Subsystems in a stable planetary system I. A classification

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Abstract

Our planetary system is dynamically stable for the lifetime of the solar system according to the long-term numerical integrations of planetary orbits (Ito & Tanikawa, 2002). we discuss various forms of subsystems of a stable planetary system which may contribute to maintain the system stability. It is well known that resonances play an important role in such a long time scale. We stress that contrary to the restricted problem such as the stability of asteroids and comets, multi-planet subsystems may have variety of mechanisms for keeping stability.

§1. Introduction

The solar system is dynamically stable for at least five billion years in the past and four billion years in the future (Ito & Tanikawa, 2002; hereater IT2002). The orbital elements of nine planets are almost constant within a small range of variability for 9 billion years. Only the eccentricity and/or inclination of Mercury might have a slight touch of secular change. We expect that if the solar system of nine planets would become unstable, then Mercury would be the first that makes a close encounter with Venus or Earth and possibly dives into the Sun or is scattered away from the solar system.

For the moment, we are safe to say that the solar system is stable. We are apt to ask: Why is the solar system stable? This is a dangerous question because the question requires a 'true' reason of stability. We instead ask: How is the solar system stable? This is a feasible problem. We can ask more specifically. 'What kind of stabilization mechanims are working? 'What kind of realization and what kind of characteristics do these interactions have?' 'Have these interactions continued to exist from the formation era of the system? 'Are these interactions common to extrasolar systems?

In the present paper and the one follows this, we address these questions and answer some of them or at least show the direction of study to be taken. Five-billion-year integrations of our solar system provided us information not only on the stability of the system but also on mechanisms acting on various subgroups of the system. These are the clues for unveiling the mechanisms of keeping stability. The purpose of the present paper is to classify the groupings of planets, to exmanine the stabilizing mechanisms of our solar system and justify the validity of the classification. As is well-known one of the stabilizing mechanisms is the grouping of individual bodies. The whole system becomes stable upon making subgroups if otherwise unstable. One famous and typical example is the hierarchical structure of gravitating systems. We will see in this report other types of grouping in the solar system.

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We start with classifying and counting subsystems. We do not claim that we count up all subsystems. A number of subsystems may have been overlooked.

(1) (mean motion) Resonant pair
   1.1. Binary planets (Sister planets)
   1.2. Other 1:1 resonant pair
   1.3. Other (mean motion) resonant pairs
   1.4. Resonant triples

(2) Close neighbors (Cousin planets)

(3) Planetary groups

(4) Independent planetary subsystems

In the following section, we describe individual subsystems and examine their stablity where it is possible. We specifically pay attention to the last three subgroups of the above list.

§2. More about subsystems

In this section, we consider subsystems in the Solar System together with possible subsystems which can be existent in other solar systems. If the latter subsystems do not exist in other solar systems, then this gives strong constraints on the formation process or mechanism of planetary systems. In some cases, we alter the existent subsystems and compare the stability of the whole solar system with and without this alteration. In other cases, we add perturbations from outside the Solar System. This stability analysis is only possible because we are sitting outside the solar system and in front of computers.

2.1. Resonant pair planets

If plural planets are in mean motion resonance(s), then we call these *resonant multiples*. If the number of planets involved is two, these are called a resonant pair. As is well-known, there are an infinite number of resonances because there are an infinity of rational numbers. It is also known that higher resonances have smaller resonance regions. Thus, the number of resonant multiples are not so many though, up to the observational limit, every pair or triple, etc is in resonance.

2.1.1. Binary planets

A pair of planets in 1:1 mean motion resonance is called a (1:1 mean motion) resonant pair or simply *binary planets*. The member bodies have strongest interaction and connection. Examples are the Earth–Moon system and Pluto–Charon system. There may be an objection that these are not the planet–planet pairs. We do not know the reason why in our solar system there are no planet–planet binaries. We do not know why Jupiter and Saturn do not have satellites of mass comparable with that of the Earth or Venus. We may find these combinations in extrasolar planetary systems. The formation process of planetary systems somehow prohibited these combinations in our solar system.

Essentially, there are three kinds of binary planets.

1) Pluto–Charon system

In this sytem, orbits of both planets are sometimes convex and other times concave to the Sun. The semi-major axis is 17 $R_{\rm pluto}$, the eccentricity is equal to zero, and the period is 6.387 days. The masses are $M_{\rm charon} = 0.08 M_{\rm pluto}$. The acceleration of Charon due to the Sun is

$$a_{\rm charon} = GM_\odot / r^2_{(\odot - {\rm charon})} = 3.79 \times 10^{-6} {\rm m/s^2}, \tag{1}$$

whereas the acceleration due to Pluto is

$$a_{\rm charon} = \quad GM_{\rm pluto} / r^2_{({\rm pluto} - {\rm charon})} = 2.63 \times 10^{-3} {\rm m/s^2}. \tag{2}$$

The acceleration from Pluto is far larger than that from the sun. So, the orbit of Charon is always concave to Pluto.

2) Earth–Moon system

The orbital semi-major axis is $60 R_\oplus$, the eccentricity is 0.0549, and the orbital period is 27.3 days. The masses satisfy $M_{\rm moon} = 0.020 M_\oplus$. The acceleration of the Moon due to the Sun is

$$a_{\rm moon} = GM_\odot / r^2_{(\odot - {\rm moon})} = 5.93 \times 10^{-3} {\rm m/s^2}, \tag{3}$$

whereas the acceleration due to the Earth is

$$a_{\rm moon} = GM_\oplus / r^2_{(\oplus - {\rm moon})} = 2.70 \times 10^{-3} {\rm m/s^2}. \tag{4}$$

The orbit of the Moon is always concave to the sun. The interaction between components is weaker than the Pluto–Charon system.

3) Retrograde binary planets

Mikkola & Innanen (1997) coined the term 'quasi satellite' for the object orbiting the mother planet with unusually large semimajor axis. Wiegert et al. (2000) surveyed the stability of these orbits around outer four giant planets and found that low inclination orbits survive for the age of the solar system. This orbit is fairly large compared with a prograde satellite orbit. So a tentative system will have a relative distance greater than the Earth–Moon system. We classify these objects as a *retrograde binary.*

2.1.2. Other 1:1 mean motion resonant pair

1) Tadpol-type pair

Typical examples are Trojan asteroids. If, instead of an asteroid, there is an object of planetary mass, the system can be called a planetary pair. There can be a pair that one of the members explores more wide area. But the position is always in front of the main member or in its back side. It seems that the stability of this pair is not well analysed.

2) Horseshoe pair

Theoretically, there can be horseshoe type pairs. However, there is a problem of long-term stability. We do not so far find any pair of comparable masses in our solar system. Small objects are found in this position. It is interesting to see the stability including perturbative forces from other planets.

