Neutron Matter, the Maximum Mass, and Neutron Star Radii

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The Importance of Neutron Stars

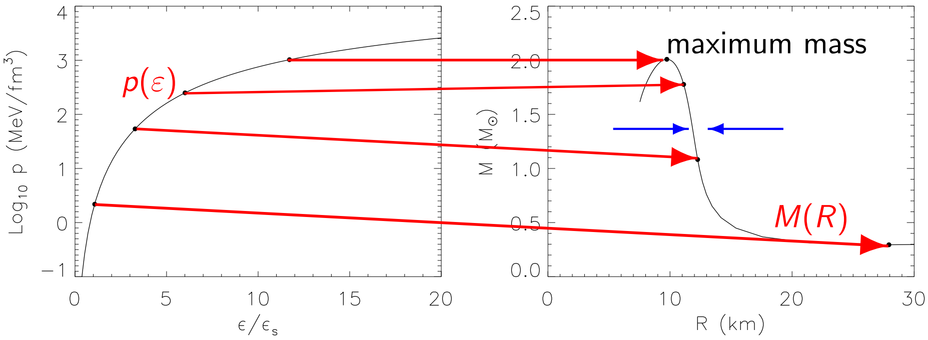
- Masses and Radii and the Dense Matter Equation of State
  - The neutron star maximum mass constrains the high-density EOS and also neutron star radii.
  - The radius of typical stars constrains the nuclear symmetry energy, the EOS between 1-3 ρ_s, and the maximum mass.
  - Universal relations relate binding energy, moment of inertia and tidal deformability to M/R.
- Symmetry Energy Constraints
  - Nuclear experiments (masses, skin thicknesses, resonances, HIC)
  - Neutron matter rheory
- Astrophysical M and R Measurements
  - pulsar timing, X-ray bursts, quiescent low-mass X-ray binaries, pulse profiles from ms pulsars, and gravitational radiation from mergers.
- NS-NS and NS-BH Mergers
  - perhaps the dominant sources of gravitational wave signals, short gamma-ray bursts, and kilonovae or macronovae;
  - perhaps the primary nucleosynthesis site of r-process heavy elements.
- Protoneutron Stars and Neutrino Signals from Supernovae
- The QCD Phase Transition
- Composition and Pairing Properties from Cooling Histories

Neutron Star Structure 101

Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G}{c^4}\frac{(mc^2+4\pi pr^3)(\varepsilon+p)}{r(r-2Gm/c^2)}$$

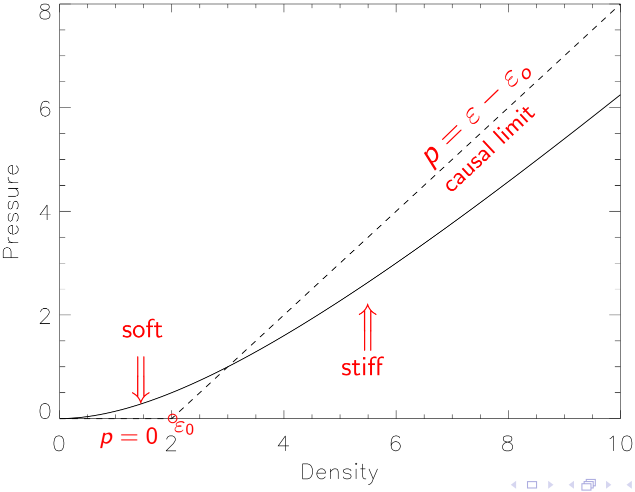
$$\frac{dm}{dr} = 4\pi\frac{\varepsilon}{c^2}r^2$$



Equation of State ← Observations

Extremes of Mass and Compactness of Neutron Stars

- The most compact and massive configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff" (Koranda, Stergioulas & Friedman 1997).



