasties. In chapters describing the system of time before the Sui 隋 dynasty, the durations of daytime and nighttime were given, but the times of sunrise and sunset were not. The durations of daytime and nighttime were described by using units of ke and fen. Suishu is the first official history book in which the times of sunrise and sunset were described by using the twelve zhis.

3.1. *Units of Time in Houhanshu*

Houhanshu is an official history book about the Houhan dynasty (AD 25–AD 220), and was compiled by Fan Ye ⟨Fan Yeh⟩ 范曄 in Song of Nanchao in c. AD 432.

In Houhanshu (志第三 律暦下 暦法) are given the durations of daytime and nighttime, the length of the shadow of a vertical pole with a standard height of 8 Chinese feet 尺 at noon, and the Sun's polar distance for the 24 qis throughout the year. We compared the recorded data with the calculated ones; the results are given in tables 2and 3. The calculation was made for the epoch of AD 100 at Yangcheng ⟨Yang-ch'eng⟩ 陽城 (latitude 34.°43) because, as we show below (table 3), there is

Table 3. Comparison of the shadow lengths and the Sun's polar distances given in Houhanshu with a calculation.

氣	Shadow length*		Rec − Cal		黄道去極†			Rec − Cal	
Qi	影		平気	定気	Sun's polar distance			平気	定気
	Rec	Cal	pingqi	dingqi	Rec		Cal	pingqi	dingqi
	(Chinese ft)		(C.ft)	(C.ft)	(C.deg)	(deg)	(deg)	(deg)	(deg)
冬至	13.	12.85	+0.15	+0.14	115	113.35	113.67	−0.32	−0.33
小寒	12.3	12.34	−0.04	−0.14	113 強	111.50	112.61	−1.11	−1.33
大寒	11.	11.13	−0.13	−0.33	110 大弱	109.10	109.86	−0.76	−1.25
立春	9.6	9.59	+0.01	−0.25	106 少強	104.79	105.74	−0.95	−1.71
雨水	7.95	8.02	−0.07	−0.34	101 強	99.67	100.63	−0.96	−1.91
驚蟄	6.5	6.56	−0.06	−0.31	95 強	93.76	94.92	−1.16	−2.21
春分	5.25	5.28	−0.03	−0.23	89 強	87.85	88.98	−1.13	−2.15
清明	4.15	4.18	−0.03	−0.19	83 少弱	81.99	83.14	−1.15	−2.04
穀雨	3.2	3.26	−0.06	−0.17	77 大強	76.70	77.73	−1.03	−1.72
立夏	2.52	2.52	0.00	−0.06	73 少弱	72.14	73.06	−0.92	−1.36
小満	1.98	1.97	+0.01	−0.03	69 大弱	68.69	69.43	−0.74	−0.96
芒種	1.68	1.63	+0.05	+0.04	67 少弱	66.22	67.12	−0.90	−0.95
夏至	1.5	1.52	−0.02	−0.02	67 強	66.16	66.32	−0.16	−0.16
小暑	1.7	1.64	+0.06	+0.06	67 大強	66.84	67.12	−0.28	−0.33
大暑	2.	1.98	+0.02	−0.01	70	69.00	69.47	−0.47	−0.65
立秋	2.55	2.54	+0.01	−0.04	73 半強	72.57	73.17	−0.60	−0.93
處暑	3.33	3.30	+0.03	−0.04	78 半強	77.50	77.96	−0.46	−0.92
白露	4.35	4.24	+0.11	+0.01	84 少強	83.10	83.52	−0.42	−0.93
秋分	5.5	5.38	+0.12	+0.02	90 半強	89.32	89.51	−0.19	−0.68
寒露	6.85	6.72	+0.13	+0.04	96 大強	95.42	95.59	−0.17	−0.55
霜降	8.4	8.22	+0.18	+0.11	102 少強	100.84	101.36	−0.52	−0.74
立冬	10.*	9.84	+0.16	+0.15	107 少強	105.77	106.45	−0.68	−0.73
小雪	11.4	11.36	+0.04	+0.07	111 弱	109.28	110.43	−1.15	−1.07
大雪	12.56	12.49	+0.07	+0.12	113 大強	112.18	112.92	−0.74	−0.65
冬至	13.	12.85	+0.15	+0.14	115	113.35	113.67	−0.32	−0.33

* The shadow length of lidong was given as 10.42 尺 in the Electronic Version of Siku Quanshu 四庫全書 (1999) and as 10 尺 in a book published by Zhonghua-shuju 中華書局 (1965).[1] Here 10 尺 by Zhonghua-shuju is adopted.
† Four values are different between Siku Quanshu and Zhonghua-shuju. Those by Zhonghua-shuju are adopted here because their variation seems to be smoother than the other.

an evidence that the observations were made at Yangcheng near the capital Luoyang ⟨Lo-yang⟩ 洛陽. To calculate the Sun's positions, we used DE406, which covers the interval 3000 BC to AD 3000 (Standish 1998).

Table 2 gives the durations of daytime and nighttime recorded in Houhanshu. Since the sum of the durations of daytime and nighttime must be equal to 100 ke, it is clear that each ke was subdivided into 10 fen.

The daytime duration here includes dawn 旦 of 2.5 ke before sunrise and dusk 昏 of 2.5 ke after sunset (2.5 ke equals 36 minutes of the current unit of time); therefore, the duration from sunrise to sunset (the 4th column in the table) is obtained by subtracting 5 ke from the daytime duration and converted into hours and minutes of the current unit in the 5th column. The fact that the sum of the duration from sunrise to sunset of dongzhi 冬至 (winter solstice) and xiazhi 夏至 (summer solstice) is 100 ke suggests that sunrise and sunset were determined by the center of the Sun's disk without atmospheric refraction, but it is not clear whether this can be applied to the other dates or not. Also, as noted in section 2, there were two methods, called pingqi (or hengqi) and dingqi, for determining the dates of 24 qis; it should be clarified which method was applied when constructing the table in Houhanshu. Therefore, a comparison is made for four cases: pingqi and dingqi without refraction and pingqi and dingqi with refraction; the differences of the recorded duration with the calculation are given in the 7th to 10th columns. The calculated duration in the 6th column is the one by pingqi without refraction. In the calculation, the dates of 24 qis by pingqi are determined by assuming that the day of xiazhi coincides with that by dingqi. In all of the cases, the calculated daytime duration is obtained by assuming that sunrise and sunset are the instances when the center of the Sun's disk coincides with the horizon.

The fact that the recorded duration from sunrise to sunset is not symmetrical with respect to dongzhi or xiazhi indicates that the 24 qis were based on pingqi. Especially the fact, for example, that the duration on the day of xiaohan 小寒 is longer than that on the day of daxue 大雪, and that the duration on the day of chunfen 春分 (vernal equinox) is longer than that on

[1] Books cited in the name of Zhonghua-shuju (year) are put under Ershisi Shi 1959–, in References.

the day of qiufen 秋分 (autumnal equinox) is consistent with the duration by pingqi. The differences between the recorded duration and the calculated ones are also on the whole smaller for the dates by pingqi. From qiufen to chunfen the recorded duration agrees well with that calculated without refraction, but the recorded duration between chunfen and qiufen agrees well with that with refraction. However, it is hard to believe that the determining method of sunrise and sunset differed according to the seasons.

The above fact might indicate that the water clocks, called louke 漏刻, used at that time had a daily variation that varied with the seasons, such that the clocks ran faster in the daytime of the summer season.

Table 3 gives the length of a shadow of a vertical pole, called gnomon (zhoubi 周髀 or biao 表 in Chinese), with a standard height of 8 Chinese feet 尺 at noon, and the Sun's polar distance for the 24 qis throughout the year. One Chinese foot 尺 was between about 20 cm and 30 cm, and it may have varied depending on the dynasties (Wu 1981). The values in the columns headed by "Cal" are the calculated ones for the epoch AD 100 at Yangcheng using DE406, and assuming that the dates of 24 qis were determined by pingqi.

The recorded shadow lengths and Sun's polar distances are not symmetrical with respect to dongzhi or xiazhi, either. When one compares the values for the dates before and after xiazhi, which are equally apart from xiazhi, one can find that the values after xiazhi are larger than those before xiazhi. These facts also indicate that the 24 qis were based on pingqi.

Let h, d, and δ denote the height and shadow length of the pole, and the Sun's declination. The relation of the latitude, ϕ, of the observation station with those values is

$$\phi = \delta + \arctan(d/h). \tag{1}$$

If we solve the latitude, ϕ, from the recorded shadow lengths using the theoretical declination, δ, of the Sun for the dates based on pingqi, and assuming the equal accuracy of the measured shadow lengths throughout the year, we then obtain the latitude as

$$\phi = 34^{\circ}\!.54 \pm 0^{\circ}\!.06 \tag{2}$$

by the least squares method.

The fact that the obtained latitude is closer to that ($34^{\circ}\!.43$) of Yangcheng than that ($34^{\circ}\!.75$) of the capital Luoyang suggests that the astronomical observations in the Han era were made in Yangcheng, as discussed by Needham and Wang (1959). The difference of about $0^{\circ}\!.1$ of the obtained latitude from that ($34^{\circ}\!.43$) of Yangcheng might be produced by a systematic error of measurements because the shadow has both umbra and penumbra due to the Sun's apparent semidiameter of about $0^{\circ}\!.25$.

The polar distances were recorded in Chinese degrees (6th column of table 3), and using the relation 365.25 Chinese degree = 360° they were converted to degrees (7th column of table 3). For the recorded polar distances of the Sun, the fraction of a Chinese degree was indicated by one or two descriptive characters, like large 大 or over 強. It is not clear what fractions these characters refer to, but the following conversions are adopted here:

$$\begin{aligned} &弱 = -1/8, 強 = 1/8, 少弱 = 3/16, 少 = 1/4, \\ &少強 = 5/16, 半弱 = 3/8, 半 = 1/2, 半強 = 5/8, \\ &大弱 = 11/16, 大 = 3/4, 大強 = 13/16. \end{aligned} \tag{3}$$

The mean values of the residuals Rec − Cal are $-0^{\circ}\!.71$, assuming that the dates were determined by pingqi, and $-1^{\circ}\!.09$ assuming that the dates were determined by dingqi. These differences are larger than the difference in latitude from the shadow lengths. This fact might indicate that the astronomical instruments used at that time to measure the angles between the pole and the Sun had systematic errors, or that the conversion table of the Sun's altitudes from the shadow lengths used at that time was inaccurate.