2.1.3. Neptune–Pluto system

This is a very special system maintaining its long-term stability using plural resonant mechanisms (Kinoshita & Nakai, 1995, 1996; IT2002). In the shortest time scale, this is a 3:2 mean motion pair. It roughly resumes the original configuration every five hundred

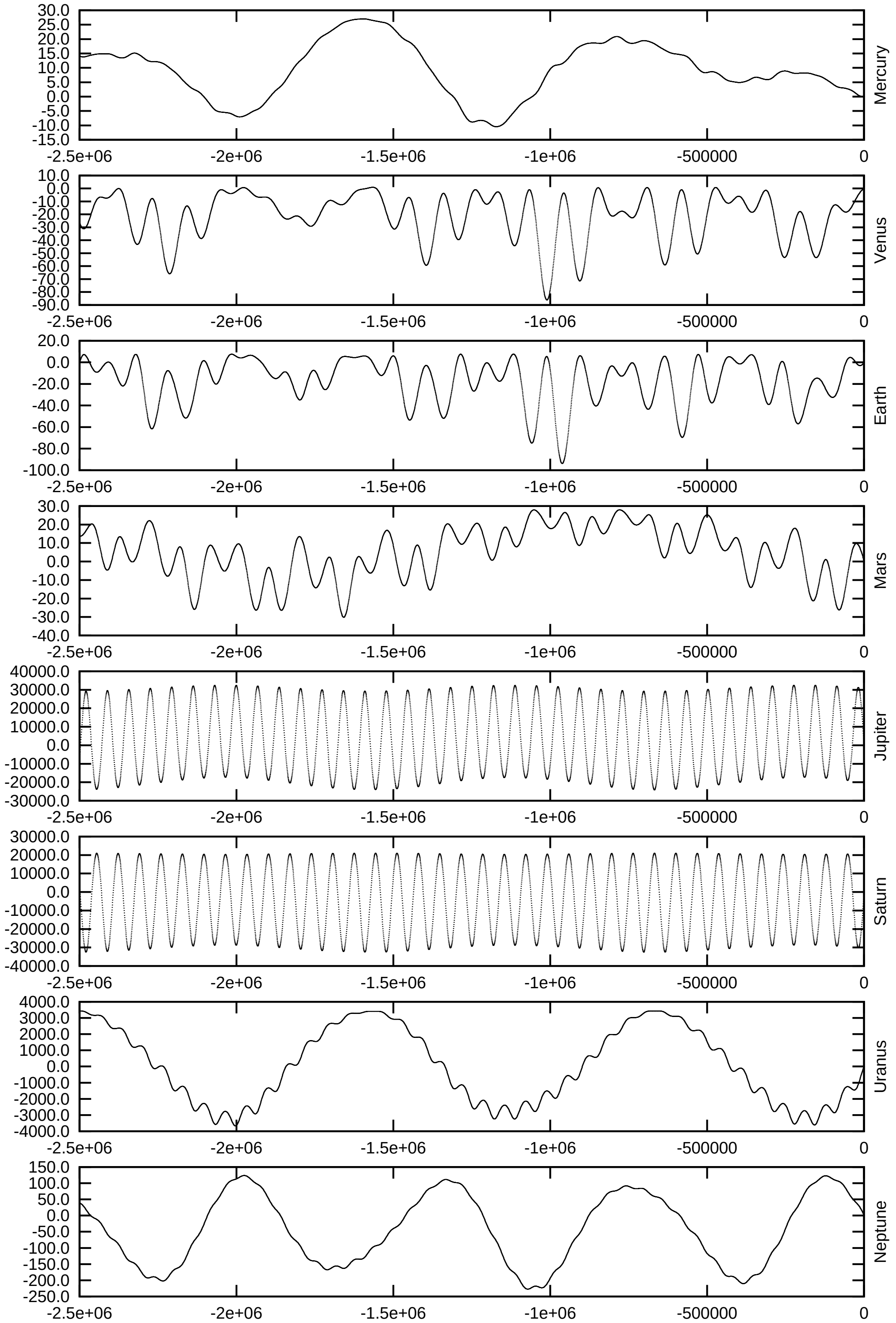

Figure 1: Variation of the angular momentum of eight planets for 2.5 million years from Brower & van Woerkom's (1950) theory of secular perturbation. From the top, Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune. The ordinate represents the total angular momentum. The unit is $10^{-12} M_{\odot} \mathrm{AU}^2 \mathrm{day}^{-1}$.

years. In a more microscopic view, the critical argument $\theta_1 = 3\lambda_P - 2\lambda_N - \varpi_P$ librates around 180°. The period is 2×10^4 year. Thus, the system returns to the original configuration every 2×10^4 years.

Pluto's argument of perihelion $\omega_P = \theta_2 = \varpi_P - \Omega_P$ librates around 180°. The period is 3.8×10^6 years. The system ressumes its original configuration, which takes into account the relative position of the ascending node, every 3.8×10^6 years. This is a very long periodicity. The longitude of Pluto's node referred to the longitude of Neptune's node, $\theta_3 = \Omega_P - \Omega_N$, circulates. The period of circulation is equal to the period of θ_2 libration. When longitudes of acending nodes of Neptune and Pluto coincide ($\theta_3 = 0$), Pluto's inclination becomes maximum, its eccentricity minimum, and argument of perihelion 90 °. When $\theta_3 = 90°$, Pluto's inclination becomes minimum, its ecentricity maximum, and argument of perihelion 90° again. This was confirmed by Milani et al. (1989).

There is a longer periodicity. An argument $\theta_4 = \varpi_P - \varpi_N + 3(\Omega_P - \Omega_N)$ librates around 180°. The period is 5.7×10^8 years. IT2002 showed that θ_4 varies between librations and circulation in $O(10^{10})$-year timescale. This is one of the longest timescales ever known.

2.2 Close neighbors

In our solar system, the Earth–Venus system, if these two can be called a system at all, occupies a special position as a subsystem. The Earth and Venus are planets of similar character. Nonetheless, these two planets have not explicitly been regarded as a dynamical pair. According to the long-term integrations of our planetary system (IT2002), these planets have interesting behaviors. These are not in any low order mean motion resonance. In the secular perturbation theory (Brouwer & van Woerkom, 1950), their orbital angular momenta have negative correlation in a short time-scale ($\sim$ million years), i.e., if one planet obtains the angular momentum, the other loses the angular momentum, and *vice versa* (the second and third panels of Fig. 1). This is also observed in our numerical integrations. In a longer time-scale (billion years), the orbital angular momenta seem to have a positive correlation (Laskar, 1994; IT2002), i.e., if, for example, one planet obtains the angular momentum, the other also obtains the angular momentum. Figure 2 (the second and third panels) shows one of the numerical results taken from IT2002. This means that in a longer time-scale, these two planets behave synchronously against the perturbation from outside. They are repulsive each other in a shorter timse scale but move together against outer forces in a longer time scale. These may be alternatively called cousin planets.

So far, our statement is of a qualitative nature. In order to quantitatively confirm the above statement and to see the difference from other conceivable pairs like Jupiter–Saturn and Uranus–Neptune, we take correlations of orbital elements using the results of long-term integrations of IT2002.

