9P56

相対論的高速流プラズマの数値実験技法 (PICシミュレーション)

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Loading Relativistic Maxwell Distributions in Particle Simulations
Zenitani, *Phys. Plasmas* **22**, 042116 (2015)

Dissipation in relativistic pair-plasma reconnection: revisited
Zenitani, *Plasma Phys. Control. Fusion* **60**, 014028 (2018)

Three technical barriers in relativistic PIC simulations

- 1. Setup
  - Loading velocity distribution functions by using random variables
  - Not clearly documented ==> Zenitani 2015

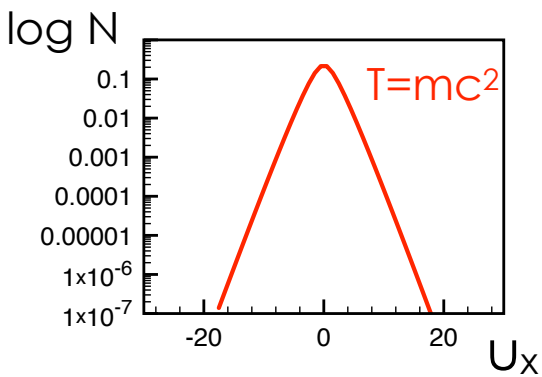
- 2. Computation
  - Electromagnetic field (Haber 1974, Vay+ 2011)
  - Particle (Vay 2008)

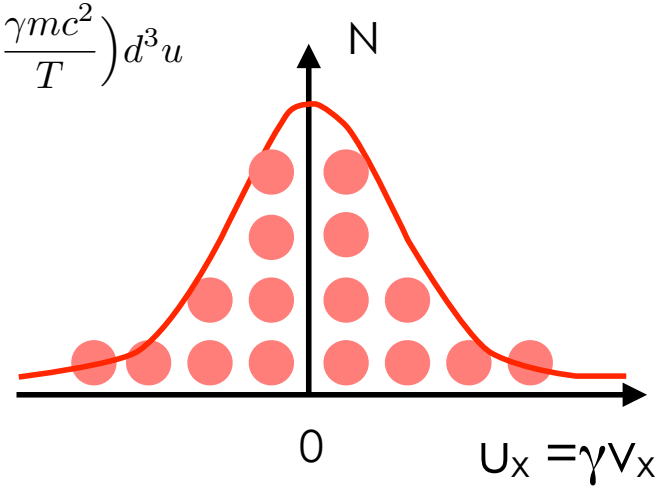
- 3. Diagnosis & Interpretation
  - Definition of “fluid” frame and further decomposition
  - Not clearly documented ==> Zenitani 2018

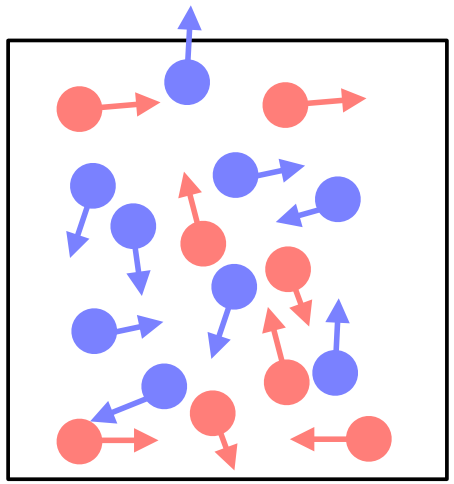
Loading relativistic particles

- Maxwell=Jüttner distribution

$$f(\boldsymbol{u})d^3u \propto \exp\left(-\frac{\gamma mc^2}{T}\right)d^3u$$

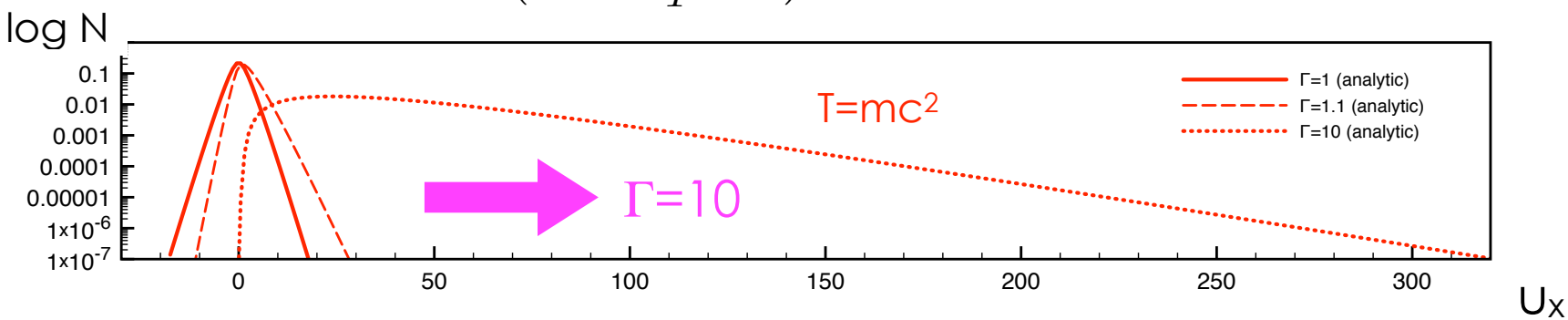






- Shifted Maxwell distribution

$$\propto \exp\left(-\frac{\Gamma(\gamma' - \beta u'_x)}{T}\right)d^3u'$$



Recent attempts

- Swisdak (2013) -- Two-step rejection method
- Melzani+ (2013) -- Cylindrical transformation + numerical table