3.2. *Units of Time in Jinshu*

Jinshu is an official history book about the Jin dynasty (AD 265–AD 420), which was compiled by Fang Xuanling ⟨Fang Hsuan-ling⟩ 房玄齢 et al. in the Tang dynasty in AD 648.

The daytime and nighttime durations, the shadow length, and the Sun's polar distance are given in Chapter 18 of Jinshu. The values of the daytime and nighttime durations and the shadow lengths are exactly the same as those in Houhanshu. The values of the Sun's polar distance in Jinshu also agree with those in Houhanshu, except for a few cases where descriptive characters indicating a fraction of a Chinese degree disagree. However, considering the fact that there were also a few cases in Houhanshu that the descriptive characters disagree between the Electronic Version of Siku Quanshu 四庫全書 (1999) and the book published by Zhonghua-shuju 中華書局 (1974), the differences can be regarded as being miswritings in copying a previous version to the next one. Therefore, we conclude that all of the data concerning the daytime and nighttime durations, the shadow length, and the Sun's polar distance in Jinshu are those copied from Houhanshu.

3.3. *Units of Time in Songshu*

Songshu is an official history book about the Song dynasty (AD 420–AD 479) of Nanchao, and was compiled by Chen Yue ⟨Shen Yueh⟩ 沈約 in the Liang ⟨Liang⟩ 梁 dynasty of Nanchao in AD 487.

The daytime and nighttime durations, and the shadow length at noon are given in Songshu Chapter 13 (巻十三 志第三暦下 元嘉暦法).

Table 4 gives the durations of daytime and nighttime recorded in Songshu, and a comparison is made with the calculated duration, as in table 2. The calculation is made for the epoch AD 400 at Yangcheng using DE406. As in table 2, a comparison is made for four cases: pingqi and dingqi without refraction and pingqi and dingqi with refraction; the differences of the recorded duration with the calculation are given in the 7th to 10th columns. The calculated duration in the 6th column is the one by pingqi without refraction. Unlike the data in Houhanshu, the recorded duration from sunrise to sunset is symmetrical with respect to dongzhi or xiazhi (see figure 1). This fact suggests that the 24 qis were based on dingqi, but a more detailed comparison leads to the fact that the recorded duration agrees better with the calculation based on pingqi. This fact seems to show that the recorded data were obtained from the observed ones based on the dates of

Table 4. Comparison of the daytime duration given in Songshu with a calculation.

氣 Qi	晝漏刻 Daytime	夜漏刻 Nighttime	Duration from Sunrise to Sunset (ke)	Rec	Cal	Rec − Cal (min) without refraction 平気 pingqi	定気 dingqi	with refraction 平気 pingqi	定気 dingqi
冬至	45 刻	55 刻	40.0	9:36.0	9:40.3	−4.3	−4.3	−10.8	−10.8
小寒	45 刻 6 分	54 刻 4 分	40.6	9:44.6	9:47.3	−2.7	−1.5	−9.1	−7.9
大寒	46 刻 7 分	53 刻 3 分	41.7	10:00.5	10:05.1	−4.6	−1.8	−10.8	−8.1
立春	48 刻 4 分	51 刻 6 分	43.4	10:25.0	10:30.5	−5.5	−1.4	−11.5	−7.5
雨水	50 刻 5 分	49 刻 5 分	45.5	10:55.2	11:00.5	−5.3	−0.3	−11.1	−6.1
驚蟄	52 刻 9 分	47 刻 1 分	47.9	11:29.8	11:32.6	−2.8	+2.6	−8.5	−3.1
春分	55 刻 5 分	44 刻 5 分	50.5	12:07.2	12:05.3	+1.9	+7.2	−3.8	+1.5
清明	58 刻	42 刻	53.0	12:43.2	12:37.6	+5.6	+10.4	−0.1	+4.7
穀雨	60 刻 3 分	39 刻 7 分	55.3	13:16.3	13:08.4	+7.9	+11.8	+2.1	+5.9
立夏	62 刻 3 分	37 刻 7 分	57.3	13:45.1	13:36.2	+8.9	+11.5	+2.8	+5.5
小満	63 刻 9 分	36 刻 1 分	58.9	14:08.2	13:59.0	+9.2	+10.5	+2.9	+4.3
芒種	64 刻 8 分	35 刻 2 分	59.8	14:21.1	14:14.3	+6.8	+7.2	+0.3	+0.7
夏至	65 刻	35 刻	60.0	14:24.0	14:19.7	+4.3	+4.3	−2.2	−2.2
小暑	64 刻 8 分	35 刻 2 分	59.8	14:21.1	14:14.3	+6.8	+7.2	+0.4	+0.7
大暑	63 刻 9 分	36 刻 1 分	58.9	14:08.2	13:58.9	+9.3	+10.5	+3.1	+4.3
立秋	62 刻 3 分	37 刻 7 分	57.3	13:45.1	13:35.7	+9.4	+11.5	+3.3	+5.5
處暑	60 刻 3 分	39 刻 7 分	55.3	13:16.3	13:07.4	+8.9	+11.8	+3.0	+5.9
白露	58 刻	42 刻	53.0	12:43.2	12:36.0	+7.2	+10.4	+1.4	+4.7
秋分	55 刻 5 分	44 刻 5 分	50.5	12:07.2	12:03.1	+4.1	+7.2	−1.6	+1.5
寒露	52 刻 9 分	47 刻 1 分	47.9	11:29.8	11:29.9	−0.1	+2.6	−5.8	−3.1
霜降	50 刻 5 分	49 刻 5 分	45.5	10:55.2	10:57.4	−2.2	−0.3	−8.0	−6.1
立冬	48 刻 4 分	51 刻 6 分	43.4	10:25.0	10:27.3	−2.3	−1.4	−8.4	−7.5
小雪	46 刻 7 分	53 刻 3 分	41.7	10:00.5	10:02.4	−1.9	−1.8	−8.1	−8.1
大雪	45 刻 6 分	54 刻 4 分	40.6	9:44.6	9:45.7	−1.1	−1.5	−7.5	−7.9
冬至	45 刻	55 刻	40.0	9:36.0	9:40.3	−4.3	−4.3	−10.8	−10.8

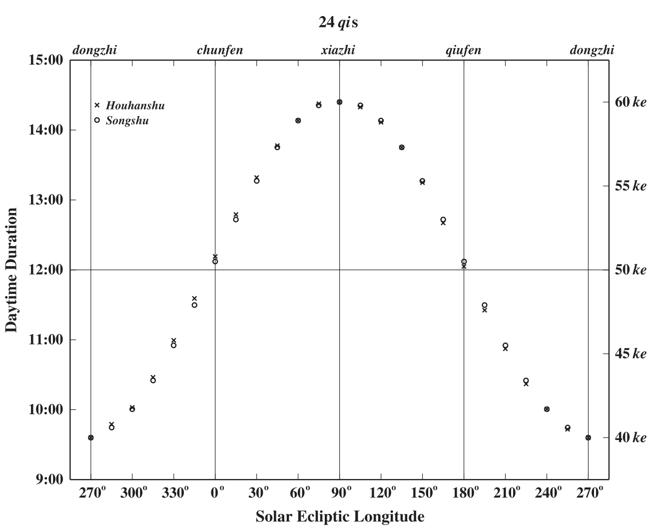

Fig. 1. Daytime duration given in Houhanshu and Songshu. The daytime duration in Songshu is symmetrical with respect to dongzhi and xiazhi.

pingqi, and compiled from a consideration that the data should be symmetrical with respect to dongzhi or xiazhi. The fact is that the recorded values of the daytime and nighttime durations of the symmetrical days with respect to dongzhi or xiazhi, such as lidong 立冬 and lichun 立春, coincide with the mean values (rounded off to the unit of fen) of those of the corresponding dates in Houhanshu or Jinshu. Therefore, we conclude that the daytime and nighttime durations in Songshu are not the original ones, but the compiled ones based on the data in Houhanshu. Consequently, one cannot discuss the determining method of the 24 qis at that time from these records.

Table 5 gives the length of the shadow of a vertical pole (zhoubi or biao) with a standard height of 8 Chinese feet at noon for the 24 qis throughout the year. The values in the columns headed by “Cal” are the calculated ones for the epoch AD 400 at Yangcheng using DE406, and assuming that the dates of 24 qis were determined by pingqi. The recorded shadow lengths are also symmetrical with respect to dongzhi or xiazhi (the values of the symmetrical dates yushui 雨水 and shuangjiang 霜降 are different but either one of them should be a miswriting). Unlike the daytime and nighttime durations, these values are not the compiled ones from the values in Houhanshu or Jinshu, and therefore these values are considered to be newly observed ones. The Rec − Cal values based

Table 5. Comparison of the shadow lengths given in Songshu with the calculation.

