# Solar Eclipses in the First Half of the Chunqiu Period

Kiyotaka TANIKAWA, Tadato YAMAMOTO, and Mitsuru SÔMA

National Astronomical Observatory of Japan, 2-21-1 Osawa, Mitaka, Tokyo 181-8588

tanikawa.ky@nao.ac.jp

(Received 2009 July 30; accepted 2010 April 15)

# Abstract

In the Chinese chronicle Chunqiu, which describes a partial (722 BC–481 BC) history of the Chunqiu Period (770 BC–403 BC), there are recorded 37 solar eclipse observations, starting in 720 BC and ending in 481 BC. Among these, there are 10 eclipse records that lack either the cyclic day number or a statement of 'the new moon', or both. Three explanatory books Zuoshi-Zhuan, Guliang-Zhuan, and Gongyang-Zhuan written in the Zhanguo Period (403 BC–221 BC) have different interpretations concerning these records. Zuoshi-Zhuan simply says that royal astronomers forgot to record these data. On the other hand, Guliang-Zhuan says that these eclipses were observed on the last day of the month or during the night. Gongyang-Zhuan says that these were mainly observed on the second day of the month. A famous astronomer, Liuxin in the Han Dynasty, argued close to the idea of Gongyang-Zhuan. These discussions are related to the problem of establishing a precise calendar system in the Chunqiu Period. Controversial arguments continue until today. A Japanese astronomer, Toshio Watanabe (1958), conjectured that these eclipses may have been observed at sunrise or sunset. In the present report, we intend to confirm Watanabe's conjecture, and after confirming this we use this property to accurately determine the range of  $\Delta T$  (see section 1 for definition) in this period. We have obtained new  $\Delta T$  ranges in seconds from five eclipses which accompanied deep contemporaneous solar eclipses:

 $20153 \leq \Delta T \leq 21094 \text{ at around } 720 \text{ BC February } 22,$   $18526 \leq \Delta T \leq 20686 \text{ at around } 676 \text{ BC April } 15,$   $19409 \leq \Delta T \leq 20402 \text{ at around } 648 \text{ BC April } 6 \text{ if the site is Paros,}$ or  $18353 \leq \Delta T \leq 19235 \text{ at around } 648 \text{ BC April } 6 \text{ if the site is Thasos,}$   $19172 \leq \Delta T \leq 20910 \text{ at around } 599 \text{ BC March } 6,$   $16134 \leq \Delta T \leq 19101 \text{ at around } 558 \text{ BC May } 31.$ 

Key words: clock correction — Earth and Moon — eclipse — histories and philosophy of astronomy

# 1. Introduction and Motivation

We wish to determine variations in the past of Earth's rotational velocity. This is important not only for astronomy and geophysics, but also for history. We use ancient solar eclipses for this purpose. Actually, we have basically two parameters by which the time and place of a particular eclipse are determined. In order to explain these, let us first introduce some notions. Homogeneously flowing time is denoted by TT, which is an abbreviation of Terrestrial Time. The time measured by the rotation of Earth is denoted by UT (Universal Time). Then, we denote the difference of these two as

$$\Delta T = \mathrm{TT} - \mathrm{UT}.\tag{1}$$

This  $\Delta T$  is one of the two parameters. We know that the difference was roughly 2 hours two thousand years ago. The other parameter is the acceleration of the motion of the Moon along its orbit. The tidal deceleration in the orbital motion of the Moon was derived as  $25.^{\prime\prime}858 \text{ cy}^{-2}$  by Chapront et al. (2002) from analyses of modern Lunar laser ranging measurements. It corresponds to the recession rate of about  $3.8 \text{ cm yr}^{-1}$  of the distance of the Moon from the Earth. In the past, the decrease rate might have been different. We expect that our research may more accurately determine the past values of these two

parameters, and in turn determine more accurately the history of Earth's rotational variations and the lunar deceleration. The results can be used in the study of ancient history.

It is known that the eclipses of the Sun and the Moon and stellar occultations by the Moon are important for estimating  $\Delta T$ . In particular, total solar eclipses play the role of a time mark in the Earth clock. We can reconstruct Earth's rotational velocity variations from historical eclipses. The number of historical records for the 8th century BC is extremely small. Stephenson (1997) expressed the variations of  $\Delta T$  during this period with a fitted spline curve using Babylonian lunar eclipses. Unfortunately, the dispersion of the data around the spline curve is so large that we suspect that the spline fitting may conceal large-amplitude variations of shorter periods.

We turn our attention to the eclipses in the Chunqiu Period (770 BC–403 BC). Chunqiu (春秋) is a Chinese chronicle that describes the history from 722 BC to 481 BC in the Chunqiu Period. The study of eclipses in Chunqiu has a long history. The first study, due to Liuxin (劉歆), had already appeared in the Wuxing-Zhi (五行志) of Hanshu (漢書), the history book for the Former Han Dynasty (202 BC–AD 8), originally written and published in the AD 1st century. In modern times, Shinjo (1928) published the 'Long Calendar of Chunqiu' (春秋長暦). Shinjo reconstructed the calendar of the Chunqiu

庚

丙

÷

Table 1. Gānzhī

(a) Tiān-Gān (天干) or Shí-Gān (十干)								
甲乙	こ戸	र्गे .	丁 戊	己	庚	辛	Ŧ	癸
jiǎ y	ĭ bĭı	ng d	īng wi	ı jĭ	gēng	g xīn	rén	guĭ
	(b) D	ì-Zhī	(地支)	or Shiè	èr-Zhī	(十二支	友)	
	子	丑:	寅	卯	辰	E	.,	
	Zľ	chǒu	yín	mǎo	chén	sì		
	午	未	申	酉	戌	亥		
	wŭ	wèi	shēn	yǒu	хū	hài		
(c) Gānzhī (干支) [60-day cycle day number]								
甲子 [1]	乙丑	[2]	丙寅 [3]	丁卯	<sup>[4]</sup>	戊辰 [5		出巳 [6]
庚午 [7]	辛未	[8]	壬申 [9]	癸酉	[10]	甲戌 [1	1] Z	」亥 [12]
丙子 [13]	丁丑	[14]	戊寅 [15	] 己卯	[16]	庚辰 [1	7] 辛	ミ巳 [18]
壬午 [19]	癸未	[20]	甲申 [21	] 乙酉	[22]	丙戌 [2	3] 丁	「亥 [24]
		F A < 7			I FOOT	CT 1	07 <del>7</del> 7	2 L L L A O L

戊子 [25] 己丑 [26] 庚寅 [27] 辛卯 [28] 壬辰 [29] 癸巳 [30] 甲午 [31] 乙未 [32] 丙申 [33] 丁酉 [34] 戊戌 [35] 己亥 [36] 庚子 [37] 辛丑 [38] 壬寅 [39] 癸卯 [40] 甲辰 [41] 乙巳 [42] 丙午 [43] 丁未 [44] 戊申 [45] 己酉 [46] 庚戌 [47] 辛亥 [48] 壬子 [49] 癸丑 [50] 甲寅 [51] 乙卯 [52] 丙辰 [53] 丁巳 [54] 戊午 [55] 己未 [56] 庚申 [57] 辛酉 [58] 壬戌 [59] 癸亥 [60]

Period based on the periodicity of 'ganzhi' (see table 1, 干支) of eclipse records.

In order to better determine the values of  $\Delta T$ , the total eclipses observed from known sites are the most useful. However, this type of record is rare. We are particularly interested in eclipse records that lack either ganzhi or a statement of the first day of the month, or both, among 37 solar eclipses recorded in Chunqiu. We call these eclipses target eclipses. These eclipses were from the beginning of the Han Period, suspected to be at sunrise or sunset (Hanshu, Wuxing-Zhi: Bangu 1962). Watanabe (1979), based on his astronomical calculations, found that some of them were eclipses at sunrise or sunset. However, the theory used in his calculation was of the 1920s, so his conclusion may contain systematic errors.

The purpose of the present paper is to actually show that these were eclipses at sunrise or sunset, and by way of this to give limits to the range of  $\Delta T$ . We also discuss the meaning of a lack of descriptions.