Our strategy

- Stationary Maxwellian
  - Inverse transform method
  - Sobol (1976) method
- Lorentz boost → relativistic shifted Maxwellian
  - Rejection method - 50% efficiency for the stationary case
  - Flipping method - 100% efficiency

Modified Sobol algorithm

[Sobol 1976, Pozdnyakov+ 1983, Zenitani 2015]

repeat

generate X_1, X_2, X_3, X_4, uniform on (0, 1]

$u \leftarrow -T \ln X_1 X_2 X_3$

$\eta \leftarrow -T \ln X_1 X_2 X_3 X_4$

until $\eta^2 - u^2 > 1$.

generate X_5, X_6, X_7, uniform on [0, 1]

$u_x \leftarrow u\ (2X_5 - 1)$

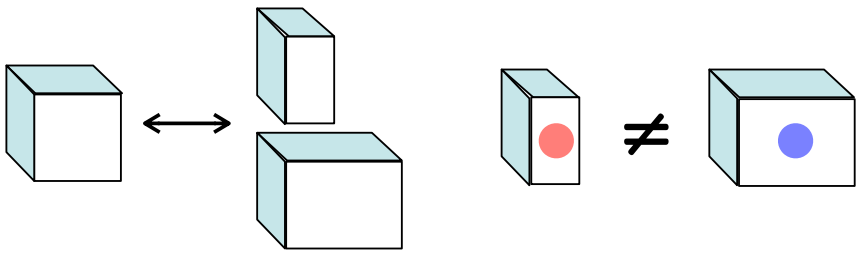
$u_y \leftarrow 2u\sqrt{X_5(1-X_5)}\cos(2\pi X_6)$

$u_z \leftarrow 2u\sqrt{X_5(1-X_5)}\sin(2\pi X_6)$

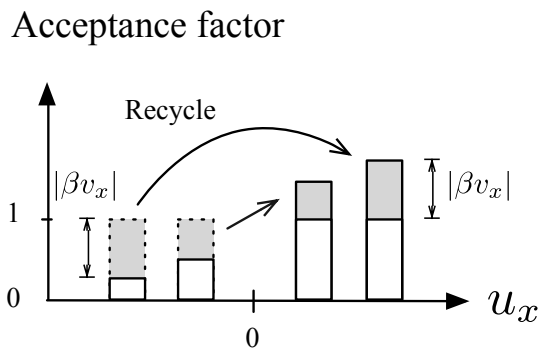
if $(-\beta v_x > X_7)$, $u_x \leftarrow -u_x$