氣 Qi	Shadow length 影 Rec	Cal	Rec − Cal 平気 pingqi	定気 dingqi
	(C.ft)		(C.ft)	(C.ft)
冬至	13.	12.84	+0.16	+0.16
小寒	12.48	12.34	+0.14	+0.06
大寒	11.34	11.15	+0.19	+0.02
立春	9.91	9.61	+0.30	+0.06
雨水	8.22	8.04	+0.18	−0.06
驚蟄	6.72	6.57	+0.15	−0.09
春分	5.39	5.29	+0.10	−0.09
清明	4.25	4.19	+0.06	−0.09
穀雨	3.25	3.26	−0.01	−0.12
立夏	2.5	2.53	−0.03	−0.09
小満	1.97	1.98	−0.01	−0.04
芒種	1.69	1.64	+0.05	+0.04
夏至	1.5	1.52	−0.02	−0.02
小暑	1.69	1.64	+0.05	+0.04
大暑	1.97	1.98	−0.01	−0.04
立秋	2.5	2.54	−0.04	−0.09
處暑	3.25	3.29	−0.04	−0.12
白露	4.25	4.24	+0.01	−0.09
秋分	5.39	5.37	+0.02	−0.09
寒露	6.72	6.69	+0.03	−0.09
霜降	8.28	8.19	+0.09	0.00
立冬	9.91	9.79	+0.12	+0.06
小雪	11.34	11.32	+0.02	+0.02
大雪	12.48	12.45	+0.03	+0.06
冬至	13.	12.84	+0.16	+0.16

on pingqi tend to be positive, as opposed to those based on dingqi. This fact suggests that, if the shadow lengths were observed based on pingqi, the observing station was different from Yangcheng. As a matter of fact, if we solve the latitude from the recorded shadow lengths assuming the equal accuracy of the measured shadow lengths throughout the year, we obtain the latitude as

$$\phi = 34.^\circ 71 \pm 0.^\circ 05 \qquad (4)$$

based on pingqi, and

$$\phi = 34.^\circ 43 \pm 0.^\circ 06 \qquad (5)$$

based on dingqi. It should be noted that the obtained latitude based on dingqi is exactly equal to that of Yangcheng, and therefore if we can assume that the recorded data of the shadow lengths were based on observations made at Yangcheng, we can infer that the variation of the Sun's speed in one year was already recognized in these days, and that the dates of the 24 qis were determined based on the apparent longitude of the Sun by taking into account the variation of the Sun's speed, although, as is explained at the end of section 2, it is believed that in China such a variation was first recognized in c. AD 550.

The capital of the Song dynasty was Jiankang ⟨Chien-k'ang⟩ 建康 (latitude 32.°05) where the daytime and nighttime durations differ from those of Yangcheng (latitude 34.°43) or Luoyang (34.°75) by as much as 14 min in the current unit of time, and the shadow length differs by as much as 1.3 Chinese feet on the day of dongzhi (winter solstice). Therefore, it is apparent that the data in Songshu were not for Jiankang, but for Yangcheng or Luoyang. In the above we show that the data of the shadow lengths were newly observed ones and, consequently, it is considered that the observations were made at Yangcheng or Luoyang. However, there might still be a possibility that the observations were made at Jiankang, and that the observed values of the shadow lengths were converted to the values at Yangcheng or Luoyang by taking into account the latitude difference of the two sites.

3.4. Units of Time in Suishu

Suishu is an official history book about the Sui dynasty (AD 581–AD 619), which was compiled by Wei Zheng ⟨Wei Cheng⟩ 魏徵 et al. in the Tang dynasty in AD 636.

According to the Suishu Chapter 19, two kinds of time system existed in the Sui dynasty; each ke was subdivided into 60 fen in the first system, while each ke was subdivided into 100 fen in the second system. It should be reminded that in the books from Houhanshu to Songshu each ke was subdivided into 10 fen.

In Suishu Chapter 17 (志第十二 律暦中) the times of sunrise and sunset for 24 qis are given as shown in table 6; these times are given in the first system. In Suishu Chapter 18 (志第十三 律暦下) the values of the half duration of the nighttime for 24 qis are given as shown in table 7, and these times are given in the second system.

Table 6 gives the times of sunrise and sunset in Suishu Chapter 17. Corrections pointed out in the book published by Zhonghua-shuju (1973) are made. In addition, the characters "雨水" and "啓蟄" for 2 of 24 qis were misplaced and are corrected here.

From this table we can see the following facts:

1. Sunrise and sunset are symmetrical with respect to wu 4 ke 10 fen (午 4 刻 10 分), which is the middle of wu. This means that the double hour wu corresponds to 11 through 13 hr.
2. 1 ke is subdivided into 60 fen 分, but the last ke (8 刻) of each double hour has only 20 fen, and therefore 1 double hour equals 500 fen.
3. Each double hour begins with 0 ke 0 fen (0 刻 0 分) and ends with 8 ke 19 fen (8 刻 19 分). 8 ke 20 fen of any double hour corresponds to 0 ke 0 fen of the next double hour.

In the 4th column of the table is given the duration in units of ke from sunrise to sunset calculated from the data in the original table. These values agree with those given in Songshu, published by Zhonghua-shuju (1974), except for the three values indicated by the ‖ mark, which differs by 0.1 ke; actually, these three values also agree with those given in the Electronic Version of Siku Quanshu of Songshu. Therefore, we conclude that the times of sunrise and sunset given in Suishu are not the observed ones in the era of Sui dynasty, but are calculated ones from the data of the daytime and nighttime durations given in Songshu.

Table 6. Sunrise and sunset given in Suishu.

Qi	Sunrise	Sunset	Duration*	Duration†
冬至	辰 50 分	申 7 刻 30 分	40.0	40.14
小寒 大雪	辰 32 分	申 7 刻 48 分	40.6	40.48
大寒 小雪	卯 8 刻 19 分‡	酉 1 分	41.7	41.48
立春 立冬	卯 7 刻 28 分	酉 52 分	43.4	43.03
雨水 霜降	卯 6 刻 25 分	酉 1 刻 55 分	45.5	45.07
啓蟄 寒露	卯 5 刻 13 分§	酉 3 刻 7 分	47.9	47.45
春分 秋分	卯 3 刻 55 分	酉 4 刻 25 分	50.5	50.00
清明 白露	卯 2 刻 37 分	酉 5 刻 43 分	53.1‖	52.55
穀雨 處暑	卯 1 刻 28 分	酉 6 刻 52 分	55.4‖	54.93
立夏 立秋	卯 28 分	酉 7 刻 52 分	57.4‖	56.97
小滿 大暑	寅 8 刻 3 分	戌 17 分	58.9	58.54
芒種 小暑	寅 7 刻 36 分	戌 44 分	59.8	59.62
夏至	寅 7 刻 30 分	戌 50 分	60.0	59.86

* Daytime duration calculated from the times of sunrise and sunset given in this table.
† Daytime duration calculated from the half nighttime duration given in table 7.
‡ This is recorded as 卯 8 刻 49 分, but the number of fen should be less than 20 for 8 刻; hence, it is regarded as a miswriting of 卯 8 刻 19 分.
§ Steele (2000) mistyped this as 卯 5 刻 30 分.
‖ These values are different from those given in Songshu published by Zhonghua-shuju 中華書局. See the text.

Table 7. Comparison of the daytime duration given in Suishu with the calculation.

氣 Qi	夜半漏 Half-nighttime	Duration from Sunrise to Sunset Rec	Duration from Sunrise to Sunset Cal	Rec − Cal (min) without refraction 平気 pingqi	Rec − Cal (min) without refraction 定気 dingqi	Rec − Cal (min) with refraction 平気 pingqi	Rec − Cal (min) with refraction 定気 dingqi
冬至	27 刻 43 分	9:38.0	9:40.5	−2.5	−2.5	−9.0	−9.0
小寒	27 刻 26 分	9:42.9	9:47.3	−4.4	−3.3	−10.8	−9.8
大寒	26 刻 76 分	9:57.3	10:04.9	−7.6	−5.2	−13.8	−11.4
立春	25 刻 98 分半	10:19.6	10:30.3	−10.7	−6.9	−16.7	−13.0
雨水	24 刻 96 分半	10:49.0	11:00.2	−11.2	−6.5	−17.0	−12.4
驚蟄	23 刻 77 分半	11:23.3	11:32.3	−9.0	−3.9	−14.7	−9.6
春分	22 刻 50 分	12:00.0	12:05.1	−5.1	0.0	−10.8	−5.7
清明	21 刻 22 分半	12:36.7	12:37.4	−0.7	+3.9	−6.4	−1.8
穀雨	20 刻 3 分半	13:11.0	13:08.2	+2.8	+6.5	−3.1	+0.7
立夏	19 刻 1 分半	13:40.4	13:36.0	+4.4	+6.9	−1.7	+0.9
小満	18 刻 23 分	14:03.0	13:58.9	+4.1	+5.5	−2.2	−0.8
芒種	17 刻 69 分	14:18.5	14:14.1	+4.4	+4.7	−2.1	−1.7
夏至	17 刻 57 分	14:22.0	14:19.5	+2.5	+2.5	−4.0	−4.0
小暑	17 刻 69 分	14:18.5	14:14.1	+4.4	+4.7	−2.1	−1.7
大暑	18 刻 23 分	14:03.0	13:58.7	+4.3	+5.5	−2.0	−0.8
立秋	19 刻 1 分半	13:40.4	13:35.7	+4.7	+6.9	−1.4	+0.9
處暑	20 刻 3 分半	13:11.0	13:07.5	+3.5	+6.5	−2.3	+0.7
白露	21 刻 22 分半	12:36.7	12:36.2	+0.5	+3.9	−5.3	−1.8
秋分	22 刻 50 分	12:00.0	12:03.4	−3.4	0.0	−9.1	−5.7
寒露	23 刻 77 分半	11:23.3	11:30.3	−7.0	−3.9	−12.7	−9.6
霜降	24 刻 96 分半	10:49.0	10:57.9	−8.9	−6.5	−14.7	−12.4
立冬	25 刻 98 分半	10:19.6	10:27.9	−8.3	−6.9	−14.3	−13.0
小雪	26 刻 76 分	9:57.3	10:02.9	−5.6	−5.2	−11.8	−11.4
大雪	27 刻 26 分	9:42.9	9:46.1	−3.2	−3.3	−9.6	−9.8
冬至	27 刻 43 分	9:38.0	9:40.5	−2.5	−2.5	−9.0	−9.0

Table 8. Data given in Jiutangshu.

