Numerical modeling of relativistic magnetic reconnection: Kinetic, two-fluid, and MHD simulations

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Outline

• 0. Introduction
• 1. Kinetic modeling
  – 2D reconnection
• 2. Fluid modeling
  – RRMHD & two-fluid simulations
  – New insight: high-speed fluid dynamics
• 3. Cross-code comparison
• 4. Summary
Magnetic reconnection

- Re-configuration of magnetic topology
- Release of magnetic energy to plasma energy
- Violation of the ideal condition is necessary

\[ E + v \times B \neq 0 \]
Magnetic reconnection in high-energy settings

- Magnetically dominated settings
- Special relativity
  - + radiation, pair creation ...

Jet model

Giannios+ 2009

Striped wind

Coroniti 1990

Gamma-ray flares

Tavani 2011
Current status of relativistic reconnection research (2015Q2)

- Theories have been discussed over two decades
- Many works came out after the Crab flare (2011)

MHD theories

- Blackman & Field 1994
- Lyutikov & Uzdensky 2003
- Lyubarsky 2005
- Tenbarge+ 2010
- Comisso & Asenjo 2014

- Basic properties are under debate

Kinetic (PIC) simulations

- Zenitani & Hoshino 2001-2008
- Jaroschek+ 2004-2009
  - Liu+ 2011
  - Cerutti+ 2012-2014
  - Kagan+ 2013
  - Sironi & Spitkovsky 2014
  - Melzani+ 2014
  - Guo+ 2014-2015
  - Zenitani in prep.

MHD simulations

- Watanabe & Yokoyama 2006
- Zenitani+ 2010, 11
- Zanotti & Dumbser 2011
- Takahashi+ 2011
- Takamoto 2013
  - (Zenitani+ 2015)

- Ideal for global modeling

Two-fluid simulations

- Zenitani+ 2009

- Meso-scale evolution

- Fast evolution
- Particle acceleration
1. Kinetic modeling
2D Particle-In-Cell (PIC) simulation

- Fast reconnection and particle acceleration occurs

DC particle acceleration

\[ E^2 - B^2 > 0 \]

Hard-energy spectra

- Power-law index in earlier works
  \( s \sim -1 \) (near acc. region), \(-2 \sim 3\) (global)
- Fermi process? Upstream magnetization may control the index (Guo+ 2014)
3D reconnection: Onset problem

Drift-kink instability dominates in another 2D plane

Turbulent Plasma heating

Zenitani & Hoshino 2008
3D large-scale evolution: RX appears to win

- Kink-mode often saturates
- Reconnection outruns kink, and then particle acceleration turns on
- 3D energy spectra is similar to 2D
2. Fluid modeling
Relativistic Petschek (PK) reconnection

- Relativistic resistive MHD (RRMHD) eqs. (Watanabe & Yokoyama 2006, Komissarov 2007)

\[
(\rho U^\mu)_{,\mu} = 0, \quad (T_{\text{gas}}^{\mu\nu} + T_{\text{em}}^{\mu\nu})_{,\mu} = 0 \\
F_{,\mu}^{\mu\nu} = -J^\nu, \quad F^{*\mu\nu} = 0 \\
F^{\mu\nu} u_\nu = \eta \left( J^\mu + (J^\nu u_\nu) u^\mu \right)
\]

- Alternative: relativistic two-fluid eqs. (Zenitani+ 2009a,b, Barkov+ 2014)
Relativistic PK reconnection - Alfvénic outflow

- Magnetization parameter

\[ \sigma_\varepsilon = \frac{B_0^2}{4\pi \gamma^2 \omega} \left( \approx \frac{8}{5} \frac{E_{EM}}{E_{fluid}} \right) \]

- Relativistic Alfvén speed

\[ \gamma_{jet} v_{jet} \approx \gamma_A c_A = \sqrt{\sigma_\varepsilon} \]

**Equation:**

\[ U_x = \gamma V_x \]

**Graph:**

Local maxima (adiabatic acc.)

Petschek outflow

Alfvénic relation

Nonrelativistic \[ \leftrightarrow \] \[ \sigma_\varepsilon \] \[ \rightarrow \] Ultrarelativistic

Zenitani+ 2010
Relativistic PK reconnection - a paradox

- Narrower opening angle (Lyubarsky 2005)
- Speed is limited by Alfvén speed

Semi-relativistic (sigma=0.4)

\[
\begin{align*}
\text{Reconnection rate (flux transfer speed) goes high} \\
\gamma^2 \frac{\Gamma}{\Gamma-1} p v
\end{align*}
\]

Nonrelativistic \( \sigma_E \) Ultrarelativistic

Relativistic (sigma=4)
Relativistic Sweet-Parker reconnection

Reconnection rate

Lundquist number (Normalized system length)

Transition to the plasmoid regime

Takahashi+ 2011

Takamoto 2013
New insight: High-speed fluid dynamics

- Branches of fluid dynamics

\[ \mathcal{M} = \frac{V}{c_s} \]

Subsonic | Transonic | Supersonic

- Incompressible fluids
- High-speed fluid dynamics
- Compressible fluid dynamics
  - Adiabatic effects
  - Shocks
  - Shock=Shock interaction