$u_x \leftarrow \Gamma(u_x + \beta\sqrt{1+u^2})$

return u_x, u_y, u_z

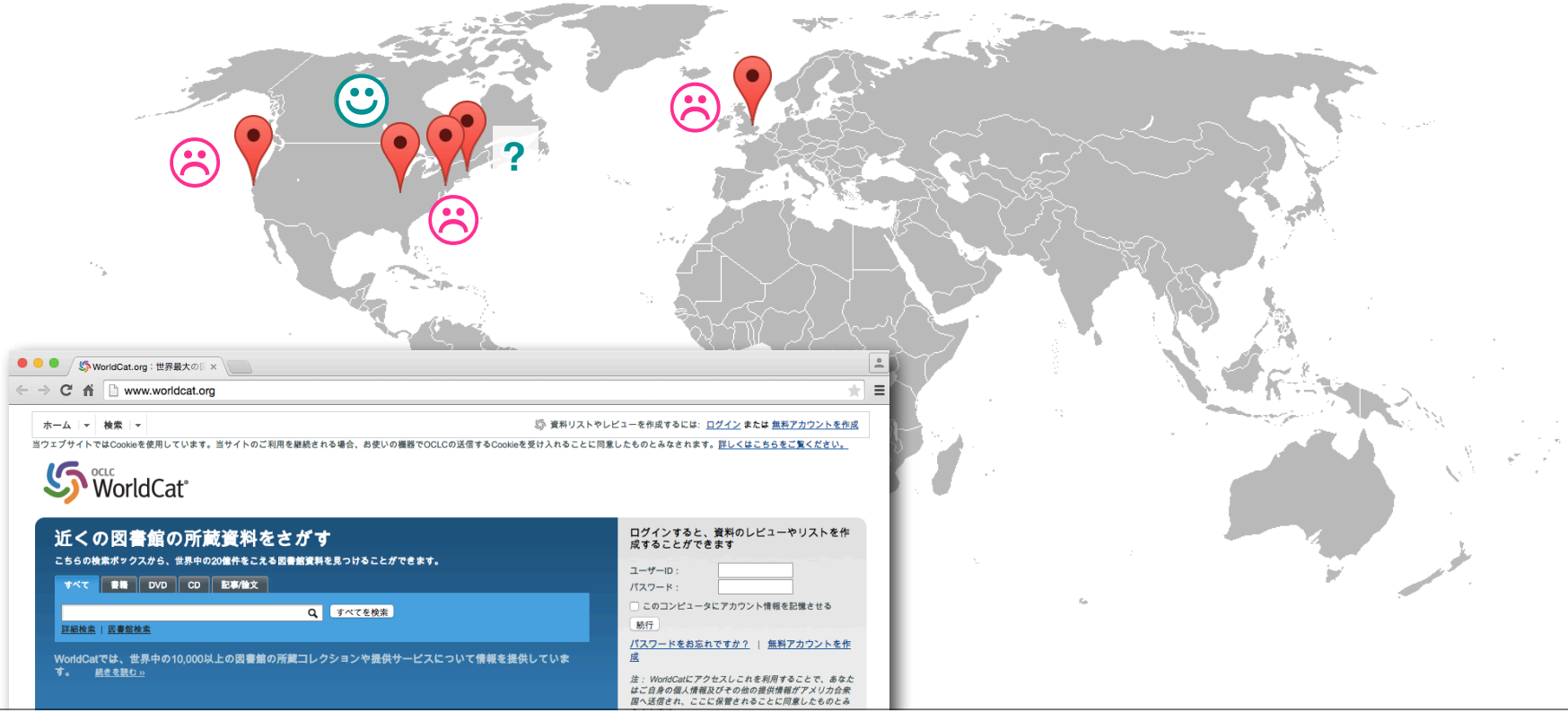


- Sobol method
  - Stationary Maxwellian
  - Reject some particles from 3rd-order Gamma distribution
- SZ's addition
  - Adjust particle density for Volume transform
  - Without this, we will see a big error (33%) in the energy flow



Quest for the original article

- OCLC/WorldCat database suggested 5 libraries
- We found it at U. Illinois Urbana-Champaign at the 4th attempt



Sobol (1976)'s article

?

И.М.Соболь

О МОДЕЛИРОВАНИИ НЕКОТОРЫХ РАСПРЕДЕЛЕНИЙ,
СХОДНЫХ С ГАММА-РАСПРЕДЕЛЕНИЕМ

Рассмотрим два нестандартных приема для моделирования случайных величин с плотностями вида

$$p(x) = B_n x^n e^{-b\sqrt{1+x^2}} \qquad (1)$$

и

$$p(x) = C_n x^n (e^{bx} - 1)^{-1}, \qquad (2)$$

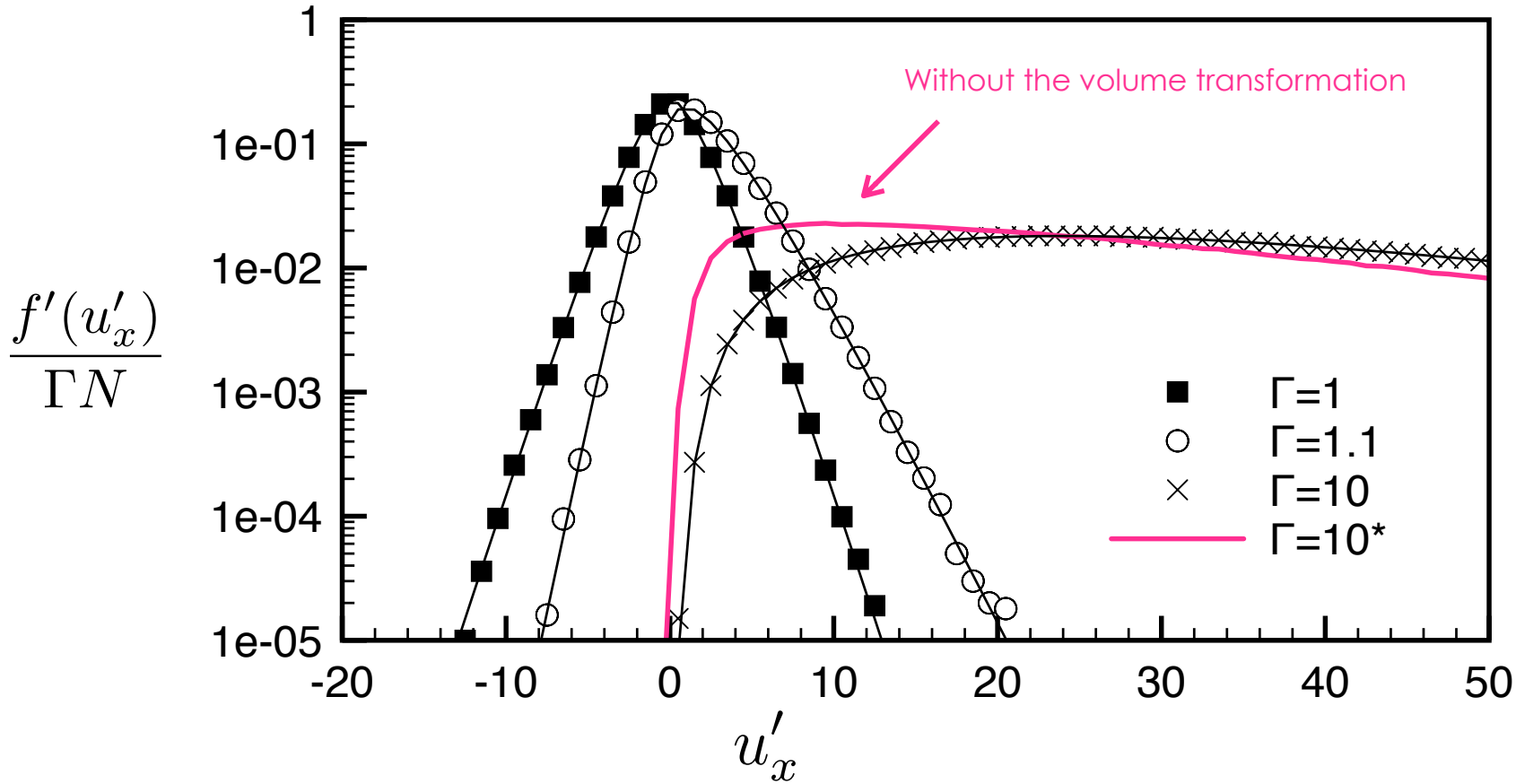
где $0 < x < \infty, b > 0$, а B_n и C_n – нормировочные постоянные. Эти приемы позволяют, в частности, моделировать импульсы релятивистских электронов и частоты фотонов; они использовались в работе [I].