Qi	Sunrise	Sunset	Half-nighttime	*
冬至	辰 20 分	申 7 刻 12 分	27 刻 12 分	
小寒同大雪	辰 13 分	申 7 刻 19 分	27 刻 5 分	
大寒同小雪	卯 8 刻 7 分	酉 1 分	26 刻 15 分	
立春同立冬	卯 7 刻 11 分	酉 21 分	25 刻 19 分‡	
雨水同霜降	卯 6 刻 10 分	酉 1 刻†22 分	24 刻 18 分	
啓蟄同寒露	卯 5 刻 5 分	酉 3 刻†3 分	23 刻 13 分	
春分同秋分	卯 3 刻 22 分	酉 4 刻 10 分	22 刻 10 分	22 刻 6 分
清明同白露	卯 2 刻 15 分	酉 5 刻 17 分	20 刻 22 分‡	20 刻 23 分
穀雨同處暑	卯 1 刻 11 分†	酉 6 刻 21 分	19 刻 19 分	
立夏同立秋	卯 12 分†	酉 7 刻 20 分†	18 刻	18 刻 20 分
小滿同大暑	寅 8 刻 1 分	戌 7 分	18 刻 1 分	
芒種同小暑	寅 7 刻 14 分	戌 18 分	17 刻 14 分	
夏至	寅 7 刻 12 分	戌 20 分	17 刻 12 分‡	

† These are different between Zhonghua-shuju (1975) and Siku Quanshu (1999). The values in Siku Quanshu are adopted here because its times of sunrise and sunset are symmetrical with respect to noon.
‡ These are different between Zhonghua-shuju and Siku Quanshu. The values in Zhonghua-shuju are given here, but if the value is different from the value calculated from sunrise and sunset, the calculated one is given under the column heading *.

Table 7 gives the half of duration of nighttime recorded in Suishu Chapter 18. As in Houhanshu through Songshu, the nighttime does not include dawn 旦 or dusk 昏 (2.5 ke each). The derived values of the duration from sunrise to sunset from half of the nighttime duration are given in the third column. As shown in table 6 (in the 4th and 5th columns) these values do not coincide with the data of sunrise and sunset given in Suishu Chapter 17. A comparison of the derived duration is made with the calculated duration, as in table 2 and in table 4. The calculation is made for the epoch AD 600 at Yangcheng using DE406. As in table 2 and in table 4, a comparison is made for four cases: pingqi and dingqi without refraction and pingqi and dingqi with refraction, and the differences of the recorded duration with the calculation are given in the 5th to 8th columns. The calculated duration in the 4th column is that by pingqi without refraction. From the comparison we can easily see that the recorded data agree better with those without refraction, but it is not clear whether the dates of 24 qis are based on pingqi or on dingqi. Like the data in Songshu, the recorded duration from sunrise to sunset is symmetrical with respect to dongzhi or xiazhi. In addition, another symmetry with respect to chunfen and qiufen exists (see also figure 2): the sum of the durations from sunrise to sunset for the dates equally apart from chunfen or qiufen (like lichun 立春 and lixia 立夏 or bailu 白露 and hanlu 寒露) are 24 hr, except for the two pairs xiaohan 小寒–mangzhong 芒種 and dahan 大寒–xiaoman 小満, whose values might be miswritings. These facts suggest that, unlike the times of sunrise and sunset given in Suishu Chapter 17 (table 6), the data of half the nighttime duration given in Suishu Chapter 18 (table 7) were obtained from new observations incorporating some theoretical consideration.

3.5. *Units of Time in Jiutangshu*

Jiutangshu is one of the official history books about the Tang dynasty (AD 618–AD 907), and was compiled by Liu Xu

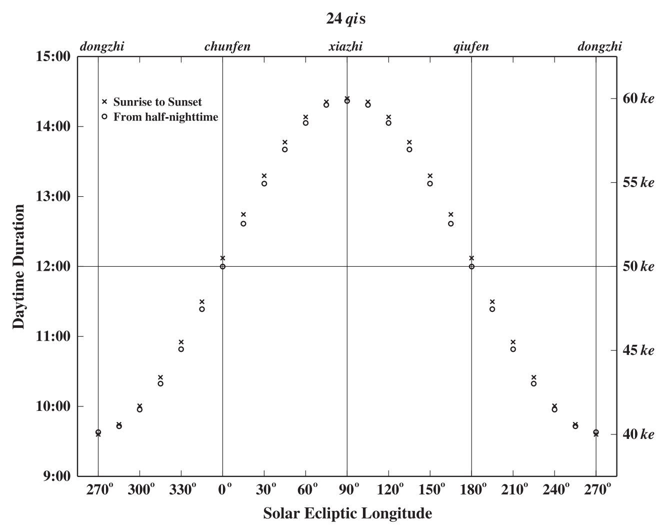

Fig. 2. Daytime duration given in Suishu. The daytime duration calculated from the times of sunrise and sunset and the daytime duration calculated from the half-nighttime are shown. Both of them are symmetrical with respect to dongzhi and xiazhi, but the latter duration has another symmetry, such that the sum of the daytime durations for the dates equally apart from chunfen or qiufen are 24 hr with a couple of exceptions.

⟨Liu Hsu⟩ 劉昫 et al. in the Jin ⟨Chin⟩ 晉 dynasty of Wudai 五代 in AD 945.

In Jiutangshu Chapter 32 (志第十二 曆一 校勘記) the times of sunrise and sunset and the half of the duration of nighttime for 24 qis are given as listed in table 8. The characters "雨水" and "啓蟄" for 2 of 24 qis were misplaced, as in Suishu Chapter 18 (隋書律曆下), and are corrected here. Note that the times of sunrise and sunset are given in two separate tables (pp. 1166–1167 and pp. 1169–1170) in Jiutangshu Chapter 32,

Table 9. Data given in Songshi Chapter 70.

Qi	Sunrise	Sunset	Daytime	Nighttime
冬至	卯 4 刻 144 半	申 3 刻 51 半	40 刻 5	59 刻 142
小寒	卯 4 刻 119 半	申 3 刻 76 半	40 刻 55	59 刻 92
大寒	卯 4 刻 34 半	申 4 刻 14 半	41 刻 78	58 刻 69
立春	卯 3 刻 56 半	申 4 刻 139 半	43 刻 34	56 刻 113
雨水	卯 2 刻 58 半	申 5 刻 137 半	45 刻 30	54 刻 117
驚蟄	卯 1 刻 40 半	申 7 刻 8 半	47 刻 66	52 刻 81
春分	卯初 空	酉初 空	50 刻 空	50 刻 空
清明	寅 7 刻 8 半*	酉 1 刻 40 半	52 刻 81	47 刻 66
穀雨	寅 5 刻 127 半	酉 2 刻 68 半	54 刻 137	45 刻 10
立夏	寅 4 刻 119 半*	酉 3 刻 76 半	57 刻 6	42 刻 141
小満	寅 3 刻 146 半	酉 4 刻 49 半	58 刻 99	41 刻 48
芒種	寅 3 刻 71 半	酉 4 刻 124 半	59 刻 102	40 刻 45
夏至	寅 3 刻 51 半	酉 4 刻 144 半	59 刻 142	40 刻 5
小暑	寅 3 刻 71 半	酉 4 刻 124 半	59 刻 102	40 刻 45
大暑	寅 3 刻 146 半	酉 4 刻 49 半	58 刻 99	41 刻 48
立秋	寅 4 刻 119 半	酉 3 刻 76 半	57 刻 6	42 刻 141
處暑	寅 5 刻 127 半	酉 2 刻 68 半	54 刻 137	45 刻 10
白露	寅 7 刻 8 半	酉 1 刻 40 半	52 刻 81	47 刻 66
秋分	卯初 空	酉初 空	50 刻 空	50 刻 空
寒露	卯 1 刻 40 半	申 7 刻 8 半	47 刻 66	52 刻 81
霜降	卯 2 刻 58 半	申 5 刻 137 半	45 刻 30	54 刻 117
立冬	卯 3 刻 56 半	申 4 刻 139 半*	43 刻 34	56 刻 113
小雪	卯 4 刻 34 半	申 4 刻 14 半	41 刻 78	58 刻 69
大雪	卯 4 刻 119 半	申 3 刻 76 半	40 刻 55	59 刻 92

* Miswriting is corrected.

but the former table, on which Steele's (2000) analysis in his Appendix B was based, contains many errors, including some missing lines; we therefore use the latter table.

From this table we can see the following facts:

1. Sunrise and sunset are symmetrical with respect to wu 4 ke 10 fen (午 4 刻 10 分), which is the middle of wu, as in Suishu.
2. 1 ke is subdivided into 24 fen, but the last ke (8 刻) of each double hour has only 8 fen, and therefore 1 double hour equals 200 fen.
3. Each double hour begins with 0 ke 0 fen (0 刻 0 分) and ends with 8 ke 7 fen (8 刻 7 分). 8 ke 8 fen of any double hour corresponds to 0 ke 0 fen of the next double hour.

The times of sunrise and sunset in Jiutangshu coincide with those given in Chapter 17 of Suishu within 1 fen in the units used in Jiutangshu (1/24 ke), which equals 0.6 min in the current units of time. Considering that these recorded times of sunrise and sunset differed from the actual times of sunrise and sunset by more than a few minutes, we can see that the table was constructed based on the data in Suishu, whose data can be traced back to Houhanshu. The values of half the nighttime duration recorded in Jiutangshu agree with those calculated from the recorded times of sunrise and sunset by taking into account the duration of dawn and dusk (2.5 ke each), except for three values, as noted in table 8, which can be regarded as miswritings or miscalculations. This means that the data in Jiutangshu ignored the data of half of the nighttime duration given in Chapter 18 of Suishu, which were based on the new observations and some theoretical consideration.

3.6. Units of Time in Songshi

Songshi is an official history book about the Song dynasty (AD 960–AD 1279), which was compiled by Tuo Tuo ⟨T'o T'o⟩ 脱脱 et al. in the Yuan dynasty in AD 1345.

In Songshi Chapter 70 (志第二十三 律暦三) the times of sunrise and sunset and the durations of daytime and nighttime for 24 qis are given as listed in table 9. The recorded values of the duration of daytime and nighttime coincide with those calculated from the recorded times of sunrise and sunset, except for the two cases of sunrise and one case for sunset, which should be miswritings, as indicated in the book by Zhonghua-shuju (1985).