A simplest method of taking correlation would be, after removing the trend from a time series, that is, after subtracting an average value, to asign + (resp. $-$) to the data at $t + \Delta t$ if the value of a parameter at $t + \Delta t$ is larger than or equal to (resp. less than) that at t. Pick up two time series and take the data according to the elapse of time. If at t both data have +, then we add 1. If at t one has + and the other has $-$, then we add -1. If at t both data have $-$, then we add 1. We sum up 1 and -1 made from two time series and divide by the number of data. This quantity ρ is the easiest index for the strength of correlation. If ρ is positive and large, then a positive correlation. If ρ is negative and large in absolute value, then a negative correlation. If the absoluet value of

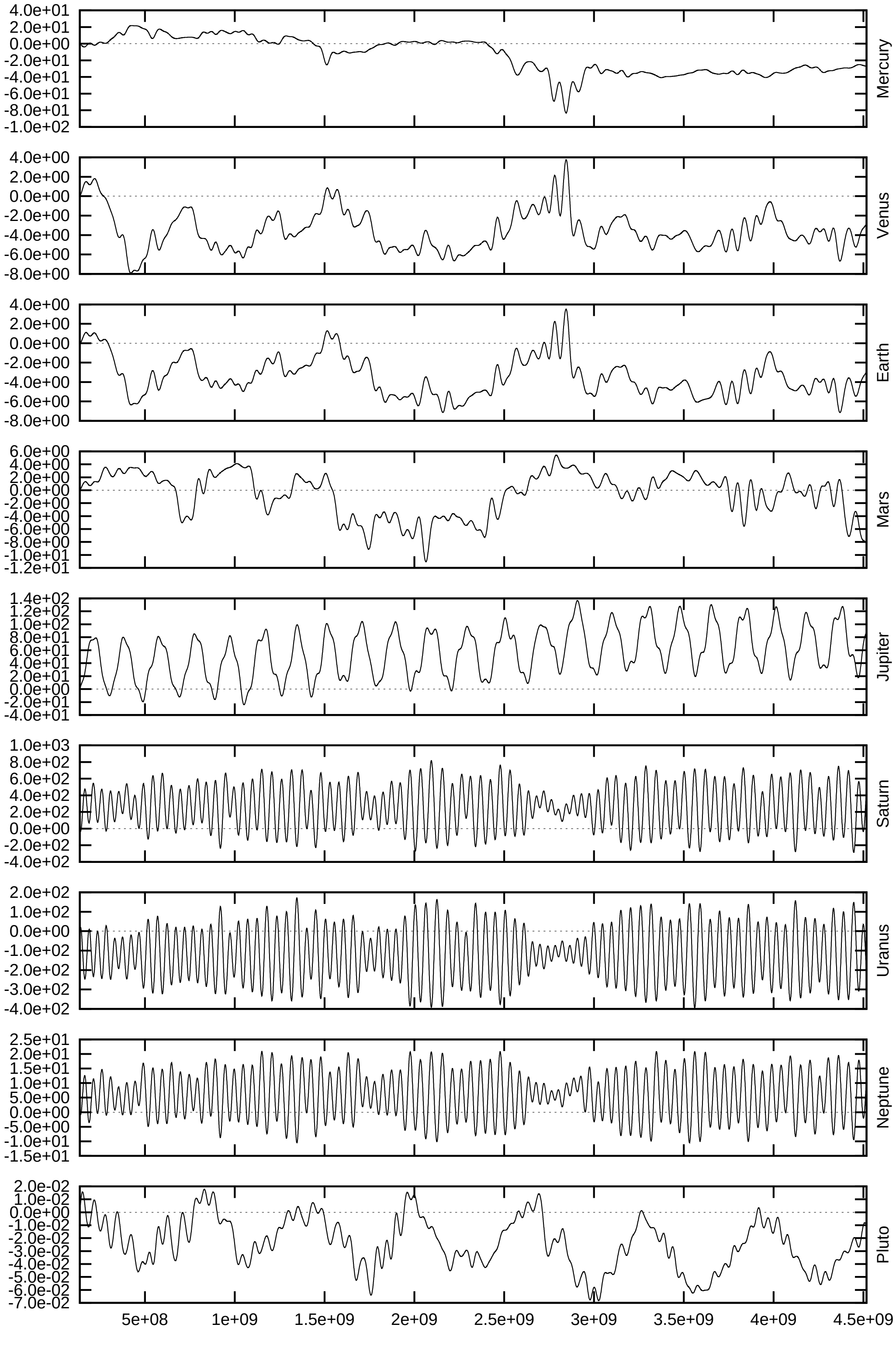

Figure 2: Variation of the angular momentum of nine planets for 4.5 billion years showing qualitatively the correlation of orbital elements (reproduced from IT2002). From the top, Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto. The ordinate represnts the total angular momentum.

ρ is small, then the correlation is weak.

The second easiest method which we have adopted is as follows. As before, we subtract the trend and obtain and average for each time series. From each data x_i, we get a normalized value $X_i = (x_i - \overline{x})/|\overline{x}|$. Then correlation ρ is defined by

$$\rho = \frac{1}{N}\sum_{i=1}^{N} X_i Y_i$$

where X_i and Y_i are normalized data from two time series.

Table I. Correlation of the orbital energy and angular momentum in Earth–Venus, Jupiter-Saturn and Uranus–Neptune pairs.

N+1	Energy	Ang. Mom.	z-component
Venus–Earth	−9.5563e-01	8.9389e-01	9.0139e-01
Jupiter–Saturn	−4.7369e-01	−1.1749e-02	−9.9988e-03
Uranus–Neptune	9.6988e-02	−9.4638e-01	−9.4363e-01
N+2			
Venus–Earth	−9.1239e-01	7.8515e-01	8.3140e-01
Jupiter–Saturn	−2.0947e-01	2.2497e-03	5.9993e-03
Uranus–Neptune	−3.7995e-02	−9.0689e-01	−9.0414e-01
N+3			
Venus–Earth	−7.5304e-01	9.9014e-01	9.9272e-01
Jupiter–Saturn	−5.7535e-01	5.5392e-01	5.6135e-01
Uranus–Neptune	−2.3225e-01	−1.5655e-01	−1.4569e-01
N-1			
Venus–Earth	−9.0385e-01	9.1585e-01	8.6786e-01
Jupiter–Saturn	−3.1795e-01	−6.6656e-03	−4.6659e-03
Uranus–Neptune	1.2198e-01	−9.0218e-01	−8.9585e-01
N-2			
Venus–Earth	−8.8844e-01	8.5159e-01	8.8302e-01
Jupiter–Saturn	−5.0593e-01	−4.3422e-02	−4.1708e-02
Uranus–Neptune	5.6563e-02	−9.6215e-01	−9.5958e-01
N-3			
Venus–Earth	−8.2674e-01	9.5758e-01	8.5702e-01
Jupiter–Saturn	−8.1674e-01	3.8252e-01	3.8395e-01
Uranus–Neptune	−3.7880e-01	−1.8083e-01	−1.9054e-01
short			
Venus–Earth	−2.0819e-01	−1.7899e-02	−1.5059e-01
Jupiter–Saturn	−6.6097e-02	−4.5458e-01	−4.5498e-01
Uranus–Neptune	−3.3998e-03	−4.2998e-03	−3.3998e-03

We have several time series of long-term integrations of our planetary system. In addition, there are several dynamical and physical quantities for which correlations can be considered. This time, we consider the correlation of the energy, angular momentum and the z-component of the angular momentum. The results are shown in Table I. Let us first explain the meaning of $N \pm i$ $(i = 1, 2, 3)$ and 'Short' in the table. $N + i$ $(i = 1, 2, 3)$ is for the future and $N - i$ $(i = 1, 2, 3)$ is for the the past. Except for $N - 3$ for which the Moon is neglected, the Earth–Moon barycenter has been adopted in the integrations (see IT2002, Table I). 'Short' is obtained from the initial 10 million years of $N + 2$. It is to be noted that data series $N \pm i$ have passed a low-pass filter, whereas 'short' data have not.