- He did it right 40 years ago...

52. "Infinite-Dimensional Uniformly Distributed Sequences in Monte Carlo Algorithms," in *Monte Carlo Methods in Computational Mathematics and Mathematical Physics* (Novosibirsk, 1974), pp. 24–31 [in Russian].

59. "On Simulation of Certain Distributions Similar to Gamma Distribution," in *Monte Carlo Methods in Computational Mathematics and Mathematical Physics* (Novosibirsk, 1976), pp. 24–29 [in Russian].

Numerical test



- Excellent agreement between numerical results vs analytic curve
- Volume transform factor corrects energy flux by ~33%

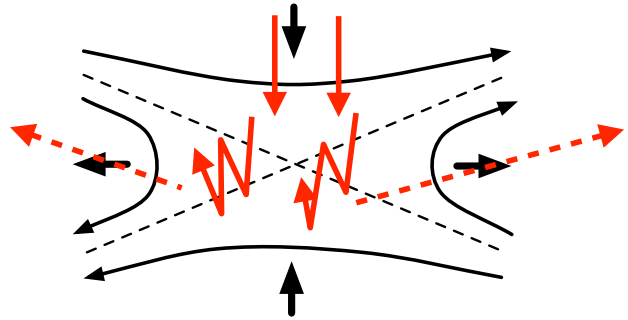
Ohm's law in a kinetic plasma

- Which term (& what physics) violates the ideal condition?

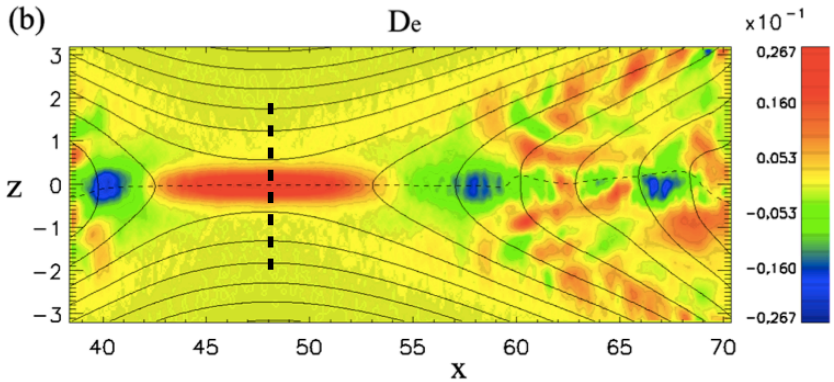
$$\boldsymbol{E} + \boldsymbol{v}_e \times \boldsymbol{B} = -\frac{1}{n_e q}\nabla \cdot \overleftrightarrow{P}_e - \frac{m_e}{q}\left(\frac{d\boldsymbol{v}_e}{dt}\right)$$

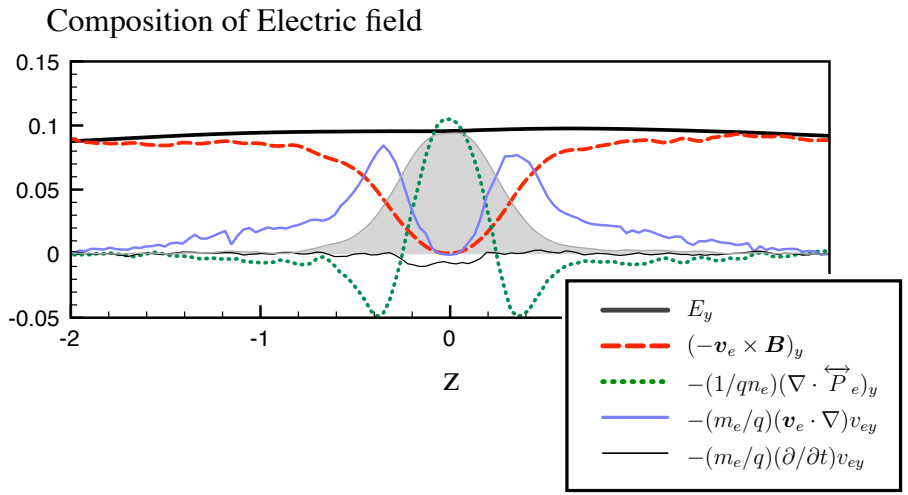
Thermal inertia
(Local momentum transport)
Hesse+ 1999, 2011