The values of  $\Delta T$  obtained from the same eclipse or contemporaneous eclipses should be identical or nearly identical. Fortunately, in some cases, there are records of total eclipses close to the years of the target eclipses. We will show that considering plural eclipses at a time we can obtain a narrower range of  $\Delta T$ . We also point out the utility of non-total eclipses.

We here give four short remarks. BC years and minus years differ by one; that is, 1 BC corresponds to 0, 2 BC to -1, and so on. The European calendar adopted in this report is Julian. The modern ephemerides of the Sun and Moon that we use are DE406 (Standish 1998). We omit the Chinese accents of the words in what follows.

#### **Target Eclipses and Auxiliary Eclipses** 2.

# 2.1. Preceding Studies

In Chinese history, eclipse records were fundamental for determining future (luni-solar) calendars. In particular, solar eclipses should take place on the first day of the month. So, royal astronomers had the duty to record the serial number of the day in the month, in addition to the 60-day cycle day number (ganzhi). A latter tradition goes back to even earlier ages. However, some Chunqiu eclipse records lack information on the serial day number in the month or the day number of the 60-day cycle, or both. These facts were already the subject of discussion in books edited in the fourth century BC, Zuoshi-Zhuan (左氏伝), Gongyang-Zhuan (公羊 伝), and Guliang-Zhuan (穀梁伝).

Hanshu, Vol. 27, Wuxing-Zhi, Vol. 7 of Zhi (Bangu 1962) says the following:

(01) Duke Yin (隠公) reign period, third year, second month, jisi [6], the Sun was eclipsed. Guliang-Zhuan says that the ganzhi was recorded, but it was not recorded whether it was on the first day of the month or not, and that this is because the eclipse took place on the last day of the month. Gongyang-Zhuan says that the eclipse was on the second day of the month. …… Shiji 史記 (Sima Qian 司馬遷, ~90 BC) says that as for (solar) eclipses (in Chunqiu), some say that these took place on the first day of the month, but actually these took place on other days; others say that even if not written as they occurred on the first day, these occurred on the first day; still others say simply that the astronomers lost ganzhi and/or the cyclic day number when these were not written.

(03) Duke Huan (桓公) reign period, seventeenth year, tenth month, first day, the Sun was eclipsed. Guliang-Zhuan says that 'shuo' (朔, the new Moon) is written in the record, but the cyclic day number is not, and hence the eclipse took place on the second day of the month.

(04) Duke Zhuang (荘公) reign period, eighteenth year, third month, the Sun was eclipsed. Guliang-Zhuan says that the record has neither the cyclic day number nor 'shuo', and that this is because the eclipse took place at night. The present historian guesses that the conjunction of the Sun and Moon was in the night of the first day of the month, and during the next morning, the Sun rose eclipsed by the Moon, and after sunrise the eclipse was over. We call this the 'night eclipse'. ..... Gongyang-Zhuan says that the eclipse was on the last day of the month. [Translated by K. Tanikawa from the Japanese translation by Kotake (1977, p. 229).]

In the above, (01), (03), and (04) are the serial numbers of eclipse data in table 2.

Now at this point, we summarize how three Zhuans thought of the target eclipses. From (01), in Guliang-Zhuan, if the (cyclic) day number exists and 'shuo' is lacking, the eclipse was taken to be that which occurred on the last day of the month, whereas in Gongyang-Zhuan, it was on the second day of the month. From (03), in Guliang-Zhuan, if 'shuo' exists and the cyclic day number is lacking, the eclipse was taken to be that which occurred on the second day of the month.

From (04), in Guliang-Zhuan, the eclipse without both 'shuo' and the cyclic day number was considered to be a night eclipse. In Gongyang-Zhuan, the eclipse was taken to be that

# Solar Eclipses in the Chunqiu Period of China

799

Table 2. List of Chunqiu eclipses.\*

Eclipse	Opp.	Juliar	n YN	1D		(	Chur	nqiu YMD		Zuoshi	Gong-	Gu-	Liu-	Watanabe
					Duke	Y	Μ	D			yang	liang	xin	
(1)	(2)	(	3)					(4)		(5)	(6)	(7)	(8)	(9)
01	1147	-719	2	22	隠公	3	2	己巳 [6]		lost	2nd	last	2nd	sunrise
02	1176	-708	7	17	桓公	3	7	壬辰 [29]	朔・既					total
03	1211	-694	10	10	桓公	17	10		朔	lost		2nd		
04	1257	-675	4	15	荘公	18	3			lost	last	night	last	
05	1275	-668	5	27	荘公	25	6	辛未 [8]	朔				2nd	
06	1278	-667	11	10	荘公	26	12	癸亥 [60]	朔				2nd	
07	1288	-663	8	28	荘公	30	9	庚午 [7]	朔					
08	1311	-654	8	19	僖公	5	9	戊申 [45]	朔					
09	1328	-647	4	6	僖公	12	3	庚午 [7]		lost	2nd	last		sunset
10		-644			僖公	15	5			lost	last	night	朔	non-eclipse
11	1383	-625	2	3	文公	1	2	癸亥 [60]		lost	2nd	last	朔	
12	1419	-611	4	28	文公	15	6	辛丑 [38]	朔				2nd	
13	1449	-600	9	20	宣公	8	7	甲子 [1]	・既	lost	2nd	last	2nd	
14	1452	-598	3	6	宣公	10	4	丙辰 [53]		lost	2nd	last		sunrise
15					宣公	17	6	癸卯 [40]		lost	2nd	last	last	non-eclipse
16	1516	-574	5	9	成公	16	6	丙寅 [3]	朔				2nd	
17	1519	-573	10	22	成公	17	12	丁巳 [54]	朔					
18	1555	-558	1	14	襄公	14	2	乙未 [32]	朔				2nd	
19	1559	-557	5	31	襄公	15	8	丁巳 [54]		lost	2nd	last	2nd	sunrise
20	1572	-552	8	31	襄公	20	10	丙辰 [53]	朔					
21	1574	-551	8	20	襄公	21	9	庚戌 [47]	朔					
22					襄公	21	10	庚辰 [17]	朔					error
23	1579	-549	1	5	襄公	23	2	癸酉 [10]	朔				2nd	
24	1582	-548	6	19	襄公	24	7	甲子 [1]	朔・既					total
25					襄公	24	8	癸巳 [30]	朔					error
26	1590	-545	10	13	襄公	27	12	乙亥 [12]	朔					
27	1616	-534	3	18	昭公	7	4	甲辰 [41]	朔					
28	1636	-526	4	18	昭公	15	6	丁巳 [54]	朔					
29	1642	-524	8	21	昭公	17	6	甲戌 [11]	朔				2nd	
30	1652	-520	6	10	昭公	21	7	壬午 [19]	朔				2nd	
31	1655	-519	11	23	昭公	22	12	癸酉 [10]	朔					
32	1659	-517	4	9	昭公	24	5	乙未 [32]	朔				2nd	
33	1678	-510	11	14	昭公	31	12	辛亥 [48]	朔				2nd	
34	1690	-504	2	16	定公	5	3	辛亥 [48]	朔				2nd	
35	1709	-497	9	22	定公	12	11	丙寅 [3]	朔				2nd	
36	1717	-494	7	22	定公	15	8	庚辰 [17]	朔					
37	1751	-480	4	19	哀公	14	5	庚申 [57]	朔				2nd	

\* The first row is the serial number, the second shows the Oppolzer number, the third is the year, month, and day of month in the Julian calendar, the fourth is the Chinese year, month, day of the Chunqiu era, the fifth, sixth, and seventh columns represent the understanding of the target eclipses due to Zuoshi-Zhuan, Gongyang-Zhuan, and Guliang-Zhuan; the eighth column shows the interpretation of Liuxin; the ninth column shows Watanabe's interpretation. ■ denotes that the data is not written. 'lost' means that the data is lost. '2nd' means the second day of the month, while 'last' the last day of the month.

which occurred on the last day of the month. These interpretations are listed in the sixth and seventh columns of table 2.

