# 共鳴島構造がある場合の
# ２次元シンプレクティックマップの簡約

**(A reduction method for weakly nonlinear 2-dim symplectic maps)**

(`http://xxx.lanl.gov/abs/nlin.CD/0309021`)
(To appear in *Physica D* )

**箱根天体力学N体力学研究会**

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1. Our purpose and the outline of this talk
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## $\boxed{1}$ Our purpose and the outline of this talk

**I would like to find a useful method to analyze symplectic maps through the RG method.**
**(RG method we use here**
**= a singular perturbation method)**

Why?

1. We cannot easily construct reduced symplectic maps.
2. The RG method is a candidate to overcome such difficulty.

In this talk, we present

- **a problem of the usual RG method for symplectic maps**
- **a method to resolve the problem**

Acoording to this new method,

**we can construct the reduced maps for given symplectic maps as well as ODEs.**

In order to do this,
we explain our RG method for ODEs on the next page.

## $\boxed{2}$ Ordinary Differential Equations

Ex. (a perturbed oscillator: Hamiltonian System))

$$\frac{d^2x}{dt^2} + \Omega^2 x = -\varepsilon(ax + bx^3),$$

$$\begin{aligned} \frac{dq}{dt} &= \frac{\partial H}{\partial p}, \qquad \frac{dp}{dt} = -\frac{\partial H}{\partial p}, \\ H(q,p) &= \frac{p^2 + \Omega^2 q^2}{2} + \varepsilon\big(\frac{a}{2}q^2 + \frac{b}{4}q^4\big). \end{aligned}$$

Here, $\varepsilon$ is the small parameter, $a, b, \Omega$ are the real paramerers.

**Regular perturbation solution**:

$$\begin{aligned} x(t) &= x^{(0)}(t) + \varepsilon x^{(1)}(t) + \cdots, \qquad (i \equiv \sqrt{-1}) \\ x(t) &= A\mathrm{e}^{i\Omega t} + \varepsilon\Big( i\ \frac{aA + 3bA|A|^2}{2\Omega} t\mathrm{e}^{i\Omega t} + \frac{bA^3}{8\Omega^2}\mathrm{e}^{3i\Omega t}\Big) \\ &\qquad + \text{c.c.} + \mathcal{O}(\varepsilon^2), \end{aligned}$$

$A \in \boldsymbol{C}$ : the integration constant

c.c. : the complex conjugate of the preceeding expression

the approximations seem to break down while "$t$" satisfies $\boxed{\varepsilon\, t \sim 1}$.

We should renormalize away the $\propto t$ term which we call "a secular term."

$\boxed{\textbf{The RG procedure}}$ : the elimination of secular terms

$\boxed{1.}$ ( Def. of an RG transformation)

$$\widetilde{A}(t) \equiv A + \varepsilon\Big(\, i\, \frac{aA + 3bA|A|^2}{2\Omega} t\,\Big) + \mathcal{O}(\varepsilon^2),$$

$$\Big(\text{ inverse transformation } : A = \widetilde{A}(t) + \mathcal{O}(\varepsilon)\Big)$$

$$x(t) = \widetilde{A}(t)\exp(i\Omega t) + \text{non-secular}(\widetilde{A}(t)) + \text{c.c.}$$

Construct the eqaution which $\widetilde{A}(t)$ satisfies

( $\mathcal{O}(\varepsilon^2)$ terms are ignored )

$\boxed{2.}$

$$\widetilde{A}(t+\tau) - \widetilde{A}(t) = \varepsilon\Big(\, i\, \frac{aA + 3bA|A|^2}{2\Omega}\tau\,\Big)$$

$\cdots$ by the definition

$$= \varepsilon\Big(\, i\, \frac{a\widetilde{A}(t) + 3b\widetilde{A}(t)|\widetilde{A}(t)|^2}{2\Omega}\tau\,\Big)$$

by the inversion of the definition

$\boxed{3.}$ (Assume that $\widetilde{A}$ can be expanded)