Bulk inertia



2D particle-in-cell (PIC) simulation





Zenitani+ 2011 PRL

Ohm's law in a relativistic kinetic plasma

- Stress-energy tensor

$$T^{\alpha\beta} = \int f(u) u^\alpha u^\beta \frac{d^3u}{\gamma}$$

$$\mathcal{E} \equiv T^{\alpha\beta} u_\alpha u_\beta$$ rest-frame energy

$$q^\alpha \equiv -\Delta^\alpha_\beta T^{\beta\gamma} u_\gamma$$ heat flow

$$P^{\alpha\beta} \equiv \Delta^\alpha_\gamma \Delta^\beta_\delta T^{\gamma\delta}$$ pressure tensor

$$\Delta^{\alpha\beta} = g^{\alpha\beta} + u^\alpha u^\beta$$ projection operator

$$Q^{\alpha\beta} \equiv q^\alpha u^\beta + q^\beta u^\alpha$$ heat flow tensor

- Eckart (1940) decomposition
  - See also Mihalas & Mihalas (1999)

$$T^{\alpha\beta} = \mathcal{E} u^\alpha u^\beta + \boxed{q^\alpha u^\beta + q^\beta u^\alpha} + P^{\alpha\beta}$$

- Energy momentum equation for relativistic plasmas

$$\partial_\beta T^{\alpha\beta}_{(e)} = -eF^{\alpha\beta} N_\beta = -enF^{\alpha\beta} u_\beta$$

- Relativistic Ohm's law (with ∂_t=0)

$$\vec{E} = -\vec{V} \times \vec{B} - \frac{1}{\gamma e n}\left(\partial_t T^{i0} + \nabla \cdot (\mathcal{E} u^i u^j + Q^{ij} + P^{ij})\right)$$

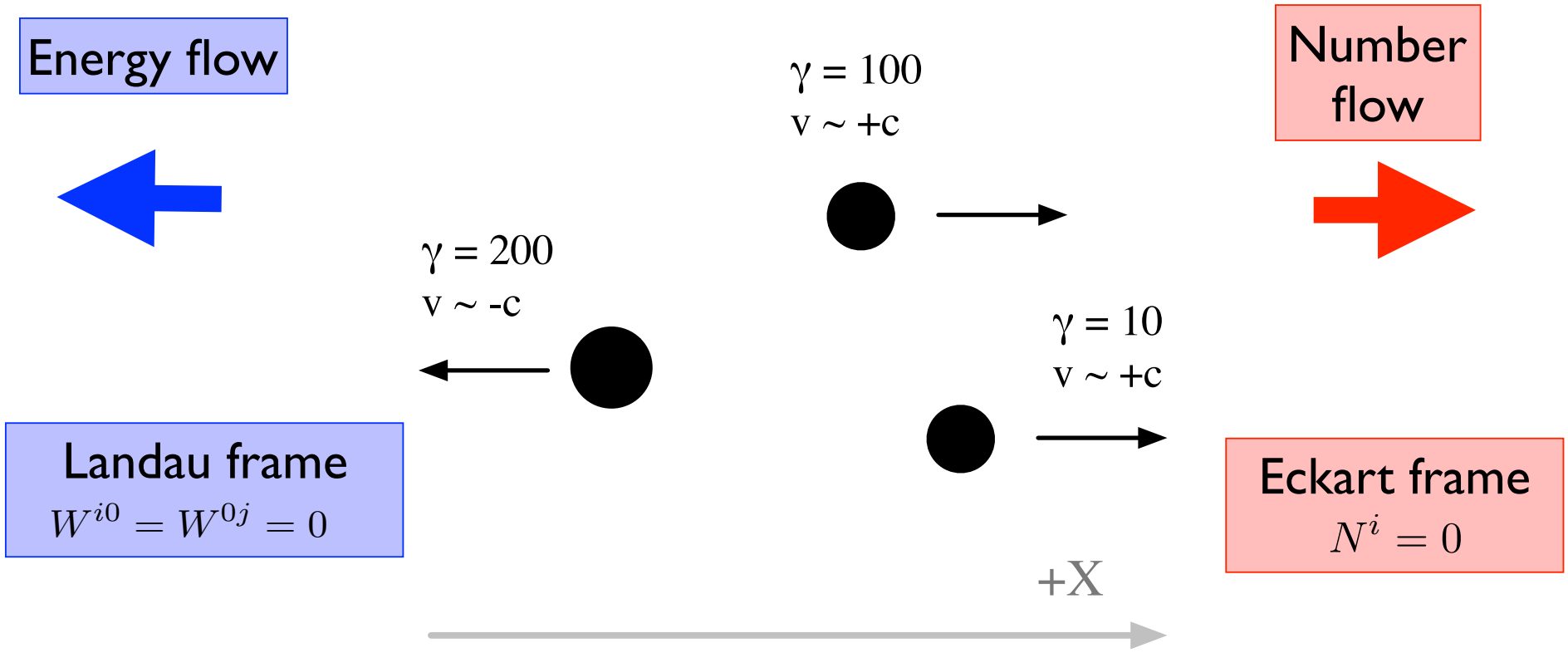
$$= -\vec{V} \times \vec{B} - \frac{1}{\gamma e n}\left(\partial_t(\gamma \mathcal{E} u^i + Q^{i0} + P^{i0}) + \nabla \cdot (\mathcal{E} u^i u^j + Q^{ij} + P^{ij})\right).$$

Heat flow inertia
(new in relativistic regime)

Thermal inertia
(Local momentum transport)

Bulk inertia
(including relativistic pressure)

What is the relativistic bulk velocity?