Steele (2000) also gives a table of these times of sunrise and sunset (his Appendix B), but he inadvertently omitted all of the characters of ban 半 (which means half fen) in sunrise and sunset. Steele also indicates that 1 ke equals 150 fen, but actually 1 ke equals 147 fen. Note that the character fen is omitted in the original table, as shown in table 9, but is added here in this text in order to clarify that the figures are the numbers of the subdivision of ke. From Houhanshu to Jiutangshu the middle of the daytime was the middle of wu, but in Songshi it is the beginning of wu; therefore, for example, the first double hour zi 子 begins at 0 hr local time instead of 23 hr.

The facts about the time system used in Songshi are summarized as follows:

Table 10. Comparison of the daytime duration given in Songshi Chapter 70 with the calculation.

氣 Qi	Duration from Sunrise to Sunset Rec	Cal	Rec − Cal (min) without refraction 平気 pingqi	定気 dingqi	with refraction 平気 pingqi	定気 dingqi	*
冬至	9:36.5	9:38.9	−2.4	−2.4	−8.9	−8.9	+2.5
小寒	9:41.4	9:45.4	−4.0	−3.3	−10.4	−9.8	+1.4
大寒	9:58.0	10:03.0	−5.0	−3.2	−11.2	−9.4	+1.0
立春	10:22.5	10:28.5	−6.0	−3.0	−12.1	−9.1	+0.3
雨水	10:50.9	10:58.8	−7.9	−3.9	−13.8	−9.8	−1.7
驚蟄	11:23.3	11:31.4	−8.1	−3.6	−13.8	−9.3	−2.4
春分	12:00.0	12:04.7	−4.7	0.0	−10.4	−5.7	0.0
清明	12:36.7	12:37.5	−0.8	+3.6	−6.6	−2.2	+2.4
穀雨	13:11.0	13:08.8	+2.2	+5.8	−3.7	−0.1	+3.6
立夏	13:41.4	13:37.0	+4.4	+6.9	−1.7	+0.8	+3.6
小満	14:04.9	14:00.2	+4.7	+6.1	−1.6	−0.2	+1.9
芒種	14:19.6	14:15.6	+4.0	+4.3	−2.5	−2.1	−0.4
夏至	14:23.5	14:21.1	+2.4	+2.4	−4.2	−4.2	−2.5
小暑	14:19.6	14:15.6	+4.0	+4.3	−2.5	−2.1	−0.4
大暑	14:04.9	14:00.1	+4.8	+6.1	−1.6	−0.2	+1.9
立秋	13:41.4	13:36.9	+4.5	+6.9	−1.6	+0.8	+3.6
處暑	13:11.0	13:08.5	+2.5	+5.8	−3.4	−0.1	+3.6
白露	12:36.7	12:37.0	−0.3	+3.6	−6.1	−2.2	+2.4
秋分	12:00.0	12:04.0	−4.0	0.0	−9.7	−5.7	0.0
寒露	11:23.3	11:30.6	−7.3	−3.6	−13.1	−9.3	−2.4
霜降	10:50.9	10:57.9	−7.0	−3.9	−12.9	−9.8	−1.7
立冬	10:22.5	10:27.6	−5.1	−3.0	−11.2	−9.1	+0.3
小雪	9:58.0	10:02.2	−4.2	−3.2	−10.4	−9.4	+1.0
大雪	9:41.4	9:44.9	−3.5	−3.3	−10.0	−9.8	+1.4
冬至	9:36.5	9:38.9	−2.4	−2.4	−8.9	−8.9	+2.5

* Residuals when the latitude is solved based on dingqi without refraction.

1. Sunrise and sunset are symmetrical with respect to the beginning of wu (wu 0 ke 0 fen 午 0 刻 0 分).
2. 1 ke is subdivided into 147 fen, but the last ke (8 刻) of each double hour has only 49 fen, and therefore 1 double hour equals 1225 fen.
3. Each double hour begins with 0 ke 0 fen (0 刻 0 分) and ends with 8 ke 48 fen (8 刻 48 分). 8 ke 49 fen of any double hour corresponds to 0 ke 0 fen of the next double hour.
4. The first double hour begins at 0 hr local time.
5. Daytime is from sunrise to sunset. Unlike the systems from Houhanshu to Jiutangshu, daytime does not include dawn or dusk.
6. The times of sunrise and sunset, and the values of the daytime and nighttime durations are symmetrical with respect to dongzhi and xiazhi.

Table 10 gives the daytime duration recorded in Songshi Chapter 70, and a comparison is made with the calculated duration. The calculation is made for the epoch AD 1000 at Bian ⟨Pien⟩ 汴 (latitude 34.°78), the capital of Song, using DE406. As in tables 2, 4, and 7, comparisons are made for four cases: pingqi and dingqi without refraction and pingqi and dingqi with refraction; the differences of the recorded duration with the calculation are given in the 4th to 7th columns. The calculated duration in the 3rd column is the one by pingqi without refraction. The fact that the daytime and nighttime durations are symmetrical with respect to dongzhi or xiazhi suggests that the dates of 24 qis are based on dingqi, and the fact that the daytime durations on chunfen and qiufen are 12 hr indicates that the recorded times of sunrise and sunset are those without refraction. The comparison of the recorded duration with the calculated one also shows that the differences based on dingqi without refraction are smaller than the others, but the systematic differences of being positive in summer and negative in winter might indicate that the observation was made at a higher latitude than Bian. If we solve the latitude, ϕ, by the least squares method from the recorded daytime duration using a theoretical obliquity, ϵ (23.°569 by the formula by Williams 1994), of the ecliptic of AD 1000, assuming that the dates were determined by dingqi and the daytime duration was obtained without refraction, then we obtain the latitude as

$$\phi = 35.^\circ 68 \pm 0.^\circ 17, \tag{6}$$

(if we solve both the latitude, ϕ, and the obliquity, ϵ, from the recorded duration, we obtain $\epsilon = 10^\circ$ and $\phi = 60^\circ$, whose

Table 11. Data given in Songshi Chapter 76.

Qi	Sunrise	Sunset	Daytime	Nighttime
冬至	卯正 5 刻分空	申正 3 刻 20 分	40 刻分空	60 刻分空
小寒	卯正 4 刻 50 分	申正 3 刻 30 分	40 刻 19 分	59 刻 41 分†
大寒	卯正 4 刻 20 分	申正 4 刻分空	41 刻 19 分	58 刻 41 分†
立春	卯正 3 刻 32 分*	申正 4 刻 48 分	42 刻 54 分	57 刻 6 分‡
雨水	卯正 2 刻 30 分	申正 5 刻 50 分	44 刻 58 分	55 刻 2 分*, ‡
驚蟄	卯正 1 刻 17 分	申正 7 刻 3 分	47 刻 24 分	52 刻 36 分‡
春分	卯正初刻分空	酉正初刻分空	50 刻分空	50 刻分空
清明	寅正 7 刻 3 分	酉正 1 刻 17 分	52 刻 35 分	47 刻 25 分§
穀雨	寅正 5 刻 50 分	酉正 2 刻 30 分*	55 刻 3 分	44 刻 57 分‖
立夏	寅正 4 刻 48 分	酉正 3 刻 32 分	57 刻 5 分	42 刻 55 分§
小満	寅正 4 刻分空	酉正 4 刻 20 分	58 刻 40 分	41 刻 20 分
芒種	寅正 3 刻 30 分	酉正 4 刻 50 分	59 刻 40 分	40 刻 20 分
夏至	寅正 3 刻 20 分	酉正 5 刻分空	60 刻分	40 刻分空
小暑	寅正 3 刻 30 分	酉正 4 刻 50 分	59 刻 40 分	40 刻 20 分
大暑	寅正 4 刻分空	酉正 4 刻 20 分	58 刻 40 分	41 刻 20 分
立秋	寅正 4 刻 48 分*	酉正 3 刻 32 分*	57 刻 5 分	42 刻 55 分§
處暑	寅正 5 刻 50 分	酉正 2 刻 30 分	55 刻 3 分	44 刻 57 分‖
白露	寅正 7 刻 3 分	酉正 1 刻 17 分	52 刻 35 分	47 刻 25 分§
秋分	卯正初刻分空	酉正初刻分空	50 刻分空	50 刻分空
寒露	卯正 1 刻 17 分	申正 7 刻 3 分	47 刻 24 分	52 刻 36 分‡
霜降	卯正 2 刻 30 分	申正 5 刻 50 分	44 刻 58 分	55 刻 2 分‡
立冬	卯正 3 刻 32 分	申正 4 刻 48 分	42 刻 54 分	57 刻 6 分‡
小雪	卯正 4 刻 20 分	申正 4 刻分空	41 刻 19 分	58 刻 41 分†
大雪	卯正 4 刻 50 分	申正 3 刻 30 分	40 刻 19 分	59 刻 41 分*, †

* Miswritings are corrected, as indicated in the book by Zhonghua-shuju.
† The daytime duration needs a correction of +1 fen and the nighttime duration needs a correction of −1 fen.
‡ The daytime duration needs a correction of +2 fen and the nighttime duration needs a correction of −2 fen.
§ The daytime duration needs a correction of −1 fen and the nighttime duration needs a correction of +1 fen.
‖ The daytime duration needs a correction of −3 fen and the nighttime duration needs a correction of +3 fen.

values are absolutely unacceptable). The residuals in the daytime duration when adopting this latitude are given in the column under the asterisk in the table. The residuals still have some systematic behavior (note that the sum of the residuals is positive, and that this fact cannot be changed whatever values we adopt for the latitude). Therefore, the differences obtained when adopting the latitude of Bian might be due to some deficiency in the clocks or in the theories used to obtain the daytime duration at that time.

In Songshi Chapter 76 (志第二十九 律暦九), the times of sunrise and sunset and the duration of daytime and nighttime for 24 qis are given as shown in table 11. In this table 1 ke equals 60 fen instead of 147 fen, and therefore one double hour equals 500 fen. The following facts are the same as in table 9:

1. The first double hour zi begins at 0 hr local time.
2. Sunrise and sunset are symmetrical with respect to the beginning of wu (wu 0 ke 0 fen 午 0 刻 0 分).
3. Daytime is the time from sunrise to sunset.