We assume $$\widetilde{A}(t+\tau) = \widetilde{A}(t) + \tau\frac{d\widetilde{A}}{dt} + \cdots$$

$\boxed{4.}$ (Compare $\boxed{2.}$ and $\boxed{3.}$ at order $\mathcal{O}(\tau^1)$ )

$$\frac{d\widetilde{A}}{dt} = \varepsilon\Big(\, i\, \frac{a\widetilde{A} + 3b\widetilde{A}|\widetilde{A}|^2}{2\Omega}\Big) \qquad \boxed{\text{RG Eq.}}$$

( corresponding to the Landau Eq. in dissipative systems)

# The validity of the RG method

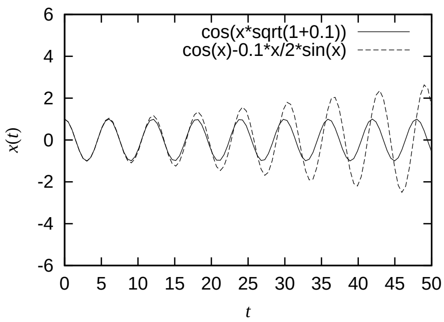
$$\widetilde{A}(t) = \widetilde{A}(0) \exp\Big\{ \, i \, \frac{\varepsilon t}{2\Omega}(a + 3b|\widetilde{A}(0)|^2)\Big\},$$

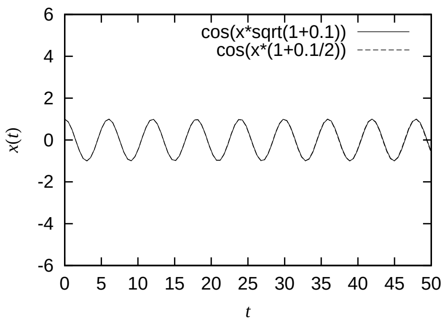
$$x(t) = \widetilde{A}(t)\exp(i\Omega t) + \text{c.c.} + (\text{non-secular terms}),$$

$$= \widetilde{A}(0)\exp\Big\{ \, it \, \Big(\Omega + \frac{\varepsilon}{2\Omega}(a + 3b|\widetilde{A}(0)|^2)\Big)\Big\}.$$

$$+\text{c.c.} + (\text{non-secular terms}).$$

<u>**Numeriacl check:**</u> $(\varepsilon = 0.1, \, \Omega = 1, \, a = 1, \, b = 1)$




(left): regular perturbation and the exact solution

(right): RG and the exact solution ,

## <u>**Short summary** : RG procedure</u>

- Get the regular perturbative solution:

$$(x = x^{(0)} + \varepsilon x^{(1)} + \cdots)$$

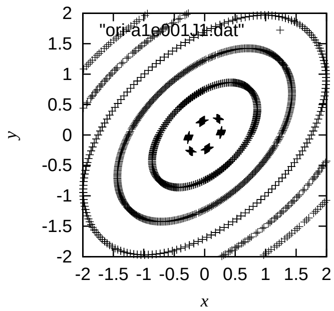
- Get the renormalization transformation: $(\widetilde{A}(t) \equiv \cdots)$
- Construct the RGE $(\dfrac{d\widetilde{A}}{dt} = \cdots)$
- (Construct the renormalized solution if you want)

# $\boxed{3}$ Symplectic maps (I) :non-resonant case

Ex. a 2-D symplectic map $(x^n, y^n) \mapsto (x^{n+1}, y^{n+1})$

$$\begin{cases} x^{n+1} = x^n + y^{n+1}, \\ y^{n+1} = y^n + ax^n + 2\varepsilon J(x^n)^3 \end{cases}$$

$\varepsilon$: the small parameter $a,\ J$ : $\mathcal{O}(\varepsilon^0)$ paremeters