The first feature we point out is that the correlation is strongest in the Earth–Venus system. This confirms our first impression looking at Figs. 1 and 2 that the Earth–Venus may form a pair. The second characteristics seen in the table is that the Earth–Venus system has reverse correlations in long and short time scales. The contrast between long and short time scales may be more conspicuous if we take a shorter time series for the shorter data. In 10 million times scale, the correlation seems in a transition, i.e., the correlation is rather weak in paticular for the angular momentum. The correlation of the energy is negative in both cases, that is, if the orbital energy of the Earth increases, then the energy of Venus decreases in any time scale, and *vice versa*. The correlation of the angular momentum has a different character. In the shorter time scale, it has the same character as that of the energy. However, in a longer time scale, the angular momentum correlation is positive. It means that both planets gain and lose their angular momenta synchronously. Both planets behave as a unit against external perturbations. The third characteristics seen in Table I is rather unexpected. The Uranus–Neptune pair has strong correlations for the angular momentum. Correlations are negative in the long time scale, whereas the correlations are very small in the short time scale. The energy correlation is weak in both time scales. Thus, Uranus and Neptune move independently in a short time scale, whereas in the long time scale, their motions are related. When Uranus gain the angular momentum, then Neptune loses and vice versa. As a pair, the connection is weaker in the Uranus–Neptune pair than in the Earth–Venus pair.

It is interesting that Mercury and Mars seem to have a positive correlation for the angular momentum in the long-term. However this is a consequence of the fact that each planet behaves with negative correlations to the Earth–Venus system. We cannot say that Mars and Mercury consitute a system.

2.3 Planetary groups

So far, most of the efforts on the stability problem of our solar system have been concentrated on the motion of small bodies like satellites, asteroids, and rings. Recently, Kuiper-belt objects are added to the list. In other word, most studies treated the stability problem as a restricted problem in the sense that bodies for which the stability of motion is examined are assumed to be massless, and give no responding force to perturbers (see the review papers by Wisdom, 1987; Lissauer, 1999; Lecar et al., 2001).

There have occasionally been numerical studies on the stability of the whole solar system. 350 years of Eckert (1951) was the first. Then, in the chronological order, 120 thousand years of Cohen & Hubbard (1965), 1 million years of Cohen et al. (1973), 5 million years of Kinoshita & Nakai (1984), 100 Myr of Nobili et al. (1989), and so on. The longest record at present (2002) is ± 5 billion years integration of 9 plantes by IT2002. IT2002 have shown that our solar system is stable 5 billion years in the past and 4 billion years in the future.

There have been a small number of systematic studies of the stability of planets as a group in our solar system. One of the first trials has been done by Gladman (1993) and later by Chambers et al. (1996). Ito & Tanikawa (1999 or IT1999) noticed the importance of these ideas to apply to the actual solar system. IT1999 were the first to examine the possibility that Jupiter form prior to the formation of terrestrial planets. The idea was that the existence of Jovian planets, especially the existence of Jupiter may affect the formation process of terrestrial planet group and even now affect the stability of this group.

Ito & Tanikawa (2001 or IT2001) also noticed that the so-called outer planetary group or Jovian planets form a subsystem which are not affected from other groups. Indeed, the motion of the Jovian planetary group may be not altered if there is no terrestrial planet group. On the other hand, the terrestrial planet group receives the secular perturbation of Jupiter ($\sim$ 300 thousand year periodicity of the motion of Jupiter's perihelion) and make it uninfluential by sharing the effect of perturbation (IT1999).

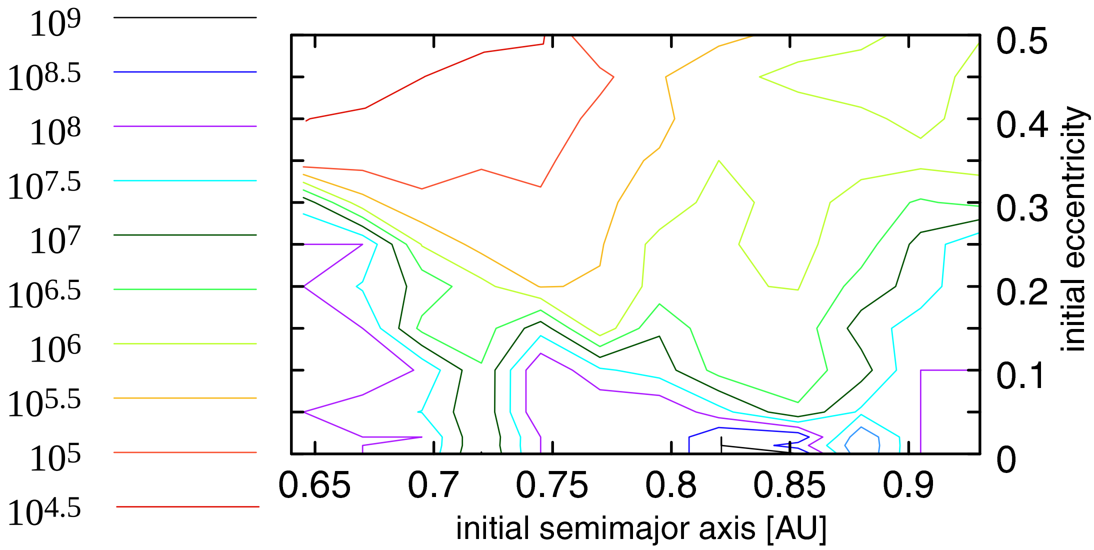

Figure 3: The contours of the same instability time. The results of the numerical integrations with the Earth and Venus merged. The ordinate and abscissa are the semimajor axis and eccentricity of a merger. The inclination is related to e by $2I = e$.

In order to check the meaning and role of this kind of subsystem, one somhow alters the terrestrial system and see what happens. Innanen et al. (1998) examined the dynamical stability of the inner solar system by removing each of terrestrial planets in turn. They found that a drastic phenomenon takes place when the Earth–Moon system is removed. The eccentricity of Venus oscillates with large amplitude. If Mercury is included, then the large oscillation of Mercury's eccentricity takes the role of Venus. Venus is nearly at the position of secular resonance from Jupiter. Innanen et al. (1998) interpreted that the Earth–Moon system suppress the secular resonance. IT1999 gave a slightly different interpretation to this phenomenon. Terrestrial planets share the effect of the secular perturbation from Jupiter. Eccentricities of all the inner planets increase. The mechanism is simple. Due to the perturbation of Jupiter, the eccentricity of Venus tends to increase. The eccentricity of the Earth increases according to the long time scale behavior of correlations in Table I. In other words, the Earth shares the increase of the eccentricity of Venus. The enhancement of the eccentricity is weakened by sharing.

Yet another method of checking stability is to merge two of the planets. To conserve the total mass, total energy and angular momentum is a too restrictive condition, so we consider two cases in which the total mass is conserved and either energy or angular momentum is conserved. We survey the stability of the altered system around the specified position. The position of the merger will be at $a \simeq 0.855$AU if the merger has the mass of the Earth and Venus and the orbital energy of the Earth and Venus neglecting the eccentricity. Indeed, we put the merger at various places between $a = 0.645$AU and $a = 0.930$AU with e ranging from 0 to 0.5. The number of different sets of initial parameters are more than 150. To reduce the number of integrations, we assumed $e = 2I$.