ε_0 is the only EOS parameter

The TOV solutions scale with ε_0

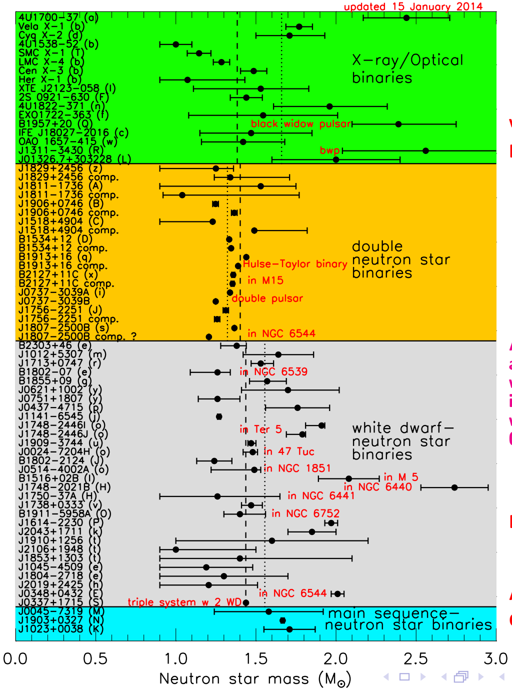
$w = \varepsilon/\varepsilon_0$
$y = p/\varepsilon_0$
$x = r\sqrt{G\varepsilon_0}/c^2$
$z = m\sqrt{G^3\varepsilon_0}/c^2$

Extremal Properties of Neutron Stars

The maximum mass configuration is achieved when
$x_R = 0.2404$, $w_c = 3.034$, $y_c = 2.034$, $z_R = 0.08513$.

A useful reference density is the nuclear saturation density (interior density of normal nuclei):
$\rho_s = 2.7 \times 10^{14}$ g cm^{-3}, $n_s = 0.16$ baryons fm^{-3}, $\varepsilon_s = 150$ MeV fm^{-3}

- $M_{\rm max} = 4.1\ (\varepsilon_s/\varepsilon_0)^{1/2} M_\odot$ (Rhoades & Ruffini 1974)
- $M_{B,\rm max} = 5.41\ (m_B c^2/\mu_o)(\varepsilon_s/\varepsilon_0)^{1/2} M_\odot$
- $R_{\rm min} = 2.82\ GM/c^2 = 4.3\ (M/M_\odot)$ km
- $\mu_{b,\rm max} = 2.09$ GeV
- $\varepsilon_{c,\rm max} = 3.034\ \varepsilon_0 \simeq 51\ (M_\odot/M_{\rm largest})^2\ \varepsilon_s$
- $p_{c,\rm max} = 2.034\ \varepsilon_0 \simeq 34\ (M_\odot/M_{\rm largest})^2\ \varepsilon_s$
- $n_{B,\rm max} \simeq 38\ (M_\odot/M_{\rm largest})^2\ n_s$
- $\mathrm{BE}_{\rm max} = 0.34\ M$
- $P_{\rm min} = 0.74\ (M_\odot/M_{\rm sph})^{1/2}(R_{\rm sph}/10\ {\rm km})^{3/2}\ {\rm ms} = 0.20\ (M_{\rm sph,max}/M_\odot)\ {\rm ms}$



vanKerkwijk 2010

Romani et al. 2012

Although simple average mass of w.d. companions is 0.23 $M_\odot$ larger, weighted average is 0.04 $M_\odot$ smaller

Demorest et al. 2010

Antoniadis et al. 2013

Champion et al. 2008

What is the Maximum Mass?

- PSR J1614+2230 (Demorest et al. 2010) $M = 1.97 \pm 0.04\ M_\odot$; a nearly edge-on system with well-measured Shapiro time delay.
- PSRJ0548+0432 (Antoniadis et al. 2013) $M = 2.01 \pm 0.04\ M_\odot$; measured using optical data and theoretical properties of companion white dwarf.
- B1957+20 (van Kerkwijk 2010) $M = 2.4 \pm 0.3\ M_\odot$; black widow pulsar (BWP).
- PSR J1311-3430 (Romani et al. 2012) $M = 2.55 \pm 0.50\ M_\odot$; BWP.
- PSR J1544+4937 (Tang et al. 2014) $M = 2.06 \pm 0.56 M_\odot$; BWP.
- PSR 2FGL J1653.6-0159 (Romani et al. 2014) $M > f(M_2)/\sin^3 i \gtrsim 1.96 M_\odot$ [largest $f(M_2)$]; BWP.
- PSR J1227-4859 (de Martino et al. 2014) $M = 2.2 \pm 0.8 M_\odot$; redback pulsar.