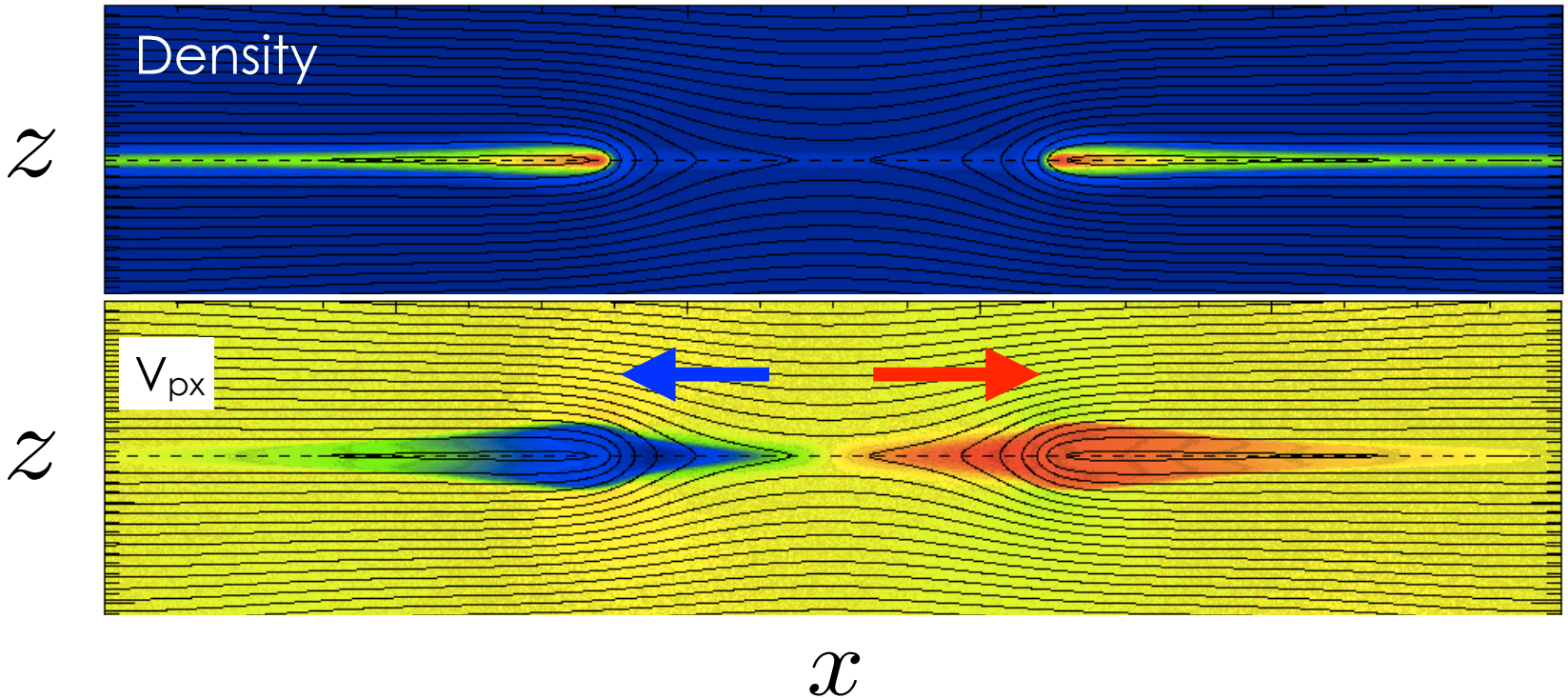


We set the bulk velocity to the Eckart velocity

$$u^{\mu}_{(E)} = N^{\mu}/\sqrt{-N^{\nu}N_{\nu}} = N^{\mu}/n$$

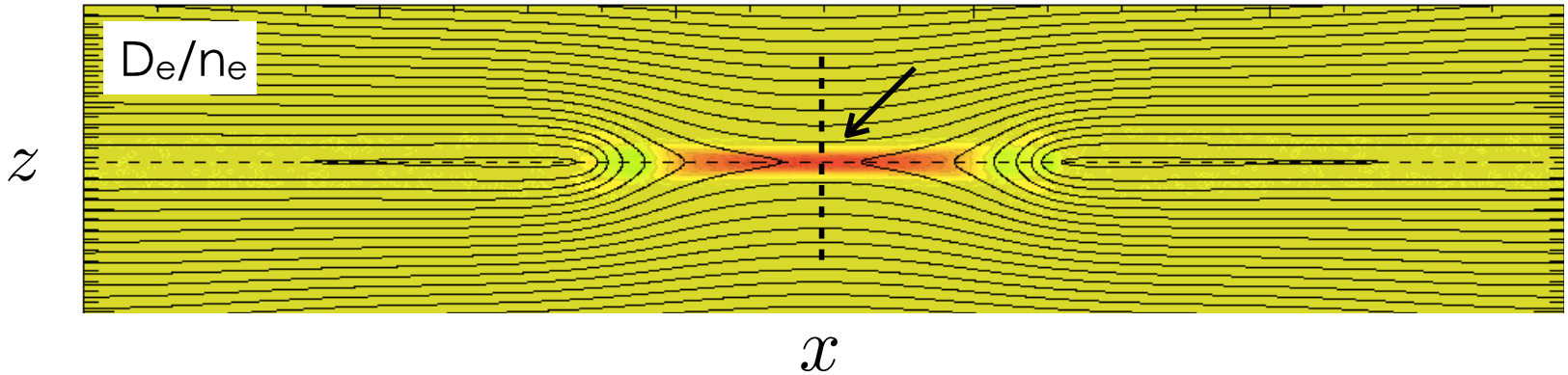
2D Particle-in-Cell simulation

- Relativistic electron-positron plasma
- $T/mc^2=1$, $n_{bg}/n_0=0.1$, $v_{drift}/c=0.3$