- Relativistic reconnection is usually in the high-speed regime

\[ c_s \leq \frac{c}{\sqrt{3}} \quad c_A = c \sqrt{\frac{\sigma_\varepsilon}{1 + \sigma_\varepsilon}} \rightarrow c \quad \sigma_\varepsilon > \frac{1}{2} \]
High-speed fluid effects (1/3): Adiabatic acceleration

- Proposed by Shimizu & Ugai 2000
- Super-Alfvénic flow was also reported by Zanotti & Dumbser 2011

Outflow 4-velocity

Laval nozzle

Magnetic island ("plasmoid")

Sound 4-velocity
High-speed fluid effects (2/3): normal shocks

Postplasmoid slow shocks

Forward slow shocks

u_x

t = 195.0

Downstream

Upstream

\begin{align*}
P & \quad \rho
\end{align*}
Normal shock (Recompression shock)

- Magnetic island = Airfoil

$V_{jet} \approx C_A$

Subsonic ($V \ll C_s$)

Transonic ($0.8C_s < V$)

Supersonic ($C_s < V$)
Compressible effects (3/3): Shock diamond

\[ v_{jet} \approx C_A > C_s \]
**Shock diamond**

- **(a) Over-expanded flow**
  
  \[ c_s < V_{\text{jet}} \quad \text{p}_{\text{jet}} < \text{p}_{\text{ambient}} \]

- **(b) Under-expanded flow**

  \[ c_s < V_{\text{jet}} \quad \text{p}_{\text{jet}} > \text{p}_{\text{ambient}} \]
Shock diamonds in an extragalactic jet

$2 \times 10^6$ light years

PKS 0637-752    Godfrey+ 2012
Shock diamonds in aeronautics

Shock diamonds in video game

Microsoft Flight Simulator X  https://www.youtube.com/watch?v=S8QGaiE4yWc
Some more shock-diamonds

1. Over-expanded shock diamonds

2. Under-expanded shock diamonds

\[ \sigma_\varepsilon > 3 \]

3. Mach disk
   This may evolve to over-expanded shock diamonds

\[ \pm 1.5\% \]
Structure of a reconnection-plasmoid system

- High-speed effects were also missing in nonrelativistic reconnection industry
- They result in complex shock structure in a low-\(\beta\) plasma

\[
\beta = \frac{2}{\Gamma} \left( \frac{c_s}{c_A} \right)^2 \sim \left( \frac{c_s}{V} \right)^2 = M^{-2}
\]

1. Petschek slow shock (Petschek 1964)
2. outer shell = slow shock (Ugai 1995)
3. intermediate shock (Abe & Koshibe 2001) or slow shock (Saito et al. 1995)
4. fast shock (Forbes & Priest 1983)
5. looptop front (Ugai 1987)
6. tangential discontinuity
7. post-plasmoid vertical slow shock (Zenitani et al. 2010)
8. outer vertical slow shock (Zenitani & Miyoshi 2011)
9. fast-mode wave front (Saito et al. 1995)
10. overexpanded shock-diamonds (Zenitani et al. 2010)
11. underexpanded shock-diamonds (Zenitani 2015)
12. contact discontinuity (Zenitani & Miyoshi 2011, 2015)
13. contact discontinuity (Zenitani 2015)
14. contact discontinuity (Zenitani 2015)
15. slow expansion wave front (Zenitani 2015)

A. reconnection inflow
B. outflow jet
C. post-plasmoid reverse flow
D. internal flow
E. flapping jet (NH instability)
3. Cross-code comparison
Relativistic kinetic Ohm’s law

- MHD simulation relies on pre-defined Ohm’s law
- What about self-consistent PIC results?

- Stress-energy tensor
  \[ W^{\alpha\beta} = \int f(u) u^\alpha u^\beta \frac{d^3u}{\gamma} \]
  \( w^\alpha : \text{heat flow} \)

- Standard decomposition
  \[ W^{\alpha\beta} = w u^\alpha u^\beta + w^\alpha u^\beta + w^\beta u^\alpha + w^{\alpha\beta} \]

- Ohm’s law (\( \partial_t = 0 \))
  \[ E + \frac{v}{c} \times B = \frac{1}{\gamma n q} \nabla \cdot (w u^i u^j + Q^{ij} + P^{ij}) \]
  - Heat flow term (Only in relativistic kinetic plasma)
  - Bulk term (incl. relativistic pressure)
  - Local momentum transport (Incl. kinetic div.P term)
Kinetic Ohm’s law (cont.)

- Normalized energy dissipation ($\sim j.E/n \sim \eta j^2/n$)

- Ohm’s law: relativistic **heat flow term** appears

Zenitani in prep.
PIC vs Two-fluid vs RRMHD - energy throughput

- Matter flow
- Bulk kinetic
- Heat flow: $Q^0_i$
- Poynting flux

$T^0_i$ (gas)

$T^0_i$ (EM)

- Kinetic term (heat flow) appears to be important
Summary

• Kinetic modeling
  – Reconnection features DC particle acceleration
  – 3D evolution - a kink instability
  – Now it is a field of active research!!

• RRMHD, Two-fluid modeling
  – Straightforward extension of nonrelativistic MHD reconnection
  – Narrower exhaust, faster reconnection rate, heat-dominated flow
  – High-speed fluid effects - adiabatic acc., shocks, shock diamonds

• Cross-code comparison
  – Heat flow appears in the Ohm’s law and dominates in the energy budget (due to accelerated particles)