$(\varepsilon = 0.01, \quad a = 1.0, \quad J = 1)$

**Regular perturbation** $x^n = x^{(0)n} + \varepsilon x^{(1)n} + \varepsilon^2 x^{(2)n} + \mathcal{O}(\varepsilon^3)$,

$$\begin{aligned} x^{(0)n} &= A\exp(i\theta n) + \text{c.c.} \\ x^{(1)n} &= \frac{-3i|A|^2 AJ}{\sin\theta} n\mathrm{e}^{i\theta n} + (\ \propto \exp(3i\theta) \quad \text{terms}) \\ x^{(2)n} &= \{\frac{-9}{2}\frac{J^2|A|^4 A}{\sin^2\theta} n^2 \\ &\quad -i\frac{J^2|A|^4 A}{\sin\theta}(\frac{3}{\cos 3\theta - \cos\theta} + \frac{9\cos\theta}{2\sin^2\theta})n\}\mathrm{e}^{i\theta n} \\ &\quad +\Big(\ \propto \exp(3i\theta),\ \ \exp(5i\theta) \text{ terms}\Big) \end{aligned}$$

$(\cos\theta \equiv 1 - a/2, \quad A \in \boldsymbol{C} :$ integration constant$)$

$\boxed{\textbf{There is a secular behavior as well as ODEs}}$

$(x^n \approx \varepsilon n, \varepsilon^2 n^2, ..)$ $\underline{\cos 3\theta \neq \cos\theta}$ is assumed.

**RG transformation** : To remove the secular behavior

$$
\begin{aligned}
A^n \quad &\equiv A + \varepsilon \frac{-3i|A|^2 AJ}{\sin\theta} n \\
&+ \varepsilon^2 \{ \frac{-9}{2} \frac{J^2 |A|^4 A}{\sin^2\theta} n^2 - i \frac{J^2|A|^4 A}{\sin\theta} (\frac{3}{\cos 3\theta - \cos\theta} + \frac{9\cos\theta}{2\sin^2\theta}) n \}.
\end{aligned}
$$

**Naive RG map**: which $A^n$ and $A^{n+1}$ satisfy

$$
\begin{aligned}
A^{n+1} \; = \; & A^n + \varepsilon \frac{-3iJ}{\sin\theta} |A^n|^2 A^n + \varepsilon^2 \{ \frac{1}{2!} (\frac{-3iJ}{\sin\theta} |A^n|^2)^2 A^n \\
& + \frac{-9i}{2} \frac{\cos\theta}{\sin^3\theta} J^2 |A^n|^4 A^n - \frac{3iJ^2|A^n|^4 A^n}{\sin\theta(\cos 3\theta - \cos\theta)} \}
\end{aligned}
$$

**The important thing is that**
**the naive RG map doesn't have symplecticity**

$$
dA_1^{n+1} \wedge dA_2^{n+1} - dA_1^n \wedge dA_2^n = \mathcal{O}(\varepsilon^3), \quad (A^n \equiv A_1^n + i A_2^n)
$$

# A downside of "naive" RG method for symplectic maps

**Breaking down of symplecticity,**
**the reduced map is a dissipative system!**

$\Downarrow$

Let's recover the symplecticity. To do this we take the continuous-limit of the RG map and use the Liouville operator relation.

What is the Liouvile operator relation?

Ans. Canonical Eqs definitely satisfy this relation

$$Z(t+\mu) = \big(1 + \mu \mathcal{L}_H + \frac{\mu^2}{2!}\mathcal{L}_H^2 + \cdots\big) Z(t) = \exp(\mu \mathcal{L}_H) Z(t),$$

$Z$ : a func. of canonical variables, $\quad H$ : a Hamiltonian.

$$\begin{aligned} \mathcal{L}_H Z &\equiv \{Z, H\} \equiv \sum_{j=1}^{N} \big(\frac{\partial Z}{\partial q_j}\frac{\partial H}{\partial p_j} - \frac{\partial Z}{\partial p_j}\frac{\partial H}{\partial q_j}\big), \\ \mathcal{L}_H^2 Z &= \mathcal{L}_H\big(\mathcal{L}_H Z\big) = \{\{Z, H\}, H\}, \end{aligned}$$