Like in Suishu Chapter 18 (志第十三 律暦下), there are two kinds of symmetry in the times of sunrise and sunset: the recorded duration from sunrise to sunset is symmetrical with respect to dongzhi or xiazhi, and the sum of the durations from sunrise to sunset for the dates equally apart from chunfen or qiufen (like lichun and lixia or bailu and hanlu) are 24 hr, which can be satisfied for the dates of dingqi without refraction.

The recorded values of the duration of daytime and nighttime do not necessarily coincide with those calculated from the recorded times of sunrise and sunset, as indicated in the notes in the table. The maximum difference is 3 fen, which equals about 0.7 min in the current units of time.

A comparison of the daytime duration calculated from the times of sunrise and sunset given in Songshi Chapter 76 was made with the calculated duration using DE406; the results are given in table 12. The calculation was made for the epoch AD 1000 at Bian (latitude $34^{\circ}\!.78$). The differences show almost the same tendency as for the data given in table 10, although the differences for some dates near lichun and lidong are slightly larger.

3.7. *Reference Latitude of the Chinese System*

As shown in the above subsections, the daytime and nighttime durations, the times of sunrise and sunset, and the variations of the length of the shadows of a pole, called zhoubi, during a year are described in the official Chinese chronicles. From these data, we confirm that all of the lengths of

Table 12. Comparison of the daytime duration given in Songshi Chapter 76 with the calculation.

氣 Qi	Duration from Sunrise to Sunset Rec	Cal	Rec − Cal (min) without refraction 平気 pingqi	定気 dingqi	with refraction 平気 pingqi	定気 dingqi
冬至	9:36.0	9:38.9	−2.9	−2.9	−9.4	−9.4
小寒	9:40.8	9:45.4	−4.6	−3.9	−11.0	−10.4
大寒	9:55.2	10:03.0	−7.8	−6.0	−14.0	−12.2
立春	10:18.2	10:28.5	−10.3	−7.3	−16.4	−13.4
雨水	10:48.0	10:58.8	−10.8	−6.8	−16.7	−12.7
驚蟄	11:23.0	11:31.4	−8.4	−3.8	−14.1	−9.6
春分	12:00.0	12:04.7	−4.7	0.0	−10.4	−5.7
清明	12:37.0	12:37.5	−0.5	+3.8	−6.3	−1.9
穀雨	13:12.0	13:08.8	+3.2	+6.8	−2.7	+0.9
立夏	13:41.8	13:37.0	+4.8	+7.3	−1.3	+1.2
小満	14:04.8	14:00.2	+4.6	+6.0	−1.7	−0.3
芒種	14:19.2	14:15.6	+3.6	+3.9	−2.9	−2.5
夏至	14:24.0	14:21.1	+2.9	+2.9	−3.7	−3.7
小暑	14:19.2	14:15.6	+3.6	+3.9	−2.9	−2.5
大暑	14:04.8	14:00.1	+4.7	+6.0	−1.7	−0.3
立秋	13:41.8	13:36.9	+4.9	+7.3	−1.2	+1.2
處暑	13:12.0	13:08.5	+3.5	+6.8	−2.4	+0.9
白露	12:37.0	12:37.0	0.0	+3.8	−5.8	−1.9
秋分	12:00.0	12:04.0	−4.0	0.0	−9.7	−5.7
寒露	11:23.0	11:30.6	−7.6	−3.8	−13.4	−9.6
霜降	10:48.0	10:57.9	−9.9	−6.8	−15.8	−12.7
立冬	10:18.2	10:27.6	−9.4	−7.3	−15.5	−13.4
小雪	9:55.2	10:02.2	−7.0	−6.0	−13.2	−12.2
大雪	9:40.8	9:44.9	−4.1	−3.9	−10.6	−10.4
冬至	9:36.0	9:38.9	−2.9	−2.9	−9.4	−9.4

daytime and nighttime, the times of sunrise and sunset, and the lengths of shadows refer to a particular latitude of about 34°.5. This strongly suggests that the reference point of the Chinese system is Yangcheng near Luoyang, irrespective of the dynasties. Descriptions in these chronicles stick to a formality that the ratios of daytime and nighttime defined by the sunrise and sunset are a simple ratio of 2/3 in the winter solstice and 3/2 in the summer solstice at this site. This site was called Center of Ground (dizhong 地中 in Chinese) from legendary years. This fact seems to indicate that the Chinese system selected a particular latitude as a reference point of the system at which the ratios of daytime and nighttime durations on the winter and summer solstices assume simple values. As shown in the next section, the Japanese system adopted this Chinese tradition.

4. Japanese Units of Time

In this section, the times of sunrise and sunset recorded in Guchureki 具注暦 and in Engishiki 延喜式 as well as some predicted timings of solar and lunar eclipses recorded in ancient and medieval Japan are analyzed to obtain the time systems that were used in those days in Japan.

4.1. *Units of Time in Guchureki*

Hirayama (1913a,b) wrote two papers on ancient and medieval Japanese systems of time units. In the first paper, he analyzed two kinds of calendars: a calendar called Guchureki and a calendar appearing in a collection of administration laws and rules, called Engishiki.

Guchureki is a kind of diary note, originally edited in the 11th month of the preceding year by the Astronomy and Meteorology Administration (On'you-ryou 陰陽寮 in Japanese), a branch of the Ministry of Home Affairs (Nakatsukasa-shou 中務省 in Japanese) of the Federal Government, and distributed to governers of prefectures and counties etc., as well as some main officers of local governments from the Ministry of Home Affairs. Users wrote diaries in blank spaces after the date for each day. It has been said that the oldest Guchureki in existence is that of the year 746 (Tenpyo 天平 reign period, 18th year), which is now in Shosoin 正倉院 in Nara. However, the National Research Institute for Cultural Properties of Nara 奈良文化財研究所 announced in 2003 that the oldest Guchureki of the year 689 (Jito 持統 reign period, 3rd year) was found in Asuka, Nara. Unfortunately, these calendars of Guchureki of the older years did not contain

Table 13. Times of sunrise and sunset in Guchureki 具注暦.*

(1) Date	(2) Day (1)	(3) Date	(4) Day (2)	(5) Sunrise	(6) Sunset	(7) Sunrise hr	(8) Sunset hr	(9) Day ke	(10) Day ke	(11) Night ke
11 月節 13 日	−3		177	辰 0 刻‡3 分	申 3 刻 4 分	7.24	16.76	39.67	40	60
12 月節 1 日	15	11 月節 1 日	165	辰 0 刻‡2 分	申 3 刻 5 分	7.16	16.84	40.33	41	59
12 月節 13 日	27	10 月節 21 日	155	辰 0 刻‡1 分	申 4 刻	7.08	16.92	41.00	42	58
12 月節 23 日	37	10 月節 13 日	147	卯 4 刻 1 分	酉 0 刻‡1 分	7.00	17.08	42.00	43	57
1 月節 1 日	45	10 月節 5 日	139	卯 3 刻 5 分	酉 0 刻‡2 分	6.84	17.16	43.00	44	56
1 月節 9 日	53	9 月節 27 日	131	卯 3 刻 4 分	酉 0 刻‡3 分	6.76	17.24	43.67	45	55
1 月節 17 日	61	9 月節 19 日	123	卯 3 刻 2 分	酉 0 刻‡5 分	6.60	17.40	45.00	46	54
1 月節 25 日	69	9 月節 11 日	115	卯 3 刻	酉 1 刻 1 分	6.44	17.56	46.33	47	53
2 月節 3 日	77	9 月節 3 日	107	卯 2 刻 4 分	酉 1 刻 3 分	6.28	17.72	47.67	48	52
2 月節 11 日	85	8 月節 25 日	99	卯 2 刻 2 分	酉 1 刻 5 分	6.12	17.88	49.00	49	51
2 月節 18 日	92	8 節 18 日	92	卯時正†	酉時正†	6.00	18.00	50.00	50	50
2 月節 25 日	99	8 月節 11 日	85	卯 1 刻 5 分	酉 2 刻 2 分	5.88	18.12	51.00	51	49
3 月節 3 日	107	8 月節 3 日	77	卯 1 刻 3 分	酉 2 刻 4 分	5.72	18.28	52.33	52	48
3 月節 11 日	115	7 月節 25 日	69	卯 1 刻 1 分	酉 3 刻	5.56	18.44	53.67	53	47
3 月節 19 日	123	7 月節 17 日	61	卯 0 刻‡5 分	酉 3 刻 2 分	5.40	18.60	55.00	54	46
3 月節 27 日	131	7 月節 9 日	53	卯 0 刻‡3 分	酉 3 刻 4 分	5.24	18.76	56.33	55	45
4 月節 5 日	139	7 月節 1 日	45	卯 0 刻‡2 分	酉 3 刻 5 分	5.16	18.84	57.00	56	44
4 月節 13 日	147	6 月節 23 日	37	卯 0 刻‡1 分	酉 4 刻 1 分	5.08	19.00	58.00	57	43
4 月節 21 日	155	6 月節 13 日	27	寅 4 刻	戌 0 刻‡1 分	4.92	19.08	59.00	58	42
5 月節 1 日	165	6 月節 1 日	15	寅 3 刻 5 分	戌 0 刻‡2 分	4.84	19.16	59.67	59	41
5 月節 13 日	177		−3	寅 3 刻 4 分	戌 0 刻‡3 分	4.76	19.24	60.33	60	40

* The 1st and 3rd columns give the date, and the 2nd and 4th columns give days counted from the winter and summer solstices, respectively, corresponding to the dates in the 1st and 3rd columns as given by Hirayama. In counting the numbers of the days in the 2nd and 4th columns, intercalary days, called mori 没日 (see section 2), are not counted, and then the whole year is equal to 360 days. The times in the 5th and 6th columns are the times of sunrise and sunset given in Guchureki, and the times in the 7th and 8th are their converted times in the current units. The 9th column gives the daytime duration (from sunrise to sunset) calculated from the times in the 5th and 6th columns in units of the second definition of ke (1/100 day). The 10th and 11th columns give the daytime and nighttime durations given in Guchureki in the second definition of ke.
† "正" here means center and therefore it corresponds to 2 刻 0.5 分.
‡ "0 刻" is written as "初刻" (initial ke) in the original table.

the data of sunrise and sunset or the daytime and nighttime durations.