The result of stability analyses is depicted in Fig. 3. Here the abscissa is the semi-major axis and the ordinate is the eccentricity or inclination of the merger. Instability is meant if the orbit of some planet crosses the orbit of another planet. In our case, always Mercury does this. In the figure, the contours of equi-instability time are drawn. The lines with purple color corresponds to 100 my, i.e., the system is stable until 100 my. We need longer integration times to see the final fate of the system. We see two unstable intervals of semi-major axis centered at around $a = 0.72$ and $a = 0.88$ with $e = 0$. These are the positions of secular resonance from outer planets. Figure 4 shows this. In Fig. 4(top), the secular resonant motion of Mercury with the merger at $a = 0.73$ is shown. The position of the merger is close to the resonance with Jupiter. Here instead of the increase of the mergers's eccentricity, Mercury's eccentricity increases. The number of points is small because Mercury soon becomes unstable. Figure 4(middle) and (bottom) show the secular resonant motion of Mercury with Jupiter and Uranus when the merger is at $a = 0.88$. We stopped the integration at $t = 10^8$ years in most cases. Between $a = 0.8$ and $a = 0.85$ and for small e, integrations are done until $t = 10^9$ years. The stable area in Fig.3 becomes smaller if we extend the integration time.

We can conclude that if the Earth and Venus merged in the early solar system, Mercury would have in high probability escaped away. The individual roles of the Earth and Venus as they occupy the present positions in the terrestrial zone contribute the maintenance of the stability of our planetary system. As pointed in IT1999, terretrial planets keep their stability by sharing and weakening the secular perturbation from Jupiter. The Earth–Venus system plays the distributor of the effects of perturbation.

2.4 Independent planetary subsystems

Innanen et al. (1997) carried out interesting numerical simulations. They put a companion star of $0.02 \sim 0.5M_\odot$ at 400AU from the Sun with a circular but inclined orbit from the invariant plane of the solar system with various inclinations. They wanted to see the stability of a planetary system in a binary. They took as a representative case the Jovian planetary system (Jupiter, Saturn, Uranus, and Neptune) and tried to see its behavior under the perturbation of a companion star. It was initially expected that Kozai mechanism will independently drive the inclination variations of planets and hence planets would soon experience close encounters. For a suitable parameter set, however, the Jovian planetary group shows a stability. The Kozai mechanism of individual planets is suppressed and the motions of the ascending node of the planets synchronize. They called this synchronous state a *dynamical rigidity*.

In our context, two experiments of Innanen et al. (1997, 1998) can be used as a tool for checking the strength of connection among planets against internal and external perturbations. The first experiment (Innanen et al., 1997) is a check for the unity of the system against external perturbation. The second experiment (Innanen et al., 1998) can be arranged to test the internal rigidity of the system. We will explain these in the corresponding subsections. In both cases, if there is no rigidity in two subsystems and these two are stable, then we can regard that these two systems are independent.

We give here a preliminary result in a sense that the number of experiments are not enough. We divide our planetary systems into three groups: [1] Mercury, Venus, Earth, Mars, and Jupter; [2] Saturn; [3] Unanus, Neptune, and Pluto. We will carry out numerical integrations of orbits with and without the second group (Saturn).

2.4.1 Rigidity against internal perturbation

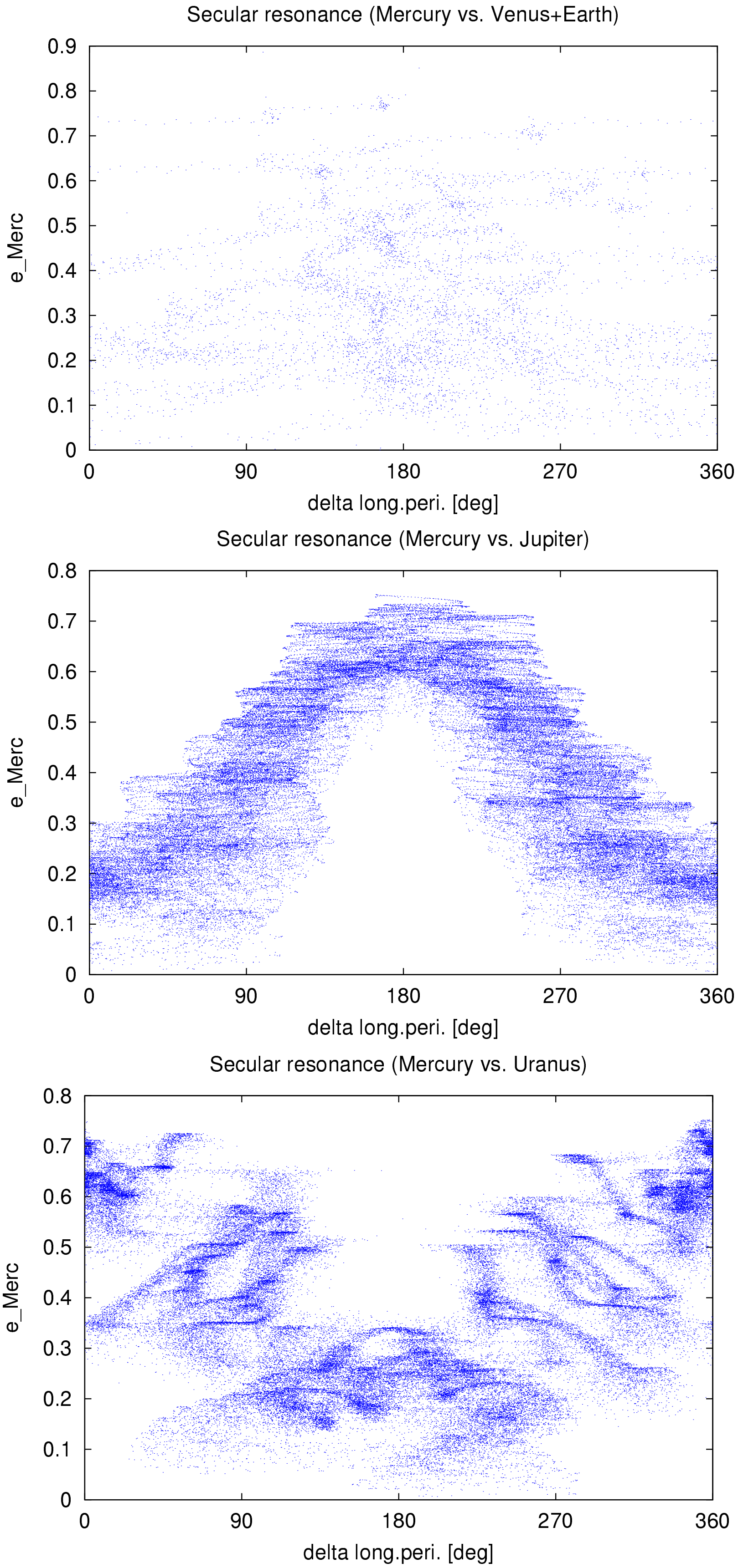

Figure 4: Secular resonance of Mercury (top) with the meregr at $a \sim 0.73$AU and (middle) with Jupiter when the merger is at $a \sim 0.88$AU, (bottom) with Uranus when the merger is at $a \sim 0.88$AU.

We carry out two integrations over 100 million years. In the first calculation, Group [1] and Groups [2] + [3] are inclined by 20°. In the second calculation, Groups [1] and [3] are inclined 20° each other, whereas Group [2] is not included (see Fig. 5(a)).