In the case of Zuoshi-Zhuan, there is a comment only on the data (01). The Zhuan says that the data without either 'shuo' or the cyclic day number, or without both is due to the fact that the royal astronomers forgot to record it. Hanshu reproduces this interpretation of Zuoshi-Zhuan. Hanshu summarizes the eclipses in Chunqiu as follows:

There were twelve Dukes in the Chunqiu Period (722 BC–481 BC), and the Period continued for two hundred and forty two years, during which there were

36 solar eclipses. Guliang-Zhuan calculates that there were 26 eclipses on the first day of the month, 7 eclipses on the last day; 2 were the night eclipses and one was on the second day. Gongyang-Zhuan calculates that there were 27 eclipses on the first day of the month, 7 eclipses on the second day; two were on the last day. Zuoshi-Zhuan calculates that there were 16 eclipses on the first day of the month, 18 eclipses on the second day; one was on the last day, and two were without a cyclic day number. (Kotake 1977, p. 229).

It is to be noted that the author of the Hanshu thought

that the number of eclipses was 36, whereas the author of Zuoshi-Zhuan thought that it was 37.

It seems that the number of eclipses on the first day in Zuoshi-Zhuan is too small. The problem proposed by the above summary is related to the calendar system of the Chunqiu Period. Gongyang-Zhuan and Guliang-Zhuan proposed a definite rule of the second-day eclipses, the last-day eclipses, and the night eclipses. Our purpose is to obtain a clear-cut conclusion concerning the astronomical reason related to this rule.

Nōda and Yabuuti (1947) introduced the calculation of Liuxin in Hanshu. We list Liuxin's interpretation in the eighth column of table 2. Watanabe (1958), based on the theory of Schoch (1927), calculated the time in the day of the Chunqiu eclipses. From this calculation, he concluded that eclipses (01), (14), and (19) were at sunrise, while eclipse (09) was at sunset. Eclipse (04) was almost at sunset based on the same calculation. However, he does not say anything about the eclipse. Later, Saito and Ozawa (1992) added eclipse (04) to the member of eclipses at sunset. They, in addition, added eclipse (22) as an eclipse at sunset by changing the date from the Duke Xiang ( $\Re$ C) reign period, twenty first year, tenth month, gengchen [17], shuo to Duke Xiang ( $\Re$ C) reign period, twenty sixth year, tenth month, gengchen [17], the last day of the month.

In the present paper, we treat eclipses (01), (04), (09), (14), and (19) by a method developed by us, and decided whether these were eclipses at sunrise or sunset, and using these results obtained better ranges of  $\Delta T$ . In the final section, we discuss the meaning of our results.

## 2.2. Target Eclipses and Observing Sites

There are ten target eclipses. These are included in table 2.

Table 3. Observation sites

Place	Long. E. (°)	Lat. N. (°)	Remark
Qufu	117.02	35.53	Capital of Lu, China
Paros	25.10	37.07	Island in Aegean sea
Thasos	24.70	40.77	Island in Aegean sea
Pteria	34.23	39.77	Asia Minor

We newly list these in table 4. Table 3 shows the latitudes and longitudes of the observation sites.

In what follows, we do not treat all of these eclipses. Our actual targets are eclipses (01), (04), (09), (14), and (19). We will explain in Discussion why we do not consider eclipses (03), (10), (11), (13), and (15).

### 2.3. Auxiliary Eclipses

We have been using plural observations of solar eclipses in order to determine narrower ranges of  $\Delta T$  (Tanikawa & Sôma 2004a, 2004b). In the present paper, we adopt the same procedure. We select one to several eclipses around a target eclipse, and call them auxiliary eclipses. We simultaneously consider the target and auxiliary eclipses, make the Sôma Diagram (see section 3 for explanation), and look for a narrower range of  $\Delta T$ . In addition, we confirm that the coefficient of the lunar tidal term is within  $\pm 1'' \text{cy}^{-2}$  of the present value obtained from lunar laser ranging observations.

### 3. Preparations

In the following section, we carry our calculations for individual eclipses, and obtain the ranges of  $\Delta T$ . We here prepare for the calculations. We assume that all observations of eclipses were done in Qufu (曲阜), the capital of Lu (魯), a feudal province, since Chunqiu is the chronicle of Lu. Watanabe (1958, p. 351) suggested that eclipse (13) (Duke Xuan reign period, eighth year, seventh month) might have been observed at a place other than Qufu, because the magnitude in Qufu was 0.87 according to his calculation based on the Schoch theory. This may be possible. However, if we admit variable observation sites, then two more free parameters (longitude and latitude of the site) are added. We do not consider this possibility. We fix the observation site to Qufu.

Let us introduce a diagram contrived by M. Sôma. We take as the abscissa the correction to the coefficient of the timesquare term in the lunar longitude (arcsec century<sup>-2</sup>) or, in short, the correction to the tidal term, and take as the ordinate the value of  $\Delta T$ . We plot on this plane curves of the boundary of eclipse(s) observed at sites of known positions. It is cumbersome to call this figure from its property. We

Table 4.	Target e	eclipses.*
----------	----------	------------

Eclipse	Oppolzer	Julian YMD	Chinese YMD	朔	Interpret	ations
			Duke		Watanabe	Wangtao
01	1147	-719 2 22	隠公 3 2 己巳[6]		sunrise	
03	1211	-694 10 10	桓公 17 10 🔳	朔		
04	1257	-675 4 15	荘公 18 3 🔳			sunset
09	1328	-647 4 6	僖公 12 3 庚午[7]		sunset	
10		-644	僖公 15 5 🔳	■・朔	non-eclipse	
11	1383	-625 2 3	文公 1 2 癸亥 [60	)]		
13	1449	-600 9 20	宣公 8 7 甲子[1]	・既	Mag. 0.87	
14	1452	-598 3 6	宣公 10 4 丙辰 [53	3]	sunrise	
15	_		宣公 17 6 癸卯 [40	)]	non-eclipse	
19	1559	-557 5 31	襄公 15 8 丁巳 [54	4]	sunrise	

\* The meaning of columns are from the left, the serial number in Chunqiu; the Oppolzer number; the year, month, and day in the Julian calendar; the year, month, day, and ganzhi of the Chunqiu Period; interpretations of Watanabe (1958), and Wangtao (Shinjo 1928). ■ denotes that the data is not written. '朔' means the first day of the month. '既' means (that the eclipse was) total.

simply call this a Sôma Diagram. The positions on Earth's surface of the eclipse band depend on these parameters. Note that the correction to the coefficient of the time-square term in the lunar longitude is half of the correction to the lunar tidal acceleration. The present value of the lunar tidal acceleration is  $-25''.86 \text{ cy}^{-2}$  (Chapront et al. 2002). The utility of the Sôma Diagram is that contemporaneous eclipses can be expressed in the same diagram, and hence the possible ranges of the correction to the tidal term and  $\Delta T$  are obtained as the intersections of plural bands.

In order to make the Sôma Diagram the boundaries of the  $\Delta T$  values for any solar eclipse at a known site are calculated for each correction,  $\Delta c$ , to the coefficient of the timesquare term in the lunar longitude by adding the correction to the calculated apparent lunar longitude,  $\lambda_0$ , of the Moon from DE406 by the following equation:

$$\lambda = \lambda_0 + \Delta c \ T^2, \tag{2}$$

where  $\lambda$  is the apparent longitude of the Moon used in the calculations of the eclipses and *T* is the time from the epoch J2000. Note that when one introduces the correction  $\Delta c T^2$  to the lunar longitude, it is possible that one also needs corrections of the terms expressed as  $\Delta a + \Delta b T$ , but since the time of the eclipses that we deal with are far from the present, the effect of the terms  $\Delta a + \Delta b T$  is insignificant compared to that of the term  $\Delta c T^2$ , and hence we can neglect the correction  $\Delta a + \Delta b T$ .

According to Sôma et al. (2004), the present value of the coefficient of the time-square term in lunar longitude may be consistently extended to the past two thousand years. Basically in the present work, we adopt the present value. Nevertheless, our analysis shows that there is an uncertainty of  $\pm 1''$  century<sup>-2</sup>. We thus keep in mind that the value of the lunar tidal acceleration is still uncertain within this error, and always use the Sôma Diagram for a confirmation.