Causality + GR Limits and the Maximum Mass

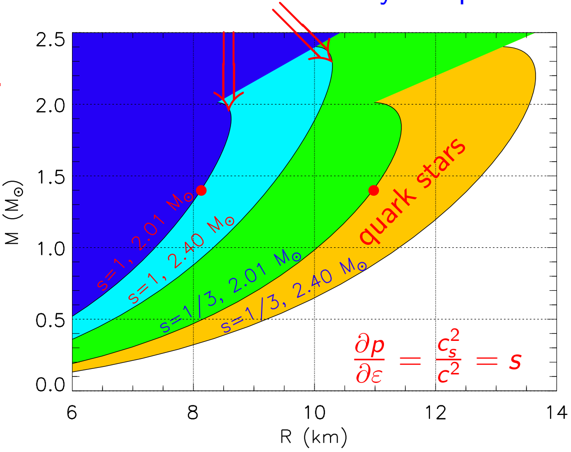
A lower limit to the maximum mass sets a lower limit to the radius for a given mass.

Similarly, a precise (M, R) measurement sets an upper limit to the maximum mass.

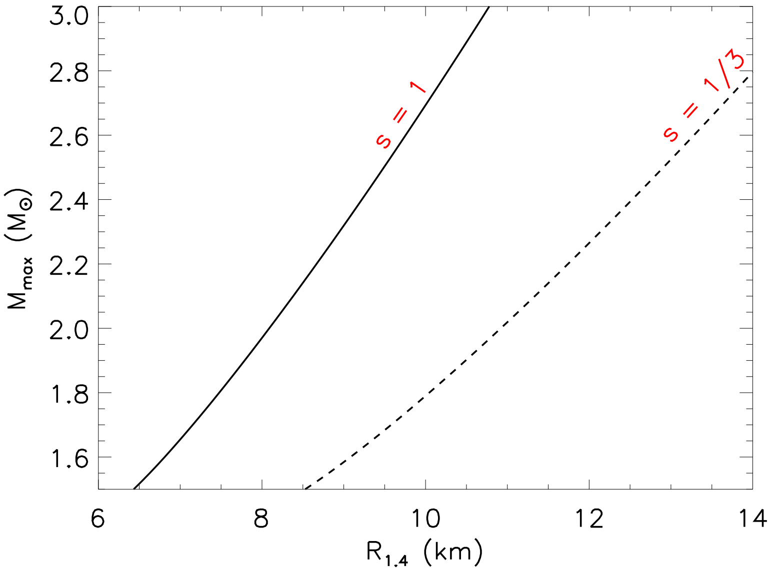
$1.4 M_\odot$ stars must have $R > 8.15 M_\odot$.

$1.4 M_\odot$ strange quark matter stars (and likely hybrid quark/hadron stars) must have $R > 11$ km.

$M - R$ curves for maximally compact EOS



Maximum Mass and Neutron Star Radii



What About Realistic EOSs?

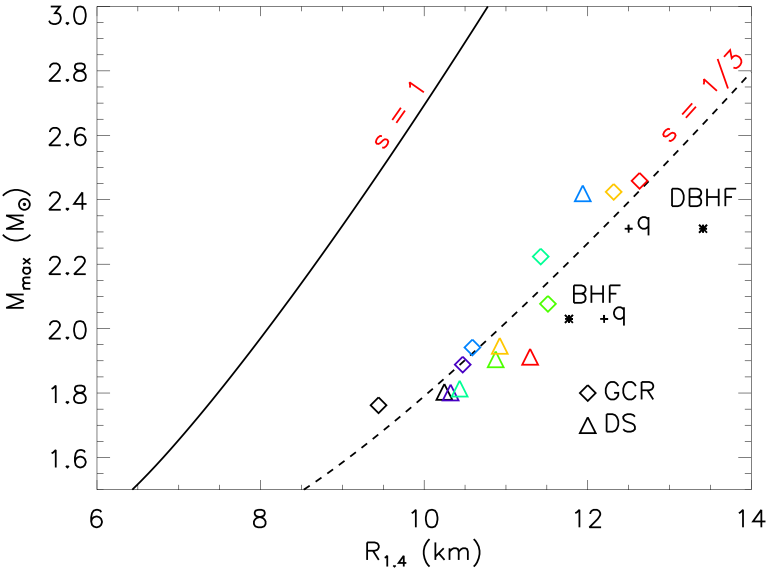
It has been proposed that the physically plausible sound speed limit is $c/\sqrt{3}$ (Bedaque & Steiner 2015), in which case $1.4M_\odot$ stars must have $R_{1.4} > 11$ km.