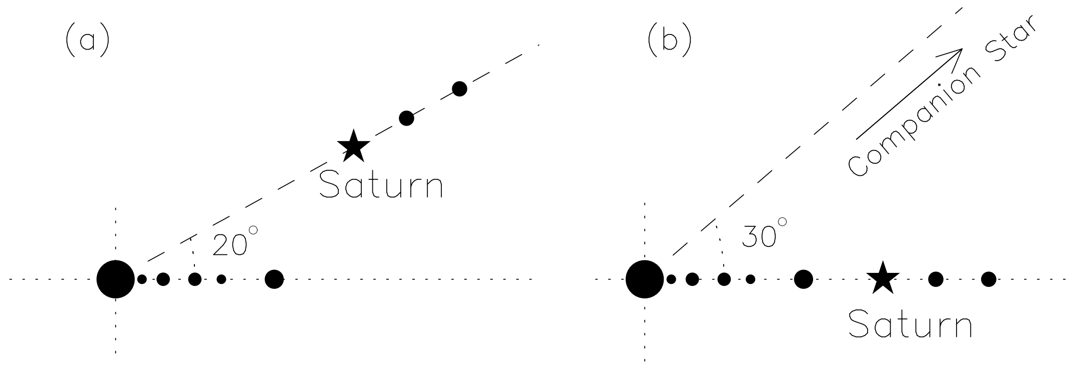

Figure 5: Initial configurations of planets for checking rigidity, (a) rigidity against internal perturbation, (b) rigidity against external perturbation.

Possible results are : (i) The sytem in the first calculation is unstable, whereas the system in the second calculation is stable; (ii) Both systems are unstable; (iii) Both systems are stable. Result (i) implies that Saturn plays the role of pivot to connect outer and inner planetary systems and the planetary system compises two independent subsystems without Saturn. Result (ii) implies that the connection among planets are strong enough irrespective of the existence or non-existence of Saturn and the planetary system is unstable with other configurations. Result (iii) implies the parameter used in this investigation is not suitable to check the independence of subsystems.

Numerical results are shown in Fig.6. At around $t = 4.4 \times 10^6$ years, the system with Saturn becomes unstable in the sense that Mercury's eccentricity becomes as large as 0.8 and more (the upper panel of Fig. 6) and its inclination approaches 60° (the lower panel of Fig. 6). In the system without Saturn, the eccentricity of Mercury oscillates stably between 0.18 and 0.23, and the inclination has similar variations (Fig. 7). Result (i) is attained. Saturn plays the role of a pivot between outer and inner planets. It is to be noted that the terrestrial planets join the rigidity of the whole planetary system. This again confirms the rigidity of terrestrial planets against the perturbation of Jupiter.

2.4.2 Rigidity against external perturbation

We use Innanen et al. (1997)'s numerical experiments as a method of measuring the strength of connection among planets against external perturbations. Rigidity implies the unity of the system. If there is no rigidity in two subsystems and these two are stable, then we can regard that these two systems are independent.

We carry out two integrations: one with Saturn and the one without Saturn. In both cases, a perturbing star of mass $0.2M_{\odot}$ moves on the circular orbit of 500AU with inclination 30°. The initial configuration is shown in Fig. 5(b). The integration time is 100 million years. The results are shown in Figs. 8 and 9. Interestingly, both systems are stable and have synchronous motion of nodes Ω. Though initially the nodes are distributed random on each orbits of planets, they converge and gather. Inclinations of planets change together (middle panels of both figures). The only noticeable difference is that the oscillation amplitude of Mercury's eccentricity is smaller when Saturn is not

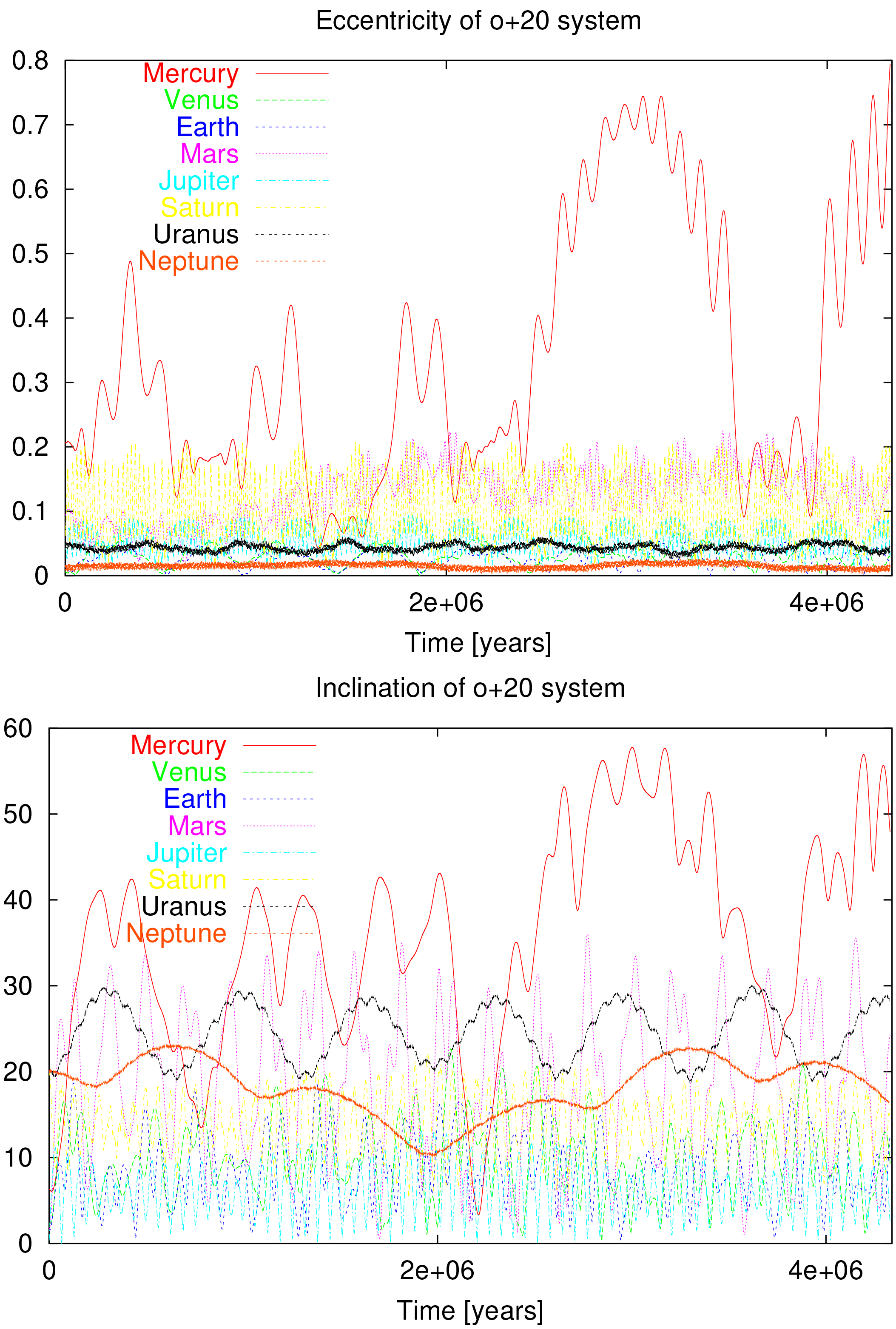

Figure 6: Rigidity against internal perturbations. The planetary system of Fig. 5 (a) is unstable if Saturn exists. Variations of eccentricities (top) and inclinations (bottom) of planets.

existent. Jupiter affects the motion of Uranus and Neptune without the connection of Saturn. This is rather a surprising fact.