The method adopted in the present paper has been developed in Tanikawa and Sôma (2002, 2004a, 2004b), Sôma et al. (2004), and Kawabata et al. (2004). In this method, plural phenomena are used to simultaneously determine the range of  $\Delta T$  for the year or years of the observation. In the present paper, the eclipses which will be the object of analysis are called *target eclipses*, and the other contemporaneous eclipses will be called *auxiliary eclipses*. Auxiliary eclipses do not necessarily exist in the close neighborhood in the sense of time of the target eclipse. Here, we estimate the errors due to the difference of years of observation. We use as a reference a curve proposed by Stephenson (1997),

$$\Delta T = 31 \times \left(\frac{\text{year} - 1820}{100}\right)^2 - 20 \quad \text{s.}$$
(3)

According to this formula,  $\Delta T$  decreases by  $16 \,\mathrm{syr}^{-1}$  on average during 700 BC through 600 BC. We adopt this value to estimate the errors of the obtained  $\Delta T$ . It is to be noted that if we use an auxiliary eclipse observed with *n* years apart, then the errors are  $n \times 16 \,\mathrm{s}$ .

An intuitive explanation of the effect of  $\Delta T$  is that, for a fixed year, month, and day, the eclipse band shifts to the east if  $\Delta T$  is larger, while it shifts to the west if  $\Delta T$  is smaller. On the other hand, the effect of a difference in the lunar tidal



**Fig. 1.** Eclipse band for a general eclipse of low latitude. The eclipse takes place at sunrise if the observation site is in a region bounded by curve ADCE, whereas it takes place at sunset if the observation site is in a region bounded by curve HKJL.

acceleration is difficult to explain intuitively. However, basically, the eclipse band moves to the west if the correction to the adopted tidal term is positive, and to the east otherwise.

We define what is the **eclipse at sunrise** and the **eclipse at sunset** in Qufu.

### **Conditions for a sunrise eclipse:**

If the observation site is in the region bounded by curve ADCE in figure 1, the Sun rises while being eclipsed. If, in addition, the site is to the right of arc ABC, the maximum obscuration is observable, whereas if the site is to the left, the maximum obscuration takes place below the horizon.

# Conditions for sunset eclipse:

If the observation site is in the region bounded by curve HKJL, the Sun sets while being eclipsed. If, in addition, the site is to the left of arc HIJ, the maximum phase of the eclipse can be observed, whereas if the site is to the right of the arc, the maximum obscuration takes place below the horizon.

In determining the range of  $\Delta T$  using the supposed sunrise eclipses or sunset eclipses, we need some remarks. In the case of an eclipse at sunrise, the upper limit of  $\Delta T$  is given rather strictly because for a larger  $\Delta T$  the eclipse ends when the Sun is fully seen. The lower limit of  $\Delta T$  is not so clear-cut. In the case of a sunset eclipse, the situation is reverse.

If the eclipse takes place in the arctic region, either arc AFH or arc CGJ does not exist. In these cases, some points of subtleties appear. We do not go into any details.

Target eclipses were surely observed. So the cases in which the Sun rises after the eclipse or the eclipse starts after the sunset are out of our consideration.

# 4. Individual Eclipses

4.1. -719 February 22: Eclipse at Sunrise

Target	-719	2	22	(01)		
	隠公	3	2	己巳[6]		
Auxiliary	-708	7	17	(02)		
·	桓公	3	7	壬辰 [29]	朔・既	
denotes a lack of data.						

As shown in the above list, the record of the target eclipse lacks a statement of 'shuo'. The auxiliary eclipse is considered

Table 5. Condition for an eclipse at sunrise for -719 February 22.

″cy <sup>-2</sup>	Ranges of $\Delta T$
-5.0	25876-27900-30230
-4.0	24526-26682-29130
-3.0	23188-25464-28016
-2.0	21862-24246-26890
-1.0	20546-23028-25752
0.0	19238-21808-24606
+1.0	17938-20588-23450
+2.0	16646-19366-22288
+3.0	15358-18146-21118
+4.0	14078-16924-19940

Table 6. Condition of a total eclipse in Qufu, -708 July 17.

$^{\prime\prime} \mathrm{cy}^{-2}$	Range of $\Delta T$
-5.0	29344-30297
-4.0	27530-28475
-3.0	25707-26646
-2.0	23871-24808
-1.0	22020-22958
0.0	20153-21094
+1.0	18266-19213
+2.0	16357-17313
+3.0	14423-15390
+4.0	12459–13442



Fig. 2. Sôma Diagram for the target eclipse -719 February 22. The band with solid curves is for the sunrise (SR) eclipse on -719 February 22, and the band with dotted curves is for the total (T) eclipse on -708 July 17. The vertical thick line indicates the candidate range of  $\Delta T$ .

to be total in Qufu. Two eclipses observed 11 years apart. So, the errors of our method amount to  $\pm 200$  s.

The range of  $\Delta T$  is given as a function of the correction to the tidal term in tables 5 and 6 for the target and auxiliary eclipses. In table 5, the values of the second, third, and fourth columns represent the values of  $\Delta T$  at which the eclipse starts at sunrise, at maximum obscuration, and the eclipse ends at sunrise.

Figure 2 shows how to determine the ranges of  $\Delta T$  and the

Table 7. The condition of sunset eclipse in Qufu for -675 April 15.

"cy-2	Min	$\Delta T$	Max
-5.0	20728-	-23906	-27143
-4.0	19490-	-22640	-25860
-3.0	18258-	-21374	-24572
-2.0	17030-	-20108	-23280
-1.0	15806-	-18842	-21986
0.0	14588-	-17578	-20686
+1.0	13378-	-16314	-19380
+2.0	12172-	-15050	-18072
+3.0	10970-	-13785	-16758
+4.0	9776-	-12522	-15438

lunar tidal coefficient (or its correction to the present value). As pointed out above, the two eclipses are 11 years apart, so errors of  $\pm 200$  s are included. The band in the Sôma Diagram for the target eclipse contains the band for the total eclipse for a wide range of (correction to) the lunar tidal term. In particular, for  $\pm 1''$ cy<sup>-2</sup> it is true. This means that as long as the auxiliary eclipse was total, the target eclipse was an eclipse at sunrise. Adopting the present value of the coefficient of the lunar tidal term (in figure 2, this corresponds to zero of the abscissa value) the range of  $\Delta T$  is given by

$$20153 \text{ s} < \Delta T < 21094 \text{ s}$$
 (width 941 s). (4)

In addition, the range is below the central dotted curve, which means that the maximum obscuration was observed after sunrise.

4.2. -675 April 15: Eclipse at Sunset

Target	-675	4	15	(04)	
	荘公	18	3		
Auxiliary	-668	5	27	(05)	
	荘公	25	6	辛未 [8]	朔
	-667	11	10	(06)	
	荘公	26	12	癸亥 [60]	朔
	-663	8	28	(07)	
	荘公	30	9	庚午 [7]	朔
	-654	8	19	(08)	
	僖公	5	9	戊申 [45]	朔

As can be seen in the above list, we have four auxiliary eclipses within 20 years of the target eclipse. Four eclipses were all partial because the word 'total' does not exist. Thus, in the Sôma Diagram, the conditions for these eclipses are out of the total-eclipse bands.

For all eclipses, the range of  $\Delta T$  for a given correction to the tidal term is given as in tables 7, 8, 9, 10, and 11. In the case of table 7, the value of  $\Delta T$  corresponds to the obscuration starting at sunset (Min), to the maximum obscuration at sunset, and to the obscuration ending at sunset (Max). The resulting Sôma Diagram is figure 3.

The candidate areas are between the thick parallel lines and out of the several parallel curves. By adopting the present value of the coefficient of the lunar tidal term, we have two separate ranges of  $\Delta T$ , as shown by the two thick lines in figure 3,

Table 8. The condition of total eclipse in Qufu for -668 May 27.