Quark matter generically has a sound speed $\lesssim c/\sqrt{3}$. Hybrid quark/hadron stars are 0.5–2 km larger than the $c/\sqrt{3}$ limit (Alford et al. 2015).

Additional constraints are imposed by our knowledge of the low-density equation of state.

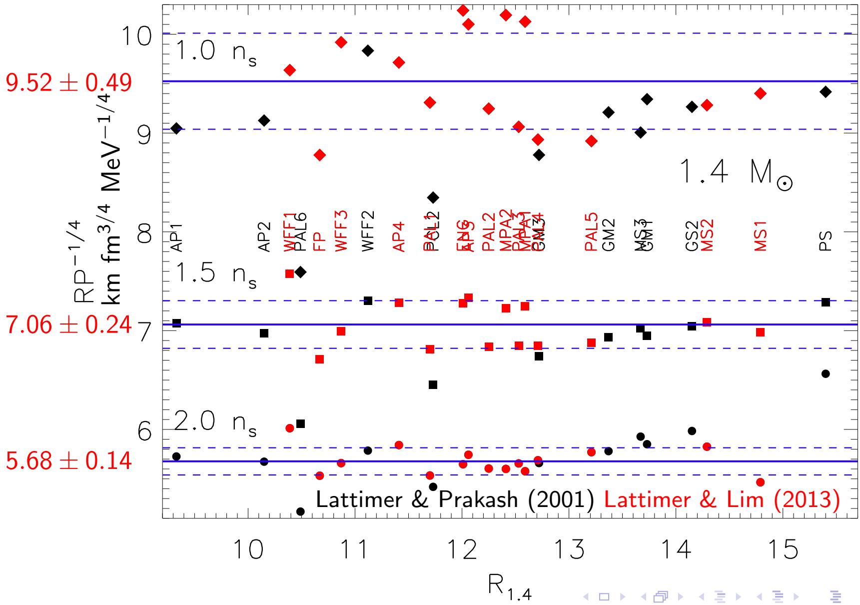
- A nuclear crust exists below about $\rho_s/3 - \rho_s/2$, with an EOS largely independent of nuclear properties.
- Chiral-Lagrangian neutron matter calculations (Hebeler & Schwenk (2010), Hebeler et al. (2010, 2013), Drischler, Somá & Schwenk (2014), $\rho \lesssim 1.25\rho_s$ (?).
- Quantum Monte Carlo neutron matter calculations (Gandolfi, Carlson & Reddy (2012), $\rho \lesssim 2\rho_s$ (?).

$M_{max} - R_{1.4}$



3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
M_{max} ($M_\odot$)
6
8
10
12
14
$R_{1.4}$ (km)
s = 1
s = 1/3
DBHF
q
BHF
q
GCR
DS

The Radius – Pressure Correlation



Nuclear Symmetry Energy and Pressure

Defined as the difference between energies of pure neutron matter ($x=0$) and symmetric ($x=1/2$) nuclear matter.

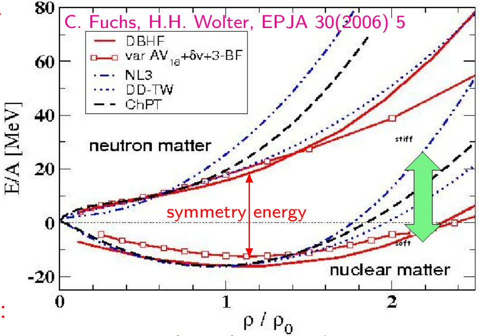
$$S(\rho) = E(\rho, x=0) - E(\rho, x=1/2)$$