Ex. canonical Eq.

$$\frac{dq}{dt} = \mathcal{L}_H q = \{q, H\}, \quad \frac{dp}{dt} = \mathcal{L}_H p = \{p, H\}$$

The solution can fomally be rewritten as

$$q(t+\mu) = \exp\,(\mu \mathcal{L}_H) q(t), \quad p(t+\mu) = \exp\,(\mu \mathcal{L}_H) p(t),$$

# A strategy for constructing a symplecticity-preserving RG map

A symplecticity-preserving RG method

[discrete-time ] [continuous-time]

$\boxed{\varepsilon = \text{"time step"}}$

$$
\begin{array}{ccc}
\text{Symp. maps} & & \\
\text{naive RG } (\heartsuit) \downarrow & & \\
\text{naive RG maps (dissipative maps)} & \xrightarrow{\text{Liouville operator}(\spadesuit)} & \text{RG Eqs. (canonical Eqs.)} \\
\text{Symp.-Pres. RG maps} & \xleftarrow{\text{Symp.-Pres. discretizations}} &
\end{array}
$$

Equating the "naive" RG map with the relation ($\spadesuit$), we get the corresponding continuous Hamiltonian system.

$$
\begin{aligned}
A^{n+1} &= A^n + \varepsilon(\cdots) + \varepsilon^2(\cdots), \quad \cdots(\heartsuit) \\
A(t+\varepsilon) &= (1 + \varepsilon\mathcal{L}_H + \frac{\varepsilon^2}{2!}\mathcal{L}_H^2 + \cdots)A(t) \quad \cdots(\spadesuit) \\
&= (1 + \varepsilon\mathcal{L}_{H^{(1)}} + \varepsilon^2(\mathcal{L}_{H^{(2)}} + \frac{\mathcal{L}_{H^{(1)}}^2}{2!}) + \cdots)A(t),
\end{aligned}
$$

**This identification gives us a canonical differential Eq. with the Hamiltonian** $H = H^{(1)} + \varepsilon H^{(2)} + \cdots$

$\boxed{\text{But how do we find the Hamiltonian?}}$

# How to find an appropriate Hamiltonian

$\boxed{H^{(1)}}$ can firstly be obtained by taking the $(\varepsilon \to 0)$ of the naive RG map:

$$\lim_{\varepsilon \to 0} \frac{A^{n+1} - A^n}{\varepsilon} \equiv \{H^{(1)}, A\} \equiv \mathcal{L}_H^{(1)} A$$

$\boxed{H^{(2)}}$ can be determined by comparing both the Liouville relation with the naive RG map and using $H^{(1)}$

$$\begin{aligned} A(t+\varepsilon) &= A(t) + \varepsilon \mathcal{L}_{H^{(1)}} A(t) + \varepsilon^2 (\mathcal{L}_{H^{(2)}} + \frac{\mathcal{L}^2_{H^{(1)}}}{2!}) A(t) + \cdots \\ A^{n+1} &= A^n + \varepsilon (A^n \text{ term}) + \varepsilon^2 (A^n \text{ term}) \end{aligned}$$

$\boxed{H^{(3)}, H^{(4)}, ..}$ same as $H^{(2)}$.