$^{\prime\prime} \mathrm{cy}^{-2}$	$\operatorname{Min} \Delta T \operatorname{Max}$		$\operatorname{Min} \Delta T \operatorname{Max}$
-3.34	17817	_	17978
-3.32	17411	_	18317
-3.3	17194	_	18466
-3.0	15469	_	19178
-2.02	11925	_	19412
-2.01	11893–15680	&	16123-19410
-2.0	11861-15501	&	16268-19409
-1.0	8829-10965	&	17436-19061
0.0	5980- 7838	&	17193-18526
+1.0	3218- 3050	&	16701-17902
+2.0	500-2184	&	16099-17232
+3.0	-2198534	&	15439–16538
+4.0	-48923225	&	14750-15837

Table 9. Condition of a total eclipse in Qufu for -667 November 10.\*

$^{\prime\prime} \mathrm{cy}^{-2}$	Min $\Delta T$ Max		$\operatorname{Min} \Delta T \operatorname{Max}$
-1.0519	11295	_	11358
-1.05	11162	_	11484
-1.0	10417	_	12070
-0.5	7727	_	13168
-0.3	6940	_	13316
-0.25	6753 - 10273	&	10500-13344
-0.24	6716 – 9996	&	10745-13349
-0.2	6569 - 9500	&	11114-13369
0.0	5856 - 8193	&	11787-13443
+1.0	2647 - 4313	&	12496-13459
+1.838	184 – 1697	&	12450-13239
+2.0	(-78)- 1227	&	12416-13182
+2.82	(-1137)1137	&	12162-12849
+3.0			12093-12768
+4.0			11650-12275

\* Parentheses indicate that the eclipse ended at sunset.

Table 10. Condition of a total eclipse in Qufu for -663 August 28.\*

$^{\prime\prime} \mathrm{cy}^{-2}$	$\operatorname{Min} \Delta T \operatorname{Max}$
-4.537	(14749)-14749
-4.0	(14050)-15198
-3.77	13804 -15344
-3.0	14321 -15686
-2.0	14671 -15896
-1.0	14788 -15919
0.0	14746 -15810
+1.0	14587 -15600
+2.0	14337 -15310
+3.0	14014 -14957
+4.0	13633 -14552

\* Parentheses indicate that the eclipse ended at sunset.

Table 11. Condition of a total eclipse in Qufu for -654 August 19.

"cy-2	$\operatorname{Min} \Delta T \operatorname{Max}$
-5.0	26098-26893
-4.0	24413-25219
-3.0	22711-23530
-2.0	20991-21824
-1.0	19249-20099
0.0	17482-18353
+1.0	15687-16582
+2.0	13859-14782
+3.0	11994-12950
+4.0	10083-11079



Fig. 3. The Sôma Diagram for the target eclipse -675 April 15. The band with solid curves is for the sunrise eclipse on -675 April 15. The vertical thick segments indicate the candidate ranges of  $\Delta T$ . Dashed lines are for the annular eclipse -668 May 27, dotted long-dash lines are for the annular eclipse -667 November 10, dotted short-dash lines are for the total eclipse -663 August 28, dotted lines are for the total eclipse -654 August 18. Here, SS stands for sunset, NA for non-annular, and NT for non-total.

$$15810 \,\mathrm{s} < \Delta T < 17193 \,\mathrm{s}$$
 or (5)

 $18526 \,\mathrm{s} < \Delta T \le 20686 \,\mathrm{s}. \tag{6}$ 

Let us check which of these ranges are appropriate. We obtained the range of  $\Delta T$  for the year -708,  $20153 \text{ s} < \Delta T < 21094 \text{ s}$ , as given in inequality (4). The time difference is 30 years. Thus, the difference of  $\Delta T$  may be at most 500 s. Then, the second range is preferable, since otherwise we need to consider a 4000 s jump during 30 years.

The auxiliary eclipse in -654 gives almost the same condition for  $\Delta T$  as the eclipse in -668 for the zero correction to the LTT (Lunar Tidal Term). We can thus dispense with the eclipse in -654. In this case, the maximum time difference becomes twelve years, and the results become more reliable.

As a conclusion, we obtain as a candidate range of  $\Delta T$ :

$$18526 \,\mathrm{s} < \Delta T \le 20686 \,\mathrm{s}. \tag{7}$$

This range is consistent with the ranges for the other target eclipses. This means that the probability that the eclipse was at sunset is very high.

Table 12. Condition of a sunset eclipse in Qufu for -647 April 6.

″cy <sup>-2</sup>	Min	$\Delta T$	Max
-5.0	21535-	-24175	-26720
-4.0	20440-	-23008	-25475
-3.0	19350-	-21838	-24220
-2.0	18273-	-20670	-22958
-1.0	17200-	-19502	-21685
0.0	16142-	-18334	-20402
+1.0	15094-	-17166	-19108
+2.0	14062-	-15998	-17798
+3.0	13046-	-14830	-16470
+4.0	12056-	-13660	-15118

# 4.3. -647 April 6: Eclipse at Sunset

—647 僖公	4 12	6 3	(09) 庚午 [7]	
-654	8	19	(08)*	
僖公	5	9	戊申 [45]	朔
-647	4	6		
Total ir	n Paro	s or 7	Thasos	
	647 僖公 654 僖公 647 Total ir	-647       4         僖公       12         -654       8         僖公       5         -647       4         Total in Parce	-647       4       6         僖公       12       3         -654       8       19         僖公       5       9         -647       4       6         Total in Paros or T       7	-647       4       6       (09)         僖公       12       3       庚午[7]         -654       8       19       (08)*         僖公       5       9       戊申[45]         -647       4       6         Total in Paros or Thasos

\* Eclipse in -654 is used in subsection 4.2.

The above table gives two auxiliary eclipses. The first one (08) was already used in the preceding subsection. The second one is the record of an eclipse observed either in Paros or Thasos. Both are cities in islands of the Aegean sea. The most interesting feature of the eclipse is that the eclipse was observed both in China and Greece.

This was recorded in a poem of Archilochus. No date was written. We need to identify this eclipse with the one in Oppolzer's Canon (Oppolzer 1887). Fortunately, according to Fotheringham (1920), the poet was known to have stayed in Paros or Thasos on the day of the eclipse. Newton (1970) and Stephenson (1997) listed several eclipses as its candidates. Stephenson (1997) calculated  $\Delta T$  for these eclipses. Unfortunately, he had no further criterion to choose one among others. He concluded that in either Paros or Thasos, there took place a total solar eclipse, and selected the eclipse on -647 April 6, as the most probable candidate. Our conclusion is more definite.

For each given value of the correction to lunar tidal term, the range of  $\Delta T$  is given in tables 12 and 13. For the data of eclipse (08), see table 11. The format of the table is the same as before. The corresponding Sôma Diagram is shown in figure 4.

It is apparent that we could not have total eclipses in both Paros and Thasos. At zero correction to the present LTT, We have two separate ranges of  $\Delta T$ . If we adopt Thasos, then we can impose on additional restriction on the range of  $\Delta T$  that the eclipse in -654 was not total in Qufu.

The two ranges are:

$$19409 \text{ s} \le \Delta T \le 20402 \text{ s} \text{ for Paros}, \tag{8}$$

$$18353 \text{ s} < \Delta T \le 19235 \text{ s} \text{ for Thasos.}$$

$$\tag{9}$$

In either case, the target eclipse satisfies the condition of sunset and the range of  $\Delta T$  is less than 1000 s. In addition, both cases are plausible. We cannot say which is better.

**Table 13.** Condition of a total eclipse in Paros and Thasos for -647 April 6.

$^{\prime\prime} \mathrm{cy}^{-2}$	$\operatorname{Min} \Delta T \operatorname{Max}$	$\operatorname{Min} \Delta T \operatorname{Max}$
	Paros	Thasos
-5.0	21605-22983	19811-21260
-4.0	21118-22521	19367-20816
-3.0	20648-22087	18924-20384
-2.0	20201-21689	18490-19971
-1.0	19785-21338	18070–19585
0.0	19409-21047	17671-19235
+1.0	19085-20836	17304–18933
+2.0	18830-20733	16977-18693
+3.0	18667-20785	16703-18536
+4.0	18632-21081	16501-18494



Fig. 4. Sôma Diagram for the target eclipse -647 April 6. The band with solid lines is for the eclipse on -647 April 6, in Qufu. The dashed band is for Paros, and the dot-dash line is for Thasos; the band with dot-short dash is for the eclipse on -654 August 19 in Qufu. The vertical thick lines show the candidate ranges of  $\Delta T$ . Here, SS stands for sunset, T for total and NT for non-total.