Expanding around the saturation density (ρ_s) and symmetric matter ($x=1/2$)

$$E(\rho, x) = E(\rho, 1/2) + (1-2x)^2 S_2(\rho) + \ldots$$

$$S_2(\rho) = \mathbf{S_v} + \frac{\mathbf{L}}{3}\frac{\rho-\rho_s}{\rho_s} + \ldots$$

$$\mathbf{S_v} \simeq 31 \text{ MeV}, \quad \mathbf{L} \simeq 50 \text{ MeV}$$



Connections to pure neutron matter:

$$E(\rho_s, 0) \approx S_v + E(\rho_s, 1/2) \equiv S_v - B, \qquad p(\rho_s, 0) = L\rho_s/3$$

Neutron star matter (in beta equilibrium):

$$\frac{\partial(E+E_e)}{\partial x} = 0, \quad p(\rho_s, x_\beta) \simeq \frac{L\rho_s}{3}\left[1 - \left(\frac{4S_v}{\hbar c}\right)^3 \frac{4-3S_v/L}{3\pi^2\rho_s}\right]$$

Nuclear Structure 101

Liquid Droplet Model (Myers and Swiatecki)

$$\frac{E_{A,Z}}{A} = -B + E_{s0}A^{-1/3} + E_c\left(\frac{Z}{A}\right)^2 A^{2/3} + \frac{S_v I^2}{1 + \frac{S_s}{S_v}A^{-1/3}}$$

Ignoring Coulomb effects:

$$E_{sym} = S_v A I^2 \left[1 + \frac{S_s}{S_v}A^{-1/3}\right]^{-1} \Rightarrow \frac{S_s}{S_v} \propto S_v,$$

$$\alpha_D = \frac{r_o^2 A^{5/3}}{20 S_v}\left[1 + \frac{5}{3}\frac{S_s}{S_v}A^{-1/3}\right] \Rightarrow \frac{S_s}{S_v} \propto S_v,$$

$$r_{np} = \sqrt{\frac{3}{5}}\frac{2r_o}{3}\frac{S_s}{S_v} I \left[1 + \frac{S_s}{S_v}A^{-1/3}\right]^{-1} \Rightarrow \frac{S_s}{S_v} \propto \text{ const.}$$

$$\frac{S_s}{S_v} \propto \int_{-\infty}^{\infty} \rho \left[\frac{S_v}{S_2(\rho)} - 1\right] dx \approx f(S_v, L) \propto L$$

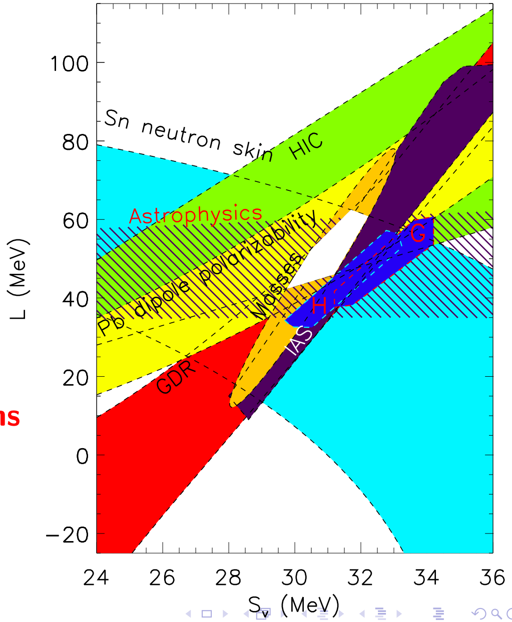
Experimental and Neutron Matter Constraints

Nuclear Experiment

Masses:
  Kortelainen et al. 2010
Neutron skin
  Chen et al. 2010
Pb dipole polarizability
  Tamii et al. 2008
  Piekarewicz et al. 2012
Giant dipole resonance
  Trippa et al. 2008
Heavy ion collisions
  Tsang et al. 2009
Isobaric analog states
  Danielewicz & Lee 2009