- $10^{9.5}$ particles: **10^4** pairs in a cell ($\Leftrightarrow$ 10^2 in typical works)

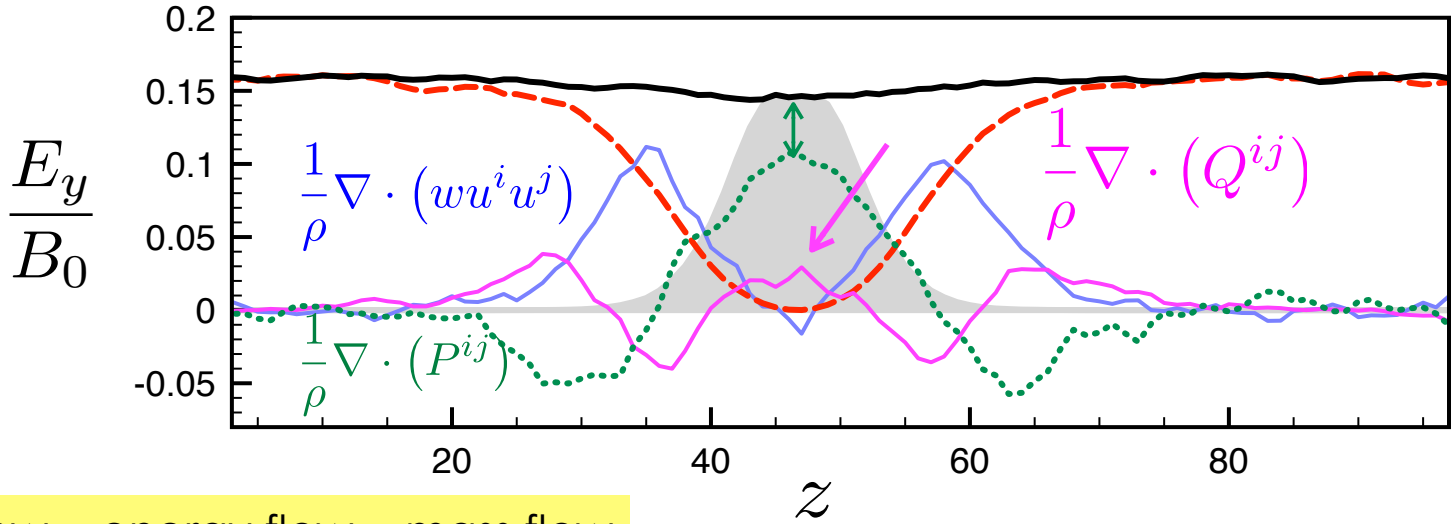


2D Particle-in-Cell simulation

- Normalized energy dissipation (~ j.E/n ~ $\eta j^2/n$)



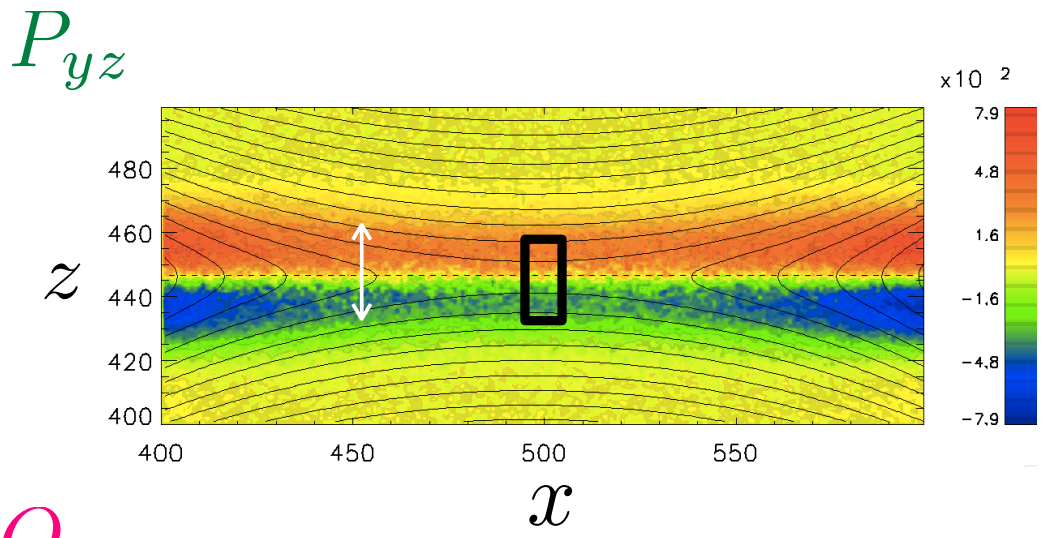
- Composition of Ohm's law: **heat flow term** appears