4.4. –598 March 6: Eclipse at Sunrise

Target	-598	3	6	(14)	
	宣公	10	4	丙辰 [53]	
Auxiliary	-600	9	20	(13)	
-	宣公	8	7	甲子 [1]	・既
	-584	5	28		
	Total ir	n Pter	ia (As	sia Minor)	

As shown in the above list, we have two auxiliary eclipses within 14 years.

The record says that eclipse (13) was total. Watanabe (1958) and Saito and Ozawa (1992) said that this was not total. The total eclipse in -584 in Asia Minor was a famous eclipse, which was said to be predicted by Thales. Stephenson (1997) does not specify the observation site. Özel and Kacar (2007) said that the observation site is Pteria. We adopt this idea.

For each given value of the correction to the LTT, the ranges of  $\Delta T$  are given in tables 14, 15, and 16. The format is the same as before. We show the corresponding Sôma Diagram in figure 5.

Table 14. Condition of a sunrise eclipse in Qufu for -598 March 6

$'' cy^{-2}$	Min $\Delta T$ Max
-5.0	20512-24280-28083
-4.0	19128-22872-26658
-3.0	17750-21464-25230
-2.0	16380-20055-23795
-1.0	15012-18648-22354
0.0	13652-17241-20910
+1.0	12300-15835-19457
+2.0	10952-14428-18000
+3.0	9612-13022-16538
+4.0	8280-11615-15068

Table 15. Condition of a total eclipse in Qufu for -600 September 20.

$'' cy^{-2}$	$\operatorname{Min} \Delta T \operatorname{Max}$
-5.0	29372-30076
-4.0	27752-28472
-3.0	26114-26853
-2.0	24456-25217
-1.0	22772-23560
0.0	21059-21878
+1.0	19309-20167
+2.0	17516-18421
+3.0	15668-16630
+4.0	13748-14786

**Table 16.** Condition of a total eclipse in Pteria for -584 May 28.

$^{\prime\prime} \mathrm{cy}^{-2}$	$\operatorname{Min} \Delta T \operatorname{Max}$
-5.0	20410-21932
-4.0	20154-21714
-3.0	19898-21505
-2.0	19646-21312
-1.0	19402-21143
0.0	19172-21008
+1.0	18962-20927
+2.0	18782-20928
+3.0	18648-21072
+4.0	18582-21518

As can be seen in the figure, the range for the target eclipse and the range for eclipse (13) do not overlap for the zero correction to LTT. This means that if the eclipse in -600 was total, then the eclipse in -598 could not be observed, since the eclipse took place just below the horizon. Conversely, if the eclipse in -598 was observable at sunrise, then the eclipse in -600 was not total in Qufu, but almost total. The eclipse in -598 was surely observed. We thus consider that the eclipse in -600 was almost total. This idea is strengthened if we add the eclipse in -584 observed in Pteria in the figure. In fact, the



**Fig. 5.** Sôma Diagram for the target eclipse -598 April 15. The band with solid lines shows the sunrise eclipse of -598 March 6 in Qufu. The band with dashed lines is for a total eclipse on -600 September 20 in Qufu. The band with a dot-long dash lines is for -584 May 28 in Pteria. This vertical interval is the candidate range of  $\Delta T$ . For a reference, the total eclipse band for the eclipse -548 June 19 is added (a folded band in the figure). Here, SR stands for sunrise, and T for total.

**Table 17.** Calculated maximum magnitude in Qufu of the eclipse on -600 September 20.

$\Delta T$	Maximum magnitude
20910	0.992
19172	0.902

total eclipse band of this eclipse in Pteria overlaps the sunrise condition of the eclipse in -598. The errors are about  $\pm 200$  s due to the year difference 14.

Let us consider the possibility that these three eclipses were observed as the records tell us. In order to realize it, the bands for -598 and -600 must overlap. If we move to the right in figure 5, that is, if we take the correction to the LTT to be large, this can be realized. However, the eclipse band for (24) superposed in figure 5 indicates that the correction should be less than  $1''cy^{-2}$  in order that this eclipse was total in Qufu. The overlap is negligibly small. The correction to the LTT is affected little in this case.

The eclipse in -647 satisfies the condition of zero correction. This also strengthens our conclusion. Another possibility is that the observation site may have been different. We do not have any information on this, so we neglect this case. We may conclude that the eclipse in -600 was not total in Qufu.

If the correction to the LTT is zero, the eclipse in -600 was almost total, and the eclipse in -584 was total in Pteria; then, the range of  $\Delta T$  is given by

$$19172 \text{ s} \le \Delta T \le 20910 \text{ s}. \tag{10}$$

The calculated maximum magnitude in Qufu of the eclipse of -600 September 20 is given in table 17.

The eclipse in -584 in Pteria can be considered to be total. This is consistent with the sunrise eclipse in -598 in Qufu.

Table 18. Condition of a total eclipse in Qufu for -557 May 31.

$^{\prime\prime} \mathrm{cy}^{-2}$	Min $\Delta T$ Max
-5.0	20815-23420-26326
-4.0	19765-22307-25165
-3.0	18725-21193-24000
-2.0	17687-20080-22830
-1.0	16658-18968-21650
0.0	15635-17853-20465
+1.0	14625-16740-19275
+2.0	13620-15628-18072
+3.0	12630-14515-16860
+4.0	11652-13403-15633

Table 19. Condition of an annular eclipse in Qufu for -549 January 5.\*

$^{\prime\prime} \mathrm{cy}^{-2}$	Min $\Delta T$ Max
-4.73	28962-(29005)
-4.7	28899-(28982)
-4.6	28686-(28880)
-4.5	28474-(28764)
-4.0	27417-(28138)
-3.0	25317-(26737)
-2.94	25192-26633
-2.0	23234-24650
-1.0	21164-22559
0.0	19101-20485
+1.0	17033- 18416
+2.0	14958- 16353
+3.0	12869- 14289
+4.0	10754-12219

\* Values in parentheses imply the cases that the eclipse ended at sunrise.

4.5. -557 May 31: Eclipse at Sunrise

Target	<b>55</b> 7 襄公	5 15	31 8	(19) 丁巳 [54]	
Auxiliary	-549 襄公 -548 襄公	1 23 6 24	5 2 19 7	(23) 癸酉 [10] (24) 甲子 [1]	朔 朔・既

As shown in the above list, there were two auxiliary eclipses within nine years. According to the records, eclipse (24) was total, whereas eclipse (23) was partial.

For each value of the corrections to the LTT, the range of  $\Delta T$  is given in tables 18, 19, and 20. The meaning of the tables is the same as before. The corresponding Sôma Diagram is shown in figure 6.

According to figure 6, the area of the intersection of the eclipse bands of the target eclipse and the total eclipse of -548 is rather large due to the form of the eclipse band of eclipse (24). As before, the zero correction to the LTT is included in the area. That the correction should be less than  $1''cy^{-2}$  has already been mentioned before. The non-totality of eclipse (23) gives additional restriction to the range of  $\Delta T$ 

Table 20. Condition of a total eclipse in Qufu for -548 June 19.