The difference of the results of sections 2.4.1 and 2.4.2 indicates that the independence of planetary groups should be examined more carefully. The different checking methods may discriminate subtle differences of the systems otherwise overseen. The existence or the non-existence of Jupiter may be more explicitly related to the independence of outer and inner planetary systems.

3. Discussions and Summary

We examined several groupings of planets. Some of the groupings are not real in our solar system. The existence of a particular grouping of planets should have reflected the formation process of planets and planetary systems. Thus for example, there are no binary planets in our solar system. There are binary asteroids and binary Kuiper-belt objects. Pluto and Charon may be interpreted as a binary Kuiper-belt objects. The Earth–Moon system may be conceivable as binary planets. However, the hypothesis of a giant impact which is most successful at present in explaining the formation of the Moon presupposes the existence the Earth prior to the impact. The hypothesis is not compatible with binary planets. We may need to consider different formation processes to obtain a binary of comparable masses. Can a binary of comparable size be produced through a giant impact?

Binary planets are in a sense a paradoxical objects. Suppose a multi-planet planetary system. In general, the system becomes unstable if two of the members make a close approach. However, if the two are close enough and continued to be close enough, then these two constitute a subsystem and the whole system becomes stable once again. The difference is that through many body interactions, gravitational potential energy is released from the binary and is given to the remaining constitients of the system as kinetic energy. If the whole system has negative enough energy so that kinetic energy does not cancel out the total energy, then the system remains stable. So binaries of small masses can be possible to form. Dissipative media may help to form binaries by absorbing the energy and scatter itself away. Planetesimals are one of the candidates. Thus, a planetary system can release energy when it forms. The Oort cloud may be interpreted as an object resulted from the energy release.

As a subsystem, cousin planets sit between binaries (sister planets) and planetary groups. Binaries can be regarded as a single body because the motion of the center of gravity frequently replaces the individual motions of the components. Cousin planets turn out in the present study to be important first because one component suppresses the orbital instability of the other component and secondly because they transmit the perturbation to other members of a larger subgroup to stabilize this larger subgroup. The Earth–Venus is a good example. Uranus–Neptune may be weak cousin planets. Numerical experiments confirm that the N-body considerations are important. Secular perturbation theory predicts the position of secular resonances of massless particles. In the non-massless pertubed system, secular perturbations may be nullified or the position of secular resonance may be moved off the system.

A planetary group is a collection of loosely connected mutually dynamically dependent planets. A typical example is the terrestrial planets. As Innanen et al. (1998) showed, if one of the important members is void, the terrestrial system becomes unstable against external perturbation. In the present paper, we showed that if two of the members merge

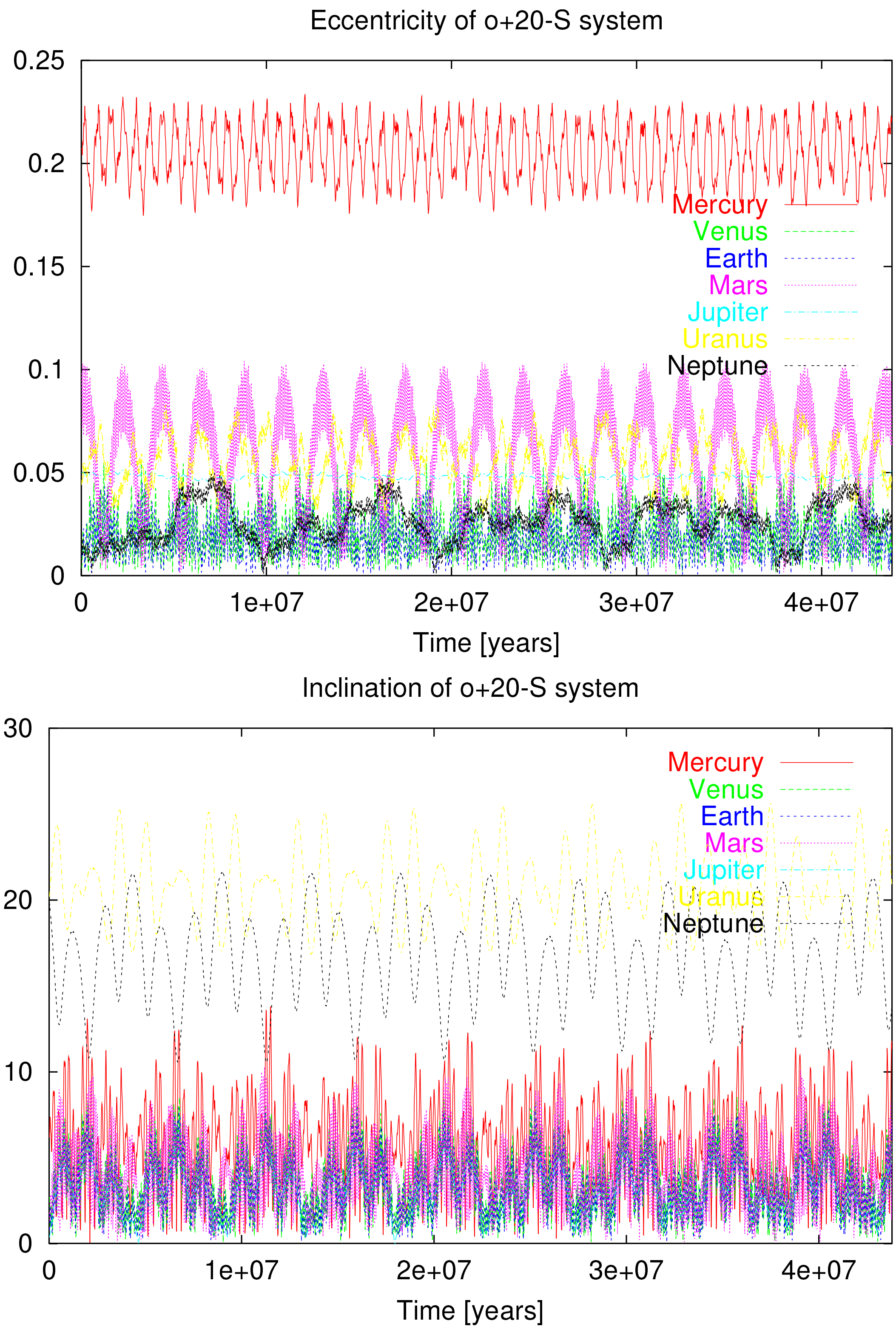

Figure 7: Rigidity against internal perturbations. The planetary system of Fig. 5(a) is stable if Saturn does not exist. Variations of eccentricities (top) and inclinations (bottom) of planets.

