相対論的高速流プラズマの数値実験技法 (PICシミュレーション)

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Loading Relativistic Maxwell Distributions in Particle Simulations

Dissipation in relativistic pair-plasma reconnection: revisited
Three technical barriers in relativistic PIC simulations

• 1. Setup
  – Loading velocity distribution functions by using random variables
  – Not clearly documented ==> Zenitani 2015

• 2. Computation
  – Electromagnetic field (Haber 1974, Vay+ 2011)
  – Particle (Vay 2008)

• 3. Diagnosis & Interpretation
  – Definition of “fluid” frame and further decomposition
  – Not clearly documented ==> Zenitani 2018
Loading relativistic particles

- **Maxwell=Jüttner distribution**

\[ f(u) d^3u \propto \exp\left(-\frac{\gamma mc^2}{T}\right) d^3u \]

- **Shifted Maxwell distribution**

\[ \propto \exp\left(-\frac{\Gamma(\gamma' - \beta u'_x)}{T}\right) d^3u' \]
Recent attempts

• Swisdak (2013) -- Two-step rejection method
• Melzani+ (2013) -- Cylindrical transformation + numerical table

Our strategy

• Stationary Maxwellian
  – Inverse transform method
  – Sobol (1976) method
• Lorentz boost $\rightarrow$ relativistic shifted Maxwellian
  – Rejection method - 50% efficiency for the stationary case
  – Flipping method - 100% efficiency
Modified Sobol algorithm

```plaintext
repeat
    generate \( X_1, X_2, X_3, X_4 \), uniform on \((0, 1]\)
    \( u \leftarrow -T \ln X_1 X_2 X_3 \)
    \( \eta \leftarrow -T \ln X_1 X_2 X_3 X_4 \)
until \( \eta^2 - u^2 > 1 \).

generate \( X_5, X_6, X_7, \) uniform on \([0, 1]\)
\( u_x \leftarrow u \cdot (2X_5 - 1) \)
\( u_y \leftarrow 2u \sqrt{X_5(1 - X_5)} \cos(2\pi X_6) \)
\( u_z \leftarrow 2u \sqrt{X_5(1 - X_5)} \sin(2\pi X_6) \)

if \( (-\beta v_x > X_7) \), \( u_x \leftarrow -u_x \)
\( u_x \leftarrow \Gamma(u_x + \beta \sqrt{1 + u^2}) \)

return \( u_x, u_y, u_z \)
```

- **Sobol method**
  - Stationary Maxwellian
  - Reject some particles from 3rd-order Gamma distribution

- **SZ's addition**
  - Adjust particle density for Volume transform
  - Without this, we will see a big error (33%) in the energy flow

Acceptance factor

![Diagram](image-url)
Quest for the original article

• OCLC/WorldCat database suggested 5 libraries
• We found it at U. Illinois Urbana-Champaign at the 4th attempt
Sobol (1976)'s article

He did it right 40 years ago...


Without the volume transformation

- Excellent agreement between numerical results vs analytic curve
- Volume transform factor corrects energy flux by ~33%
Ohm’s law in a kinetic plasma

- Which term (& what physics) violates the ideal condition?

\[ E + v_e \times B = -\frac{1}{n_e q} \nabla \cdot \mathbf{P}_e - \frac{m_e}{q} \left( \frac{d v_e}{dt} \right) \]

- Thermal inertia (Local momentum transport) Hesse+ 1999, 2011
- Bulk inertia

2D particle-in-cell (PIC) simulation

Zenitani+ 2011 PRL
Ohm’s law in a relativistic kinetic plasma

- Stress-energy tensor
  \[ T^{\alpha\beta} = \int f(u) u^\alpha u^\beta \frac{d^3u}{\gamma} \]

- Eckart (1940) decomposition
  - See also Mihalas & Mihalas (1999)
  \[ T^{\alpha\beta} = \mathcal{E} u^\alpha u^\beta + q^\alpha u^\beta + q^\beta u^\alpha + P^{\alpha\beta} \]

- Energy momentum equation for relativistic plasmas
  \[ \partial_\beta T^{\alpha\beta}_{(e)} = -eF^{\alpha\beta} N_\beta = -enF^{\alpha\beta} u_\beta \]

- Relativistic Ohm’s law (with \( \partial_t = 0 \))
  \[ \mathbf{E} = -\mathbf{V} \times \mathbf{B} - \frac{1}{\gamma en} \left( \partial_t T^{i0} + \nabla \cdot (\mathcal{E} u^i u^j + Q^{ij} + P^{ij}) \right) \]
  \[ = -\mathbf{V} \times \mathbf{B} - \frac{1}{\gamma en} \left( \partial_t (\gamma \mathcal{E} u^i + Q^{i0} + P^{i0}) + \nabla \cdot (\mathcal{E} u^i u^j + Q^{ij} + P^{ij}) \right) \]
What is the relativistic bulk velocity?

We set the bulk velocity to the Eckart velocity

\[ u^\mu_{(E)} = \frac{N^\mu}{\sqrt{-N^\nu N_\nu}} = \frac{N^\mu}{n} \]
2D Particle-in-Cell simulation

- Relativistic electron-positron plasma
- $T/mc^2=1$, $n_{bg}/n_0=0.1$, $v_{\text{drift}}/c=0.3$
- $10^{9.5}$ particles: $10^4$ pairs in a cell ($\Leftrightarrow 10^2$ in typical works)
2D Particle-in-Cell simulation

- Normalized energy dissipation ($\sim j.E/n \sim j^2/n$)

\[ \frac{D_e}{n_e} \]

- Composition of Ohm’s law: **heat flow term** appears

Heat flow = energy flow = mass flow
Momentum transport

- Strong particle acceleration gives rise to $Q_{yz}$
- Scale height: $Q$ is more confined than $P$
Summary

• Part 1: we have proposed basic algorithms to load relativistic velocity distributions
  – We reintroduced Sobol (1976) method.
  – We developed a volume-transform method.

• Part 2: we have analyzed kinetic Ohm’s law in relativistic magnetic reconnection
  – We can evaluate relativistic fluid properties from the stress-energy tensor by using Eckart decomposition.
  – In addition to thermal inertia, new dissipation term (heat flow inertia) appears
  – They are related to energetic particles.