″cy <sup>-2</sup>	$\operatorname{Min}\Delta T \operatorname{Max}$		$\operatorname{Min}\Delta T \operatorname{Max}$
-5.0	16317-18105	&	29917-31505
-4.0	16104-18058	&	27966-29712
-3.0	15933-18156	&	25867-27875
-2.0	15831-18598	&	23422-25966
-1.5	15821-19278	&	21740-24970
-1.4	15823-19589	&	21229-24766
-1.35	15825-19856	&	20862-24664
-1.320	15826-20283	&	20374-24602
-1.319	15826		24600
-1.3	15827		24561
-1.0	15851		23933
0.0	16134		21634
+0.5	16540		20220
+0.8	17106		19048
+0.9	17603		18349
+0.91	17723		18209
+0.915	17823		18099
+0.916	17855		18064
+0.917	17905		18013



Fig. 6. Sôma Diagram for the target eclipse -557 May 31. The band with solid lines shows the sunrise eclipse of -557 May 31 in Qufu. The band with dashed lines is for the total eclipse on -548 June 19 in Qufu. The band with dot-long dash lines is for -549 January 5 in Qufu. The vertical interval is the candidate range of  $\Delta T$ . Here, SR stands for sunrise, T for total, and NA for non-annular.

by 1000 s. We, as before, adopt the range of  $\Delta T$  for the zero correction to the LTT as

$$16134 \,\mathrm{s} \le \Delta T < 19101 \,\mathrm{s}. \tag{11}$$

#### 4.6. $\Delta T$ Curve

Let us compare our ranges of  $\Delta T$  with those fitted by Stephenson (1997) [equation (3) of section 3]. Figure 7 shows that two results almost coincide. However, it is apparent that the change of  $\Delta T$  is not along a parabolic curve. As can be seen in the figure,  $\Delta T$  is closer to Stephenson's value if we adopt Thasos, whereas the value deviates from Stephenson's if we adopt Paros. The value of  $\Delta T$  in -598 might prefer Paros in -648.



**Fig. 7.** Ranges of  $\Delta T$  for the Chunqiu Period together with a curve fitted by Stephenson (1997).



**Fig. 8.** Range of  $\Delta T$  for a wider period determined with solar eclipses (dark and light, in e-version red and green, vertical bars): a preliminary result. The dotted curve is a fit by Stephenson (1997). The solid curve represents the change of  $\Delta T$  expected due to the angular-momentum conservation of the Earth–Moon system.

We have already pointed out on several occasions that there can be variations of a shorter period in the  $\Delta T$  curve. We show in figure 8 our preliminary result for the  $\Delta T$  variations of wider periods.

# 5. Discussions

# 5.1. Other Eclipses Which Have a Lack of Data

There remain five eclipses that lack information on the date or 'shuo'. These are eclipses (03), (10), (11), (13), and (15) in tables 1 and 20. Let us examine these eclipses one by one.

Eclipse (03) took place on -694 October 10. Watanabe (1958) considered that the cyclic day number was simply missed. According to our calculation, this eclipse would have occurred at sunset if  $\Delta T \simeq 17000$  s. This value is inconsistent with the ranges of  $\Delta T$  given by the eclipses on -719



Fig. 9. Sôma Diagram for the target eclipse -601 May 8. The band with solid lines shows the sunrise eclipse of -598 March 6 in Qufu. The band with dashed lines is for a total eclipse on -600 September 20 in Qufu. The band with dot-long dash lines is for -584 May 28 in Pteria. The vertical interval is the candidate range of  $\Delta T$ .

February 22 and on -675 April 15, though it was close to the sunset eclipse.

Eclipse (10) is judged to be a non-eclipse by Watanabe, because no corresponding Oppolzer's eclipse exists. Saito and Ozawa (1992) changed the date to -650 June 7, and suggested that this eclipse was at sunset. We can confirm this with the range of  $\Delta T$  being close to that for eclipse (09).

Eclipse (11) took place on -625 February 3. The record has 'shuo' in Gongyang-Zhuan. Our calculation says that this eclipse was neither at sunrise nor at sunset. According to Shinjo (1928, p.250), an old version of Gongyang-Zhuan has no 'shuo' for this eclipse. He adopts that the eclipse took place on the last day of the month, contrary to the descriptions of Zuoshi-Zhuan. These ideas have already been proposed by Chinese scholars, such as Jiangyong (江永) of Qing (清) dynasty (Shinjo 1928, p. 256).

Eclipse (11) took place in the daytime according to our calculation. In the record, 'shuo' is missing, whereas 'total' exists. We will make use of the latter fact in Conclusion.

Eclipse (15) does not have a corresponding Oppolzer eclipse. According to Shinjo (1928, p. 284), Wangtao (王韜) of the late Qing dynasty considered that the record should read Duke Xuan reign period, seventh year, sixth month, guimao [40], shuo instead of Duke Xuan reign period, seventeenth year, sixth month, guimao [40]:

Wangtao says that the record is a misplacement of the eclipse during the Duke Xuan reign period, seventh year, sixth month, guimao [40], shuo (-601 May 8). According to a calculation (Chalmers, 1865), there was an eclipse on that day in China, but I do not like to rescue the record by changing the year of the record. (Translation from Japanese to English is by K. Tanikawa.)

Shinjo was not sympathetic to this alteration. Let us examine whether this eclipse was either at sunrise or sunset. Figure 9 shows the result. The figure is almost the same as the figures in subsection 4.4 for eclipse (14). The figure strongly suggests that the eclipse was at sunrise. Therefore, Wangtao's

Table 21. Interpretation of the results.\*

Eclipse	Julian YMD	Duke**	Chinese YMD	朔	Interpretations			
					Watanabe 1958	Wangtao 王韜 (清)	S & O <sup>†</sup> 1992	Ours
01	-719.02.22	隠公	3 2 己巳[6]		sunrise			sunrise
03	-694.10.10	桓公	17 10	朔				before s.s. <sup>‡</sup>
04	-675.04.15	荘公	18 3			sunset		sunset
09	-647.04.06	僖公	12 3 庚午[7]		sunset			sunset
10	-644	僖公	15 5		non-eclipse		sunset§	(sunset)
11	-625.02.03	文公	1 2 癸亥 [60]					high noon
13	-600.09.20	宣公	8 7 甲子 [1]	・既	0.87			afternoon
14	-598.03.06	宣公	10 4 丙辰 [53]		sunrise			sunrise
15		宣公	17 6 癸卯 [40]		non-eclipse	宣公 7#		(sunrise)
19	-557.05.31	襄公	15 8 丁巳 [54]		sunrise			sunrise

\* denotes that the data is not written.

<sup>†</sup> Saito and Ozawa (1992).

<sup>‡</sup> Sunset.

§ -650.06.07.

<sup>II</sup> Guliang-Zhuan has 'shuo' (朔).

# -601.05.07.

\*\* 隠 Yīn; 桓 Huán; 荘 Zhuāng; 僖 Xǐ; 文 Wén; 宣 Xuān; 襄 Xiāng.

alteration is justifiable.

As a tentative conclusion, we may say that eclipses without the word 'shuo' are those at sunrise or sunset. There is two exceptions among nine: eclipses (11) and (13). Facts advantageous to our conclusion are that eclipse (13) accompanies word 'total' and eclipse (11) has 'shuo' according to Guliang-Zhuan.

### 5.2. Paros or Thasos

In subsection 4.3, we mentioned that Stephenson (1997, p. 341) raised several candidates for the eclipse versified by Archilochus, taking into account the preceding studies. Stephenson (1997, p. 341) listed other than -647 April 6, the eclipses on -690 July 28, -656 April 15, -645 September 8, and -636 August 29 which were seen as total in Paros or Thasos, and gave the ranges of  $\Delta T$ .

The additional four eclipses give quite different ranges of  $\Delta T$ . We thus conclude that these were not the eclipse that Archilochus saw. Newton (1970) listed eclipses on -688 January 11, -661 January 12, -660 January 27, and -656 April 15. Among them, the eclipse on -688 January 11 gives us a wide range of  $\Delta T$ . Stephenson (1997) omitted this eclipse because it was annular. The remaining eclipses do not give appropriate ranges of  $\Delta T$ . Thus, the eclipse on -647 April 6 is a unique candidate for the eclipse of Archilochus.

#### 5.3. Relations between Lack of Data and Sunrise, Sunset

Starting from the fact that there were almost no lack of data in the latter half of the Chunqiu records, Shinjo discussed the setting up of the Chinese calendar system during this period. According to our calculation, however, eclipses of the latter half of the Chunqiu are not eclipses at sunrise or sunset. This may mean that the lack of information is not related to the setting up of the calendar system.