**In this way, we can systematically find the appropriate Hamiltonian**

In this case, the Hamiltonian we have been finding is

$$\begin{aligned} H &= \alpha (A_1^2 + A_2^2)^2 + \beta (A_1^2 + A_2^2)^3. \\ \alpha &\equiv \frac{3J}{4 \sin\theta}, \quad \beta \equiv \varepsilon \{ \frac{9 \cos\theta}{2 \sin^3\theta} + \frac{3}{\sin\theta(\cos 3\theta - \cos\theta)} \} \frac{J^2}{6}. \end{aligned}$$

# The Symp.-Pres RG map and its solution

The solution of $\quad dA/dt = \mathcal{L}_{H^{(1)}+\varepsilon H^{(2)}}A \quad$ is

$$
\begin{aligned}
A(t) \;=\; & A(0)\exp\Big[it\frac{-3J}{\sin\theta}|A(0)|^2 \\
& -i\varepsilon^2 J^2|A(0)|^4\big(\frac{9\cos\theta}{2\sin^3\theta}+\frac{3}{\sin\theta(\cos 3\theta-\cos\theta)}\big)\Big],
\end{aligned}
$$

The Symp.-Pres. RG map is obtained here by defining $A^{n+1} \equiv A(t+\varepsilon), A^n \equiv A(t)$,

$$
\begin{aligned}
A^{n+1} \;&= A^n\exp\Big[i\varepsilon\frac{-3J|A^n|^2}{\sin\theta} \\
& -i\varepsilon^2 J^2|A^n|^4\big(\frac{9\cos\theta}{2\sin^3\theta}+\frac{3}{\sin\theta(\cos 3\theta-\cos\theta)}\big)\Big].
\end{aligned}
$$

We can get the Symp.-Pres. RG map

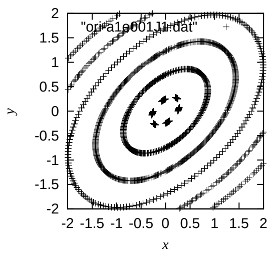
This RG map is solvable,

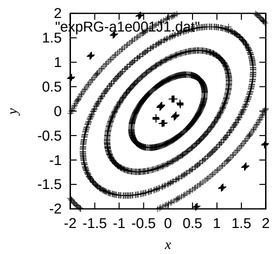
$$
A^n = A^0\exp\Big[i\varepsilon(\frac{-3J|A^0|^2}{\sin\theta})n + i\varepsilon^2(\cdots)n\Big],
$$

Then,

$$
\begin{aligned}
x^n &\approx A^0[\exp(i\theta n + i\varepsilon(\frac{-3J|A^0|^2}{\sin\theta})n + i\varepsilon^2(\cdots)n], \\
y^n &= x^n - x^{n-1}.
\end{aligned}
$$

# The validity of our method and a short summary




origianl map Symp.-Pres. map

($x$ and $y$ are constructed by $A^n$ with its definition )

**Short summary** : the RG method for Symp. maps

- Get the regular perturbative solution:
  $(x = x^{(0)} + \varepsilon x^{(1)} + \cdots)$
- Get the RG transformation: $A^n \equiv A + \mathcal{O}(\varepsilon) \cdots$
- Construct the "naive" RG map:
  $(A^{n+1} = A^n + \mathcal{O}(\varepsilon))$
- Construct the canonical Eq. by naive RG map:
  $\frac{dA}{dt} = \mathcal{L}_{H^{(1)} + \varepsilon H^{(2)}} A \cdots (\spadesuit)$
- Discretize the solution of $(\spadesuit)$:
  $A^{n+1} = A^n \exp\left(\mathcal{O}(\varepsilon)\right)$
  This is definitely symplectic.

# $\boxed{4}$ Symplectic maps (II) : resonant case

Ex. The 2dim symplectic map we analyzed

$$\begin{cases} x^{n+1} = x^n + y^{n+1}, \\ y^{n+1} = y^n + ax^n + 2\varepsilon J(x^n)^3 \end{cases}$$

$\varepsilon$: the small parameter $a,\ J$ : $\mathcal{O}(\varepsilon^0)$ parameters

This can be written as

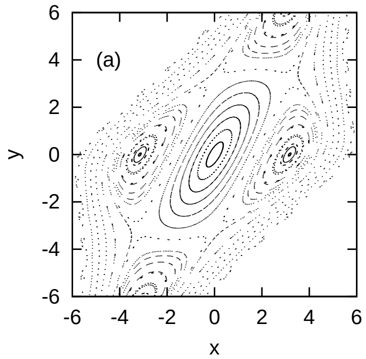
$$x^{n+1} - 2x^n \cos\theta + x^{n-1} = 2\varepsilon J(x^n)^3,$$