The times of sunrise and sunset and the daytime and nighttime durations were written on particular fixed dates in Guchureki for the years after the calendar system called Xuanmingli ⟨Hsuan-ming-li⟩ 宣明暦 was officially introduced in Japan in the year 862. Table 13 summarizes the times of sunrise and sunset and the daytime and nighttime durations written in Guchureki according to the table by Hashimoto (1966). It is apparent that the time system used for the times of sunrise and sunset in Guchureki was not the same as Xuanmingli, although many of the predicted times of eclipses in Guchureki are in the system of Xuanmingli (see subsection 4.4).

Hirayama (1913a) also gives the same table, but the times of sunrise and sunset of the days of the first and last lines (days 5 月節 13 日 and 11 月節 13 日) by Hirayama are different from the times by Hashimoto by 0.5 fen and 1 fen, respectively. Since the times of sunrise and sunset by Hashimoto are symmetrical with respect to noon, we adopt the times by Hashimoto. The times of sunrise and sunset of the other dates are symmetrical with respect to noon (wu 2 ke 0.5 fen 午 2 刻 0.5 分), except for those of days 37 and 147 (days 12 月節 23 日, 4 月節 13 日, 6 月節 23 日, and 10 月節 13 日), for which the middle of the times of sunrise and sunset differs from noon by 0.5 fen, which might be due to miscalculations or miswritings in Guchureki.

Hirayama (1913a) writes that, since the dates in the table are the same as the first days of the intervals given in the table of Engishiki, as shown in table 14 below, there is a possibility that the dates are actually the first days of the intervals to which the data in the 5th to 11th columns are applicable (for example, the sunrise and sunset given in the first line are for the days −3 to +14). We approve of his suggestion based on another evidence, as follows.

A symmetry exists in the times of sunrise and sunset in the table: the daytime (time from sunrise to sunset) and nighttime durations of dates equally apart from the day of the central line of the table are reversed, but the dates in the 1st and 3rd columns are not symmetrical with respect to the central line (for example, the day of the first line is 95 days before the day of the central line, but the day of the last line is 85 days after the day of the central line). However, if we assume that the dates given in the table are the first days of the intervals,

Table 14. Time of sunrise and sunset in Engishiki 延喜式.*

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Interval		*L*	Interval		*L*	Sunrise	Sunset	Sunrise	Sunset
1st day	last day	°	1st day	last day	°			ST h	ST h
大雪 13 日	冬至 15 日	267				辰 1 刻 2 分†	申 4 刻 6 分	7.10	16.80
小寒 1	小寒 12	285	大雪 1	大雪 12	255	辰 1 刻 1 分	申 4 刻 7 分	7.05	16.85
小寒 13	大寒 7	297	小雪 6	小雪 15	245	卯 4 刻終‡	酉 1 刻 1 分	7.00	17.05
大寒 8	大寒 15	307	立冬 13	小雪 5	237	卯 4 刻 7 分	酉 1 刻 2 分	6.85	17.10
立春 1	立春 8	315	立冬 5	立冬 12	229	卯 4 刻 5 分	酉 1 刻 5 分	6.75	17.25
立春 9	雨水 1	323	霜降 12	立冬 4	221	卯 4 刻 2 分	酉 1 刻 7 分	6.60	17.35
雨水 2	雨水 9	331	霜降 4	霜降 11	213	卯 4 刻	酉 2 刻 1 分	6.50	17.55
雨水 10	驚蟄 2	339	寒露 11	霜降 3	205	卯 3 刻 7 分	酉 2 刻 2 分	6.35	17.60
驚蟄 3	驚蟄 10	345	寒露 3	寒露 10	197	卯 3 刻 5 分	酉 2 刻 5 分	6.25	17.75
驚蟄 11	春分 2	355	秋分 10	寒露 2	189	卯 3 刻 2 分	酉 2 刻 7 分	6.10	17.85
春分 3	春分 9	2	秋分 3	秋分 9	182	卯 3 刻	酉 3 刻	6.00	18.00
春分 10	清明 2	9	白露 11	秋分 2	175	卯 2 刻 7 分	酉 3 刻 2 分	5.85	18.10
清明 3	清明 10	17	白露 3	白露 10	167	卯 2 刻 5 分	酉 3 刻 5 分	5.75	18.25
清明 11	穀雨 3	25	処暑 10	白露 2	159	卯 2 刻 2 分	酉 3 刻 7 分	5.60	18.35
穀雨 4	穀雨 11	33	処暑 2	処暑 9	151	卯 2 刻 1 分	酉 4 刻	5.55	18.50
穀雨 12	立夏 4	41	立秋 9	処暑 1	143	卯 1 刻 7 分	酉 4 刻 2 分	5.35	18.60
立夏 5	立夏 12	49	立秋 1	立秋 8	135	卯 1 刻 5 分	酉 4 刻 5 分	5.25	18.75
立夏 13	小満 5	57	大暑 8	大暑 15	127	卯 1 刻 2 分	酉 4 刻 7 分	5.10	18.85
小満 6	小満 15	65	小暑 13	大暑 7	117	卯 1 刻 1 分	酉 4 刻終‡	5.05	19.00
芒種 1	芒種 12	75	小暑 1	小暑 12	105	寅 4 刻 7 分	戌 1 刻 1 分	4.85	19.05
芒種 13	夏至 15	87				寅 4 刻 6 分	戌 1 刻 2 分	4.80	19.10

* The 1st, 2nd, 4th, and 5th columns show the intervals of dates, and the 3rd and 6th columns give the ecliptic longitude, *L*, of the Sun, corresponding to the first days of the intervals. The times in the 7th and 8th columns are those of sunrise and sunset given in Engishiki, and the times in the 9th and 10th are their converted times in the local apparent solar time ST in the current units.

† Hirayama (1913a) gives "辰 1 刻 3 分" for this, but Hashimoto's sunrise and sunset have symmetry (the numerals of ke and fen of sunrise ranged downward from the top are the same as those of sunset ranged upward from the bottom), and hence we adopt Hashimoto's data.

‡ "終" means final, and we regard it as 9 分.

the central dates of the intervals in the table become symmetrical with respect to the central line. Therefore, we conclude that the dates given in the table are the first days of the intervals, as are given in Engishiki.

From this table we can see the following facts about the time system for sunrise and sunset in Guchureki:

1. The first double hour zi is centered at midnight, and is equivalent to 23:00–01:00 in the local apparent solar time. This agrees with the Chinese system until Jiutangshu.
2. Daytime does not include dawn or dusk. As described in section 3, daytime included dawn and dusk until Jiutangshu in China, and therefore this system in Guchureki seems to be the original one in Japan at that time.
3. 1 ke is subdivided into 6 fen, and each double hour equals 4 ke 1 fen (25 fen); therefore, 1 day equals 50 ke.
4. Each ke begins with 0 fen, and 4 ke 1 fen (4 刻 1 分) of each double hour corresponds to 0 ke 0 fen of the next double hour, but as was discussed by Hashimoto (1966), the description 4 ke 1 fen appears in the table of sunrise and sunset, which should be replaced by 0 ke 0 fen of the next double hour, because, otherwise, the middle of the sunrise and sunset would no longer be the center of wu.

There are also the values of daytime and nighttime durations in Guchureki using another kind of time unit with the same character, ke, as given in the 10 th and 11 th columns of table 13. In this case, 1 ke is 1/100 days, but these values do not coincide with the duration (9th column of the table) calculated from sunrise and sunset; we therefore assume that these values of the duration indicated only the approximate duration.

4.2. *Units of Time in Engishiki*

Compilation of Engishiki started in AD 905 (Engi 延喜 reign period 5th year) and almost finished in 927. Revisions of the collections still continued, and were enforced in 967. Similar collections of administration laws and rules for 701–824 and for 824–877 were already compiled as Kouninshiki 弘仁式 and Jouganshiki 貞観式, respectively. Engishiki collected all of the administration laws and rules from the Taiho 大宝 reign period (701–703) to the Engi reign period (901–922) in separate parts for individual Ministries and Administrations. In the part of the Astronomy and Meteorology Administration, the times of sunrise and sunset, the times to open and close gates of the

Palace and the closing hours of government office are given. Incidentally, the main members of this Administration were Doctors of Astronomy, System, Time Keeping etc. and the main work of this Administration was astronomical and meteorological observations, editings almanacs, time keeping etc.

Administrative rules on opening of gates, the closing time of government office, and the closing time of gates are given in Engishiki, for example, as follows. In the English translation L means the solar ecliptic longitude and ST means the local apparent solar time.

起大雪 13 日至冬至 15 日　日出辰 1 刻 2 分日入申 4 刻 6 分
卯 4 刻 6 分開諸門鼓
辰 2 刻 7 分開大門鼓
午 1 刻 6 分退朝鼓
酉 1 刻 2 分閉門鼓
起小寒 1 日至 12 日　日出辰 1 刻 1 分日入申 4 刻 7 分
卯 4 刻 5 分開諸門鼓
辰 2 刻 6 分開大門鼓
午 1 刻 5 分退朝鼓
酉 1 刻 3 分閉門鼓

and so on.

For days of $L = 267°–284°$: The times of sunrise and sunset are chen 1 ke 2 fen (07:06 ST) and shen 4 ke 6 fen (16:48 ST), respectively.
Open the gates at mao 4 ke 6 fen (06:48 ST) and beat a drum.
Open the main gate at chen 2 ke 7 fen (07:51 ST) and beat a drum.
Close the Government Office at wu 1 ke 6 fen (11:06 ST) and beat a drum.
Close all of the gates at you 1 ke 2 fen (17:06) and beat a drum.