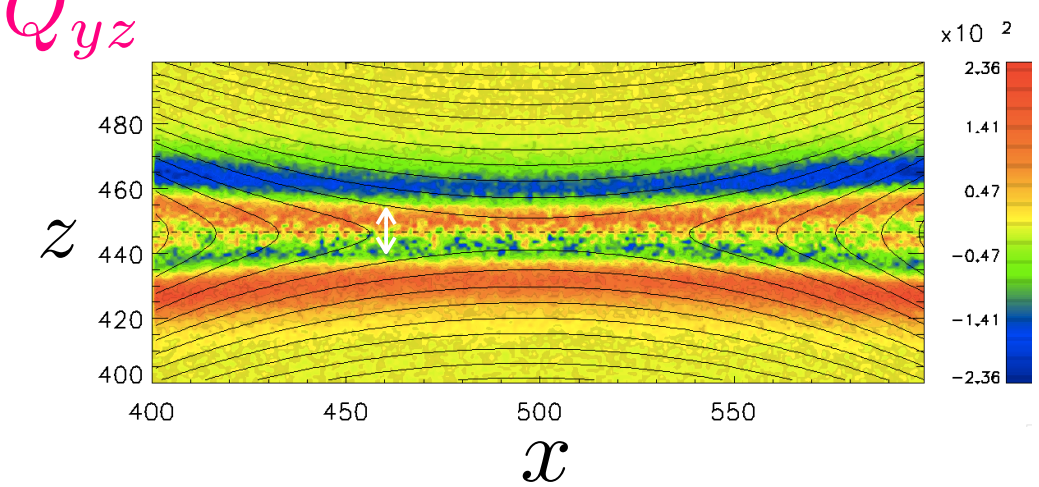


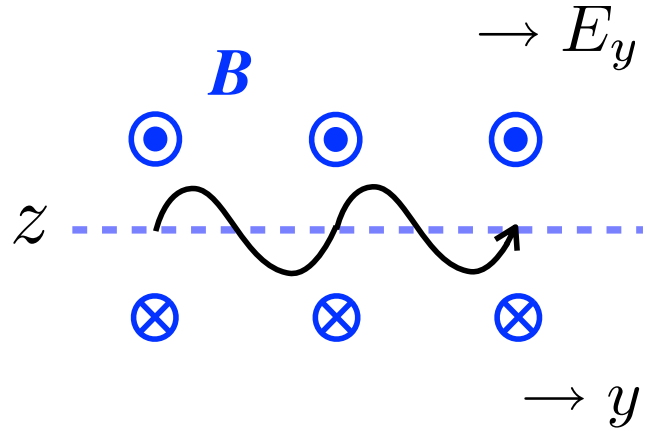
Heat flow = energy flow = mass flow

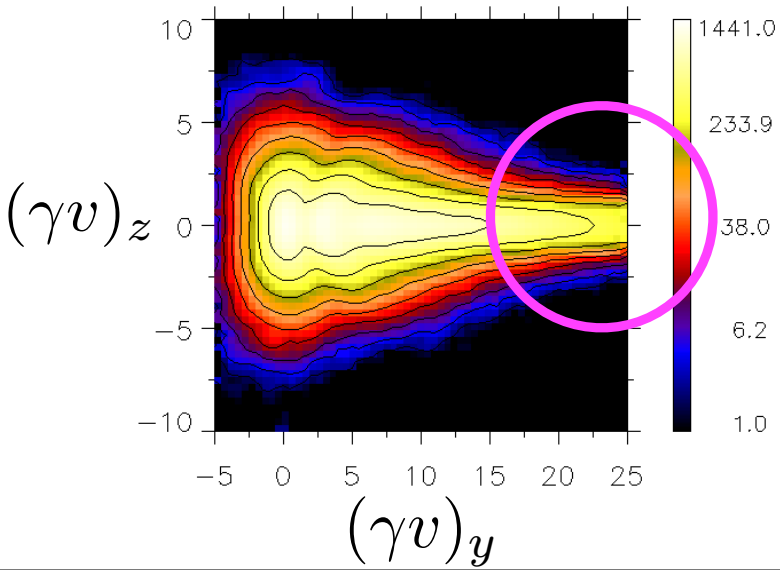
Momentum transport

- Strong particle acceleration gives rise to Q_{yz}
- Scale hight: Q is more confined than P









Summary

- Part 1: we have proposed basic algorithms to load relativistic velocity distributions
  - We reintroduced Sobol (1976) method.
  - We developed a volume-transform method.

- Part 2: we have analyzed kinetic Ohm's law in relativistic magnetic reconnection
  - We can evaluate relativistic fluid properties from the stress-energy tensor by using Eckart decomposition.
  - In addition to **thermal inertia**, new dissipation term (**heat flow inertia**) appears
  - They are related to energetic particles.