Here, $\cos\theta \equiv 1 - \frac{a}{2}$,

**Resonance we analyze**: $\boxed{\cos 3\theta - \cos\theta = \mathcal{O}(\varepsilon)}$,

$$\theta = \frac{\pi}{2} + \varepsilon\theta^{(1)} + \varepsilon^2\theta^{(2)} + \cdots$$

$\theta^{(1)}$ : parameter




$(\varepsilon = 0.01, \quad J = 1.0, \quad \theta^{(1)} = 10.0)$

## The construction of the symplectic RG map even when a resonant island appears

$\boxed{\textbf{How is this done?}}$ $\cdots$ **Same as before!**

We can take the general strategy we have shown.

1. regular perturbation
2. naive RG method
3. Liouville Operator approach

**Regular perturbation** : $x^n = x^{(0)n} + \varepsilon x^{(1)n} + \cdots$

$$\begin{aligned}
x^{(0)n} &= Ai^n + \text{c.c.} \qquad (i \equiv \sqrt{-1}),\\
x^{(1)n} &= (-i)i^n n\big[J(A^{*3} + 3|A|^2A) - \theta^{(1)}A\big] + \text{c.c.}\\
x^{(2)n} &= i^n n^2\big[\frac{3}{2}J^2\big(-2|A|^4A + |A|^2A^{*3} + A^5\big)\\
&\quad +J\theta^{(1)}\big(3|A|^2A - A^{*3}\big) - \frac{\theta^{(1)2}}{2}A\big] + i^n n i\theta^{(2)}A + \text{c.c.}.
\end{aligned}$$

$A(\in \boldsymbol{C})$ : the integration constant

**RG transformation** : $x^n = A^n i^n + \text{c.c.}$,

**“Naive” RG map** which $A^n := A_1^n + iA_2^n$ satisfies

$$\begin{aligned}
A_1^{n+1} &= A_1^n + \varepsilon\big(4J(A_2^n)^3 - \theta^{(1)}A_2^n\big) + \varepsilon^2\big\{-24J^2(A_1^n)^3(A_2^n)^2\\
&\quad +2J\theta^{(1)}\big((A_1^n)^3 + 3A_1^n(A_2^n)^2\big) - \frac{\theta^{(1)2}}{2}A_1^n - \theta^{(2)}A_2^n\big\},\\
A_2^{n+1} &= A_2^n + \varepsilon\big(-4J(A_1^n)^3 + \theta^{(1)}A_1^n\big) + \varepsilon^2\big\{-24J^2(A_1^n)^2(A_2^n)^3\\
&\quad +2J\theta^{(1)}\big((A_1^n)^3 + 3(A_1^n)^2A_2^n\big) - \frac{\theta^{(1)2}}{2}A_2^n - \theta^{(2)}A_1^n\big\}.
\end{aligned}$$

# A symplecticity-preserving RG map when a resonant island appears

The RG map doesn't have symplectic symmetry:

$$dA_1^{n+1} \wedge dA_2^{n+1} - dA_1^n \wedge dA_2^n \neq 0$$

**Strategy diagram**:

$$\begin{array}{ccc}
\text{[discrete-time ]} & & \text{[continuous-time]} \\
 & (\varepsilon = \text{“time step”}) & \\
\text{naive RG maps} & & \\
\text{(dissipative maps)} & \xrightarrow{\text{Liouville operator}(\spadesuit)} & \text{RG Eqs.} \\
 & & \text{(canonical Eqs.)} \\
\text{Symp.-Pres. RG maps} & \xleftarrow{\text{Symp.-Pres. discretizations}} &
\end{array}$$