Gongyang-Zhuan and Guliang-Zhuan recorded as eclipses at the last day or the second day of the month the eclipses which have been judged by us to take place at sunrise or sunset. This may be important to reconstruct the calendar system of Chunqiu Period. The problem is whether dusk or dawn belongs to today, tomorrow, or yesterday. In Vol. 7, Wuxing-zhi, Vol. 27 of Hanshu (Bangu 1962), there is a record: Duke Zhuang reign period, eighteenth year, third month, eclipse [eclipse (04) in table 2]. It is written (Bangu 1962 or Kotake 1977, p. 216)

The historian guesses that the conjunction was at night. The next day, the Sun rised eclipsed, and the eclipse finished after sunrise. This is said to be a night eclipse. (Tanslation to Enghlish is due to K. Tanikawa.)

The historian of Hanshu interpreted that the night eclipse in the Guliang-Zhuan was an eclipse at sunrise. The historian did not consider the possibility that the eclipse was at sunset. In fact, this was an eclipse at sunset.

The terms 'eclipse at sunrise' and 'eclipse at sunset' appeared in Xin Wudaishi (新五代史). However, it is clear that the notion was already in the period of the Han Dynasty. Even Guliang-Zhuan might have called an 'eclipse at sunrise' or 'eclipse at sunset' a night eclipse. In that case, modern usage of 'night eclipse', meaning an eclipse that takes place on the night side of Earth may be different from the historical usage.

## 6. Conclusions

In the present paper, we have taken up eclipse records without 'shuo' or the cyclic day number or both, calculated the range of  $\Delta T$  given by these eclipses, and checked whether Watanabe's supposition that these are the eclipses at sunrise or sunset, and obtained narrower ranges of  $\Delta T$  compared with preceding studies.

We have several results:

(1) The following eclipses were eclipses at sunrise or sunset in Qufu, the capital of Lu: No. 3]

Sunrise	-719	February	22,
	-598	March	6,
	-557	May	31,
Sunset	-675	April	15,
	-647	April	6.

(2) We obtained narrower ranges of  $\Delta T$  utilizing contemporary eclipses. We give here the ranges of  $\Delta T$ , assuming the present value of the coefficient of lunar tidal term:

 $\begin{array}{rll} -719 & : & 20153 \ {\rm s} < \Delta T < 21094 \ {\rm s}, \\ -675 & : & 18526 \ {\rm s} < \Delta T < 20686 \ {\rm s}, \\ -647 & : & 19409 \ {\rm s} < \Delta T < 20402 \ {\rm s} & \mbox{for Paros}, \\ & 18353 \ {\rm s} < \Delta T < 19235 \ {\rm s} & \mbox{for Thasos}, \\ -598 & : & 19172 \ {\rm s} < \Delta T < 20910 \ {\rm s}, \\ -557 & : & 16134 \ {\rm s} < \Delta T < 19101 \ {\rm s}. \end{array}$ 

There are two candidate ranges of  $\Delta T$  for the year -647. The variations of  $\Delta T$  are smaller if we choose Paros.

(3) In the present paper, we use auxiliary eclipses to make narrower the candidate ranges of  $\Delta T$ . There is no doubt that total eclipses are most useful. However, data such as 'non-total', 'at sunrise', and 'at sunset' turn out to be useful in some cases.

(4) We again confirmed that the correction to the lunar tidal

term is less than  $1''cy^{-2}$ , since 720 BC, as shown in figure 5.

(5) The lack of data in the records of eclipses turns out to have important meaning. In particular, the lack of 'shuo' corresponds to eclipses at sunrise or sunset. The lack of a cyclic day number is not important.

As a result, the change from the 'Duke Xuan (宣公) reign period, seventeenth year, sixth month, guimao[40]' to 'Duke Xuan (宣公) reign period, seventh year, sixth month, guimao[40]' seems plausible.

Then, we finally summarise as follows:

(i) Five eclipses out of ten were at sunrise or sunset.

(ii) Two erroneous records may correspond to the real observations if the dates are changed.

(iii) Eclipses without 'shuo' (朔) may have been at sunrise or sunset.

Thanks are due to Dr. Han Yanben (National Astronomical Observatory of China) for discussions. The present work has been supported by a Grant-in-aid for Scientific Research for Exploratory Research No. 19654031 (T. Yamamoto, NAOJ), the Twenty-First century COE program (K. Ishigaki of Human Science Institute of Kyoto University), and also by Grant-in-aid for Basic Research (C) No. 20540241 (M. Sôma, NAOJ).

# References

- Bangu, Yanshigu (note) 1962, Hanshu (Beijing: Chinese Press) (in Chinese; originally written in AD 1 century)
- Chalmers, Rev. J., 1865, Astronomy of the Ancient Chinese, Appendix to the Chinese Classics by James Legge, vol. III, Part I
- Chapront, J., Chapront-Touzé, M., & Francou, G. 2002, A&A, 387, 700
- Committee for ordering thirteen Jing: 2000, Commentary for Thirteen Jing Zuoshi-Zhuan, Gongyang-Zhuan, Guliang-Zhuan), (Beijing: Beijing Univ. Press) (in Chinese).

Fotheringham, J. K. 1920, MNRAS, 81, 104

- Kawabata, K., Tanikawa, K., & Sôma, M. 2004, PASJ, 56, 869
- Kotake, T.: 1977, Hanshu (translation), Vol. 3 (Tokyo: Chikumashobo) (in Japanese)
- Newton, R. R. 1970, Ancient Astronomical Observations and the Accelerations of the Earth and Moon (Baltimore: Johns Hopkins Press)
- Nōda, C., & Yabuuti, K. 1947, Study of Lizhi of Hanshu (漢書律暦志 の研究) (Kyoto: Rinsen-Shoten) (in Japanese)
- Özel, M. E., & Kacar, Y. 2007, in ASP Conf. Ser. 370, Solar and Stellar Physics Through Eclipses (San Francisco: ASP), 319
- Oppolzer, Th. R. von 1887, Canon of Eclipses, translated by O. Gingerich (1962, New York: Dover Publications)
- Saito, K., & Ozawa, K. 1992, Checking Chinese Ancient Astronomical Records (Tokyo: Yuzankaku) (in Japanese)
- Schoch, K. 1927, Planeten-Tafeln für Jedermann zur Berechnung der geozentrischen Örter der grossen Planeten (und des Mondes) für den Zeitraum von 3400 v. Chr. bis 2600 n. Chr. ohne Anwendung der Logarithmen und trigonometrischen Funktionen bis auf ein Zehntel Grad unter besonderer Berucksichtigung der babylonischen Astronomie (Berlin-Pankow: Linser-Verlag g.m.b.h.)

- Shinjo, S. 1928, Study of the History of Astronomy in East Asia (Kyoto: Kobundo) (in Japanese)
- Simaqian 1959, Shiji (Peijin: China Printing Company) (in Chinese; Originally written in  $\sim$  90 BC)
- Sôma, M., Tanikawa, K., & Kawabata, K. 2004, in Journées 2003 Systèmes de Réference Spatio-Temporels, ed. A. Finkelstein & N. Capitaine (St. Petersburg: Inst. Applied Astron.), 122
- Standish, E. M., Jr. 1998, JPL Planetary and Lunar Ephemerides, DE405/LE405, JPL IOM 312.F-98-048 (Pasadena: JPL)
- Stephenson, F. R. 1997, Historical Eclipses and Earth's Rotation (Cambridge: Cambridge University Press)
- Tanikawa, K., & Sôma, M. 2002, Astronomical Herald, 95, 27 (in Japanese)
- Tanikawa, K., & Sôma, M. 2004a, PASJ, 56, 215

Tanikawa, K., & Sôma, M. 2004b, PASJ, 56, 879

- Watanabe, T. 1958, in Modern Astronomy: papers for Tosima ARAKI's 60th anniversary, ed. J. Hitotuyanagi (Tokyo: Kouseisha) (in Japanese)
- Watanabe, T. 1979, Canon of Eclipses in Japan, Korea, and China (Tokyo: Yuzankaku) (in Japanese)