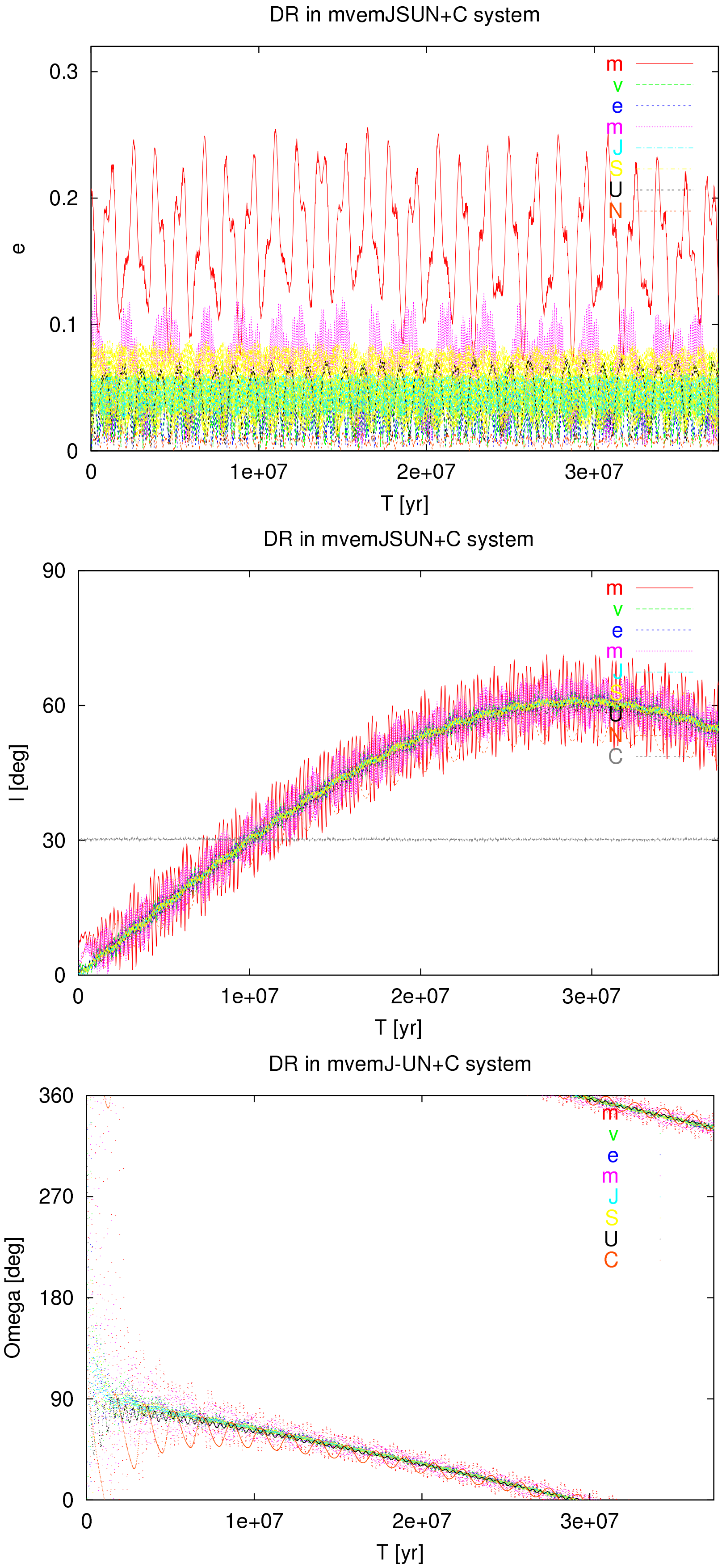

Figure 8: Rigidity of the planetary system with Saturn against external perturbations. Variations of eccentricities (top), inclinations (middle), and nodes (bottom) of planets.

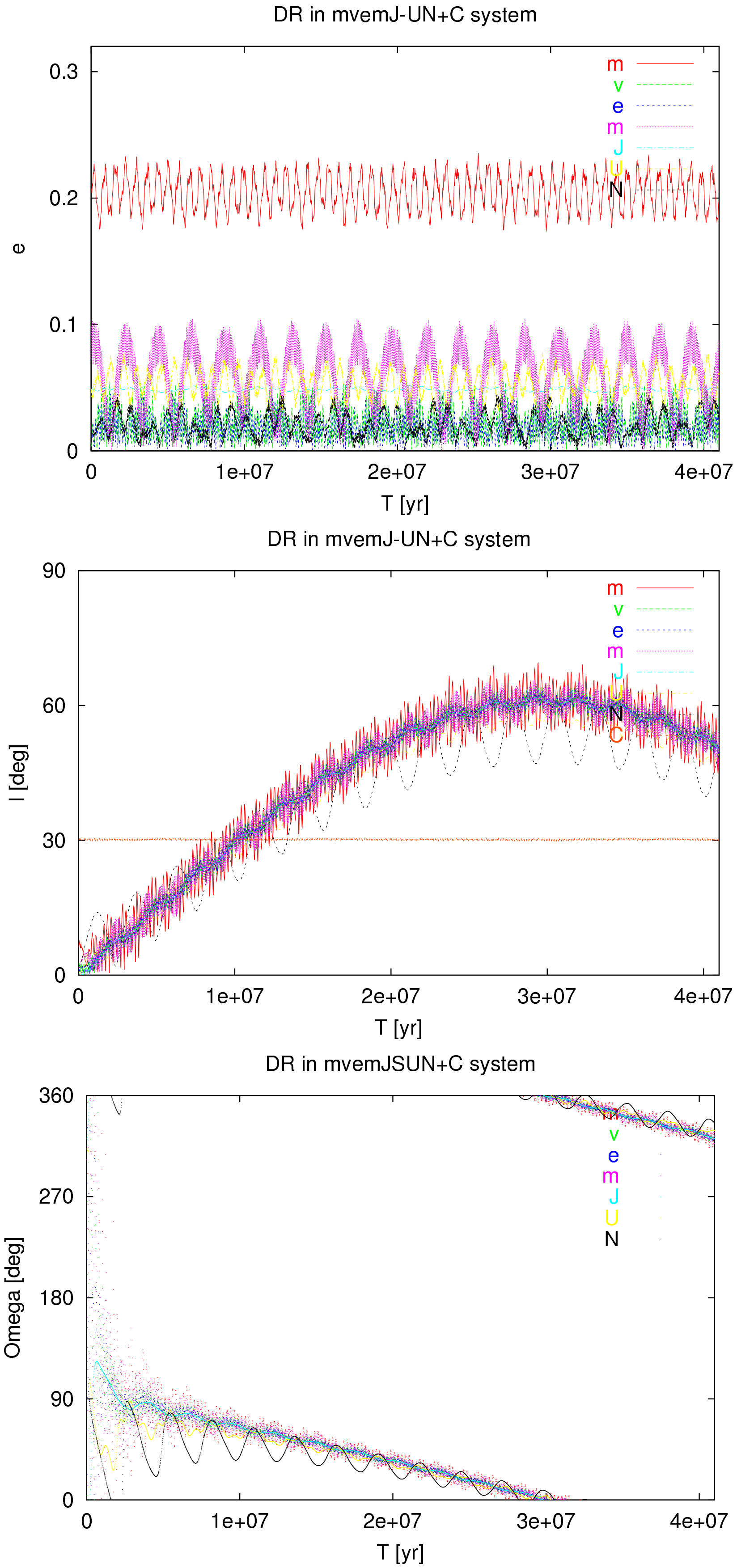

Figure 9: Rigidity of the planetary system without Saturn against external perturbations. Variations of eccentricities (top), inclinations (middle), and nodes (bottom) of planets.

into one, the system becomes unstable also against external perturbation. Even if one of the members is in strong secular resonance with perturbing bodies, all the members share the perturbation to stabilize the whole subgroup. The configuration itself contributes to the stability of the system. This gives a strong constraint to the formation process.

If there are two independent planetary subsystems around a star, We may say that planetary formation processes took place twice. These may be expected in a binary stellar system. One planetary system is around one member of the binary and the other system is around both member stars.

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