**Associated canonical Eq.**:

$$\begin{aligned}
\frac{dA_1}{dt} &= 4JA_2^3 - \theta^{(1)} A_2 = \frac{\partial H^{(1)}}{\partial A_2}, \\
\frac{dA_2}{dt} &= -4JA_1^3 + \theta^{(1)} A_1 = -\frac{\partial H^{(1)}}{\partial A_1}, \\
H^{(1)}(A_1, A_2) &= (JA_1^4 - \theta^{(1)} A_1^2/2) + (JA_2^4 - \theta^{(1)} A_2^2/2).
\end{aligned}$$

**This solution is an elliptic function. It might be difficult to discretize the soluton...**

# A symplecticity-preserving discretization of an RG flow

**Here, we use a "symplectic integrator" to discretize the flow.**

What is the symplectic integrator?

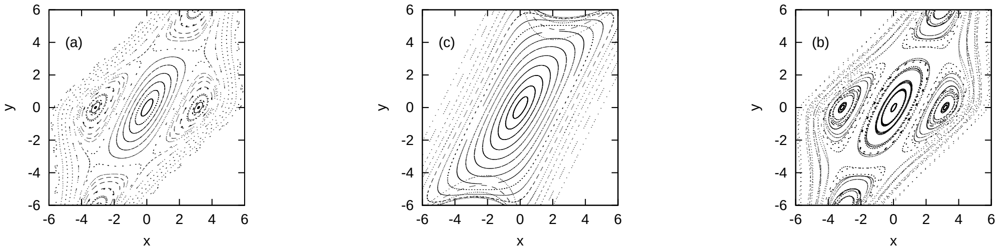
Ans. This is known to be a discritization method designed for preserving the symplecticity for Hamiltonian flows.

## Symp.-Pres. RG map

by using a symplectic integrator for the canonical Eq.

$$
\begin{aligned}
A_1^{n+1} &= A_1^n + \varepsilon\big[4J\big\{A_2^n + \frac{\varepsilon}{2}(-4JA_1^{n3} + \theta'^{(1)}A_1^n)\big\}^3 \\
&\quad -\theta'^{(1)}\big\{A_2^n + \frac{\varepsilon}{2}(-4JA_1^{n3} + \theta'^{(1)}A_1^n)\big\}\big], \\
A_2^{n+1} &= A_2^n + \frac{\varepsilon}{2}\big(-4JA_1^{n3} + \theta'^{(1)}A_1^n\big) \\
&\quad +\frac{\varepsilon}{2}\big(-4JA_1^{n+1\ 3} + \theta'^{(1)}A_1^{n+1}\big).
\end{aligned}
$$

## The validity of some RG methods and a short summary




origianl map non resonant RG map Symp.-Pres. map

($x$ and $y$ are constructed by $A^n$ with its definition )

**Short summary** : the RG method for Symp. maps

RG method is also valid even in a resonant case:

- Get the regular perturbative solution: $(x = x^{(0)} + \varepsilon x^{(1)} + \cdots)$
- Get the RG transformation: $A^n \equiv A + \mathcal{O}(\varepsilon) \cdots$
- Construct the "naive" RG map: $(A^{n+1} = A^n + \mathcal{O}(\varepsilon))$
- Construct the canonical Eq. by naive RG map: $\frac{dA}{dt} = \mathcal{L}_{H^{(1)}+\varepsilon H^{(2)}} A \cdots (\spadesuit)$
- Symplectic-discretization of $(\spadesuit)$

## $\boxed{5}$ Symplectic map chains : a discrete Nonlinear Shrödinger Eq.

**A coupled symplectic map chain** : $(x_j^n, p_j^n) \mapsto (x_j^{n+1}, p_j^{n+1})$

$$
\begin{aligned}
x_j^{n+1} - x_j^n &= \tau p_j^n \\
p_j^{n+1} - p_j^n &= \tau \Big[ -\Omega^2 x_j^n + \varepsilon \Big\{ -\alpha (x_j^n)^3 + \nu \triangle_j^2 x_j^n \Big\} \Big],
\end{aligned}
$$