For days of $L = 285°–296°$: The times of sunrise and sunset are shen 1 ke 1 fen (07:03 ST) and shen 4 ke 7 fen (16:51 ST), respectively.
Open the gates at mao 4 ke 5 fen (06:45 ST) and beat a drum.
Open the main gate at chen 2 ke 6 fen (07:48 ST) and beat a drum.
Close the Government Office at wu 1 ke 5 fen (11:15 ST) and beat a drum.
Close all of gates at you 1 ke 3 fen (17:09 ST) and beat a drum.

According to Hirayama (1913a), the following is given in the Engishiki.

> Beat a drum 9 times at zi 子 and wu 午, 8 times at chou 丑 and wei 未, 7 times at yin 寅 and shen 申, 6 times at mao 卯 and you 酉, 5 times at chen 辰 and xu 戌, and 4 times at si 巳 and hai 亥; toll a gong bell as many as the numeral of ke at the notch of ke.

Table 14 summarizes the times of sunrise and sunset described in the Engishiki. Hirayama (1913a) has obtained the following conclusions from his study of these data. The dates giving the times of sunrise and sunset are all the same as those in Guchureki, and thus these two calendars are not independent of each other. The numerals of ke and fen next to 4 ke some fen are 1 ke some fen without the time of the initial ke. This implies that each double hour starts from 1 ke instead of 0 ke. The largest numeral in fen is 9 fen, which was given at the times of opening gates, and thus the largest value of the numerals in fen must be larger than, or equal to, 9. Hirayama supposes that the largest value of fen was 9 because, otherwise, the difference would become disordered. From these considerations, he concludes that each day was divided into 48 ke, each double hour was divided into 4 ke (from 1 ke to 4 ke), and each ke was divided into 10 fen (from 0 fen to 9 fen).

As Hirayama and Hashimoto both note, the middle of sunrise and sunset ranges from wu 2 ke 9.0 fen 午 2 刻 9.0 分 to wu 3 ke 0.5 fen 午 3 刻 0.5 分, which are different from the middle of wu (wu 3 ke 0 fen 午 3 刻 0 分) by as much as 1 fen, which equals to 3 min in the current units of time. The reason is not clear.

4.3. *Comparison of the Daytime Duration with the Calculation*

The daytime duration of dates equally apart from the day of the central line of table 14 are reversed for both the data given in Guchureki and in Engishiki. The daytime duration for the dates of chunfen and qiufen is 12 hr. These facts indicate that the dates of those records are based on dingqi without refraction. The middle of the first, central, and last lines are 5.5 days later than the winter solstice (dongzhi), the equinoxes (chunfen and qiufen), and the summer solstice (xiazhi), respectively. This would be due to a deficiency in the calendar system used at that time. After correcting for this difference in the dates, a comparison is made for the daytime durations given in Guchureki and Engishiki with the calculation using DE406, assuming that the dates of 24 qis were determined by dingqi. The results are given in table 15. The calculation was made for the epoch AD 1000 at Kyoto 京都 (latitude 35°.02). From the comparison, it is shown that the recorded duration agrees with the correct ones within a few minutes in the current units of time.

4.4. *Units of Time Employed in the Predicted Times of Solar and Lunar Eclipses*

In the calendar system called Xuanmingli, which was used between the years 862 and 1684 in Japan, each day was divided into 12 double hours, and also into 100 ke. Therefore, each double hour was divided into $[8 + (1/3)]$ ke. Each ke was divided into 84 fen. Each double hour started from the initial ke 初刻, and was followed by 1 ke, 2 ke, . . . , 8 ke. Each ke started from 0 fen (actually shown by blank), and was followed by 1 fen, 2 fen, . . . , 83 fen, except for 8 ke, which ended at 27 fen. The first double hour, zi, was centered at midnight, and was equivalent to 23:00–01:00 in the local apparent solar time.

Hashimoto (1966) has investigated Japanese systems of time units from a study of the predicted times of the beginning, the maximum magnitude, and the end for 79 solar and lunar eclipses between the years 877 and 1527. Most of these data were from Kanda (1935), but 3 eclipses were from his own list. From this study, he found that the times for the eclipses of the year 1028 given in Sakeiki 左経記 and the times of all of the eclipses between the years 1150 and 1527 in various records were in the time system of Xuanmingli. However, he also

Table 15. Comparison of the daytime durations given in Guchureki and Engishiki with calculations using DE406.*

L		Cal	Guchureki	Difference	Engishiki	Difference
270°		9:38	9:31	−7	9:42	+4
285°	255°	9:44	9:41	−3	9:48	+4
296°	244°	9:55	9:50	−5	10:03	+8
305°	235°	10:08	10:05	−3	10:15	+7
313°	227°	10:22	10:19	−3	10:30	+8
321°	219°	10:37	10:29	−8	10:45	+8
329°	211°	10:52	10:48	−4	11:03	+11
337°	203°	11:10	11:07	−3	11:15	+5
345°	195°	11:27	11:26	−1	11:30	+3
353°	187°	11:44	11:46	+2	11:45	+1
0°	180°	12:00	12:00	0	12:00	0
7°	173°	12:16	12:14	−2	12:15	−1
15°	165°	12:33	12:34	+1	12:30	−3
23°	157°	12:50	12:53	+3	12:45	−5
31°	149°	13:08	13:12	+4	12:57	−11
39°	141°	13:23	13:31	+8	13:15	−8
47°	133°	13:38	13:41	+3	13:30	−8
55°	125°	13:52	13:55	+3	13:45	−7
64°	116°	14:05	14:10	+5	13:57	−8
75°	105°	14:16	14:19	+3	14:12	−4
90°		14:22	14:29	+7	14:18	−4

* *L* is the Sun's ecliptic apparent longitude. The values in the column "Cal" are the calculated values of the daytime duration. The calculation is made for the epoch AD 1000 at Kyoto 京都 (latitude 35.°02).

found that three other time systems were used for the eclipses until the year 1149. Although the details of these three time systems are not known, the numbers of ke in a day and the numbers of fen in a ke which Hashimoto presumed from the eclipse predictions are summarized in section 5.

5. Conclusions

In this paper the time systems employed in ancient China and Japan were investigated.

It has been well known that both in ancient China and Japan, 1 day was divided into 12 double hours, and the first double hour corresponded to the time from 23 hr to 1 hr in the local apparent solar time. However, it has been confirmed that in the Song dynasty in China the first double hour corresponded to the time from 0 hr to 2 hr (see Appendix). In China, 1 day was also divided into 100 equal parts, called ke, which is translated into English as "mark". This interval ke was subdivided by a time unit called fen, and the number of fen in 1 ke varied from dynasty to dynasty. In ancient Japan, the time units ke and fen were also used, but the lengths of both of them varied depending on the era. Table 16 summarizes the number of ke in a day and the number of fen in a ke in China and Japan, described in this article. The last column gives the dynasties, reference books, or periods during which the system was used.

It is shown that daytime included dawn and dusk until the Tang dynasty, but it did not include dawn or dusk in Songshi in China. In ancient Japan, daytime did not include dawn or dusk.

It is also shown that a table of daytime duration was constructed without knowing the variation of the Sun's speed in

Table 16. Length of day and ke in units of ke and fen, respectively.

ke/day	fen/ke	Period or reference book
China		
100	10	Houhan (25–220)
100	10	Jin (25–420)
100	10	Song of Nanchao (420–479)
100	60	Sui (581–619)
100	100	Sui (581–619)
100	24	Tang (618–907)
100	147	Song (960–1279)
100	60	Song (960–1279)
Japan		
(Sunrise and sunset)		
50	6	Guchureki
48	10	Engishiki
(Eclipse predictions)		
48	4?	877–1027
48	10	1029–1096*
50?	40	1096–1149
100	84	1150–1527† (Xuanmingli)

* The numbers of ke in a day and of fen in a ke used for the eclipse predictions between the years 1029 and 1096 are the same as those of Engishiki, but in the time system used in the eclipse predictions each ke starts with 1 fen and ends with 10 fen whereas in Engishiki each ke starts with 0 fen and ends with 9 fen.

† Includes eclipses of 1028 in Sakeiki.

one year in Houhanshu. The table given in Houhanshu seems to indicate that water clocks, called louke, used at that time had a daily variation that varied with the seasons. The table of daytime duration and times of sunrise and sunset was reconstructed several times until the Tang dynasty, but it is found that all of them were based on the table given in Houhanshu, except for one table in Suishu, which incorporated new observations. It is also found that the table of the daytime duration and times of sunrise and sunset given in Songshi was constructed based on knowledge concerning the variation of the Sun's speed in one year.

All of the daytimes and nighttimes, the times of sunrise and sunset, and the lengths of shadows given in the official Chinese chronicles referred to a particular latitude of about 34.°5, where the ratios of the daytime and nighttime durations, defined by sunrise and sunset, are simple values of 2/3 in the winter solstice and 3/2 in the summer solstice. The Japanese system adopted this Chinese tradition.

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Appendix. The Twelve Double Hours

Both in ancient Japan and China, 1 day was divided into 12 double hours. Their names and their corresponding time intervals are summarized in table 17.

Table 17. The 12 double hours.*

Name			Animal	Time (1)	Time (2)
				h h	h h
子	zi	⟨tzu⟩	rat	23–01	00–02
丑	chou	⟨ch'ou⟩	ox	01–03	02–04
寅	yin	⟨yin⟩	tiger	03–05	04–06
卯	mao	⟨mao⟩	hare	05–07	06–08
辰	chen	⟨ch'en⟩	dragon	07–09	08–10
巳	si	⟨szu⟩	snake	09–11	10–12
午	wu	⟨wu⟩	horse	11–13	12–14
未	wei	⟨wei⟩	sheep	13–15	14–16
申	shen	⟨shen⟩	monkey	15–17	16–18
酉	you	⟨yu⟩	cock	17–19	18–20
戌	xu	⟨hsu⟩	dog	19–21	20–22
亥	hai	⟨hai⟩	pig	21–23	22–00

* Time (1) is the one used in ancient China until Tang dynasty and ancient and medieval Japan. Time (2) is the one used in the Song dynasty of China.

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