$\varepsilon$ : small parameter, $\Omega, \alpha, \nu, \tau$ :$\mathcal{O}(1)$ parameter

$$\triangle_j^2 x_j^n \equiv x_{j+1}^n - 2x_j^n + x_{j-1}^n$$

**RG transformation**:

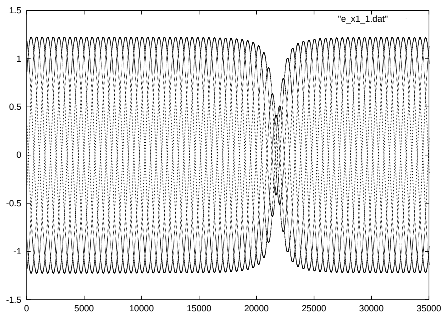
$$A_j^n \equiv A_j + \varepsilon n(\cdots), \quad x_j^n \approx A_j^n \exp(-i\theta n) + \text{c.c.}$$

$A_j \in \boldsymbol{C}$ are integration constants, $\cos\theta \equiv 1 - \Omega^2\tau^2/2$,

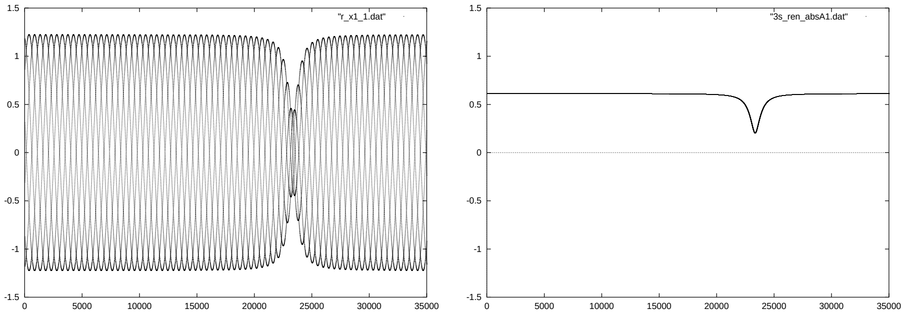
**Symplecticity preserving RG** :

$$\Big(1 - \frac{i\varepsilon\tau^2\nu}{4\sin\theta}\triangle_j^2\Big) A_j^{n+1} = \Big(1 + \frac{i\varepsilon\tau^2\nu}{4\sin\theta}\triangle_j^2\Big) \exp\Big(i\varepsilon \frac{-3\alpha\tau^2}{2\sin\theta}\Big) A_j^n$$

$\cdots$ a discrete NLS




A time sequence of $x_1^n$ in the original system.




reconstructed $x_1^n$ by $A_1^n$ $\qquad |A_1^n|$.

> The slow motion of the system is analytically extracted.

- Note that the relation

$$x_j^n \approx A_j^n \exp(-i\theta n) + \text{c.c.}$$

RG valuables $A_j^n$ are the slow variables.

## **Short summary** :

- Our method is also useful to $N$-coupled symplectic maps
- RG valuables are "slow valuables"
- Although I don't show the case of resonance, we can construct the RG map even when a resonant condition is satisfied.

## $\boxed{6}$ Conclusion

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**We can construct the correctly reduced symplectic maps even when a resonance island appears**

**In this talk, we have discussed these topics :**

**[1 ] RG method for Diff. Eqs.**

**[2 ] There exists the problem of applying [1] to Symplectic mappings.**

**[3 ] An answer to the problem [2] is that we use "the Liouville operator relation" for naive RG maps.**

**This method is applicable to study a resonance island.(see T. Maruo, S. Goto and K. Nozaki,** "*Renormalization Analysis of Resonance Structure in 2-D Symplectic Map*", `http://xxx.lanl.gov/abs/nlin.CD/0309072` **(2003).**