Neutron Matter Calculations

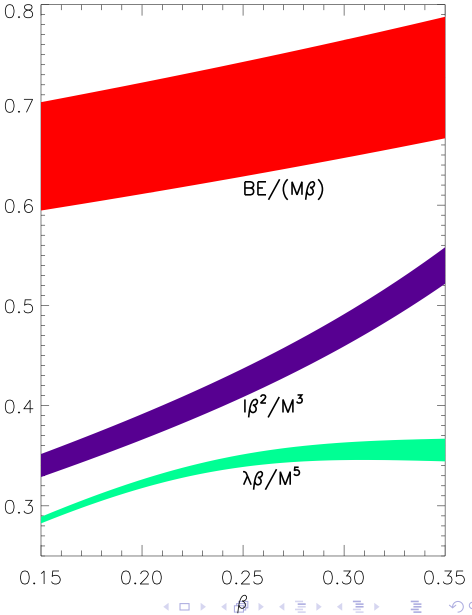
H: Chiral Lagrangian
  Hebeler & Schwenk 2010
G: Quantum Monte Carlo
  Gandolfi, Carlson & Reddy 2012
$S_v - L$ constraints:
  Hebeler et al. 2012



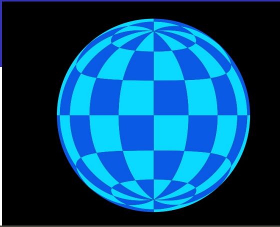
Other Universal Relations

$$\beta = \frac{GM}{Rc^2}$$

- Binding Energy
  $$\frac{\mathrm{BE}}{M} = \frac{(M_b - M)c^2}{M} = b(\beta)$$
- Moment of Inertia
  $$\frac{I}{M^3} = i(\beta)$$
- Tidal Love Number
  $$\frac{\lambda}{M^5} = k(\beta)$$

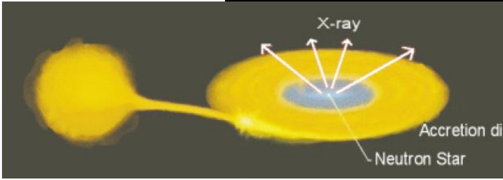


Simultaneous Mass/Radius Measurements



- Measurements of flux $F_\infty = (R_\infty/D)^2 \sigma T_{\rm eff}^4$ and color temperature $T_c \propto \lambda_{\rm max}^{-1}$ yield an apparent angular size (pseudo-BB):

$$\frac{R_\infty}{D} = \frac{R}{D}\frac{1}{\sqrt{1-2GM/Rc^2}}$$



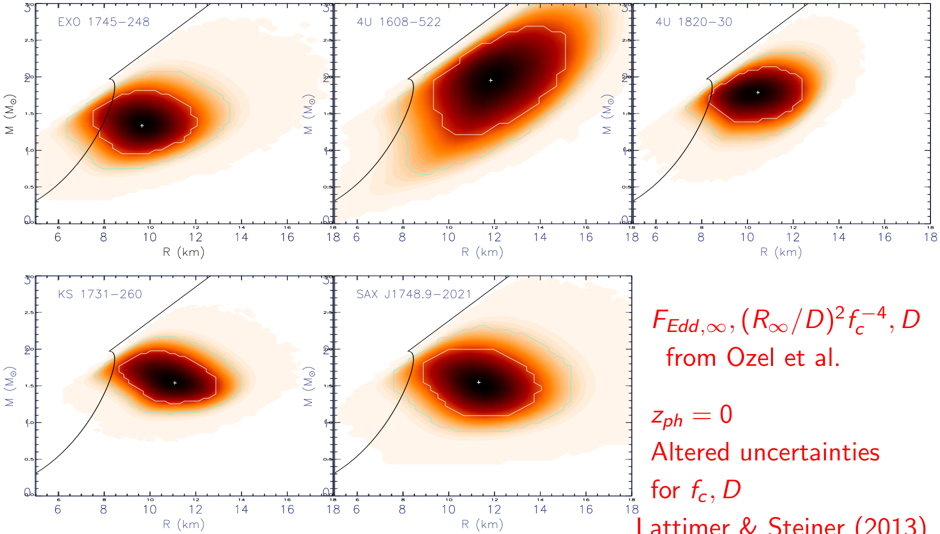
- Observational uncertainties include distance D, interstellar absorption N_H, atmospheric composition

Best chances for accurate radius measurement:

- Nearby isolated neutron stars with parallax (uncertain atmosphere)
- Quiescent low-mass X-ray binaries (QLMXBs) in globular clusters (reliable distances, low B H-atmosperes)
- Bursting sources (XRBs) with peak fluxes close to Eddington limit (where gravity balances radiation pressure)

$$F_{\rm Edd} = \frac{cGM}{\kappa D^2}\sqrt{1-2GM/Rc^2}$$

$M - R$ PRE Burst Estimates



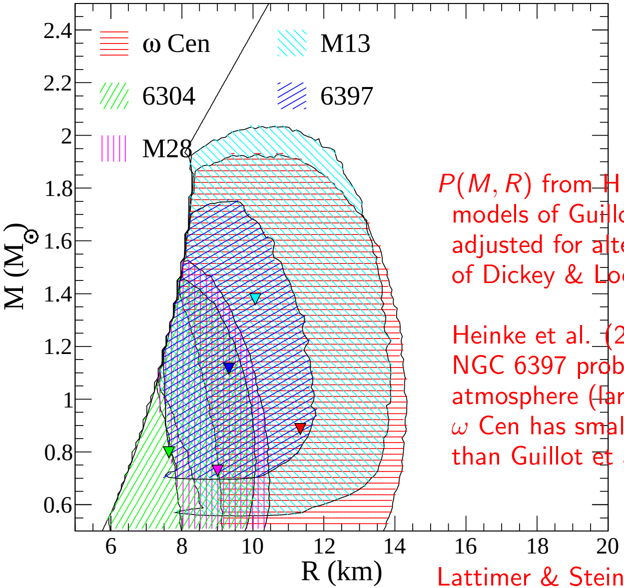
$F_{Edd,\infty}, (R_\infty/D)^2 f_c^{-4}, D$ from Ozel et al.

$z_{ph} = 0$

Altered uncertainties for f_c, D

Lattimer & Steiner (2013)

$M - R$ QLMXB Estimates



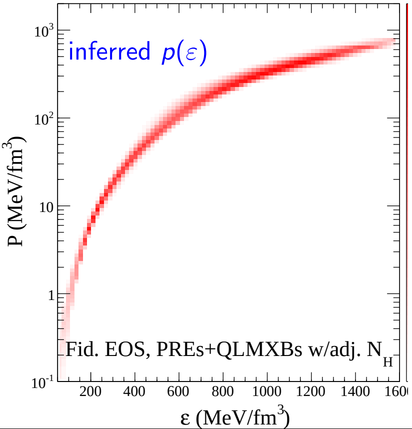
$P(M, R)$ from H atmosphere models of Guillot et al. (2013), adjusted for alternate N_H values of Dickey & Lockman (1990).

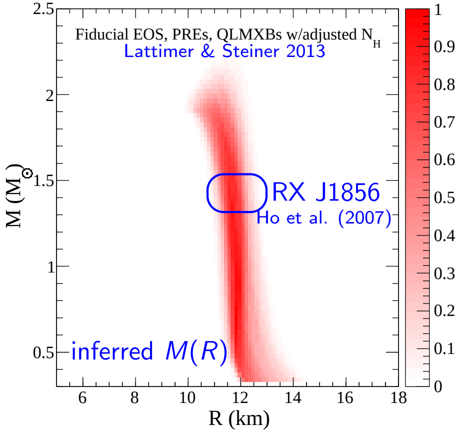
Heinke et al. (2014) found NGC 6397 probably has He atmosphere (larger R); ω Cen has smaller N_H (and R) than Guillot et al. (2013) found.

Lattimer & Steiner (2013)

Bayesian TOV Inversion

- $\varepsilon < 0.5\varepsilon_0$: Known crustal EOS
- $0.5\varepsilon_0 < \varepsilon < \varepsilon_1$: EOS parametrized by K, K', S_v, γ
- Polytropic EOS: $\varepsilon_1 < \varepsilon < \varepsilon_2$: n_1; $\varepsilon > \varepsilon_2$: n_2
- EOS parameters $K, K', S_v, \gamma, \varepsilon_1, n_1, \varepsilon_2, n_2$ uniformly distributed
- $M_{\max} \geq 1.97$ M$_\odot$, causality enforced
- All 10 stars equally weighted





Astronomy vs. Astronomy vs. Physics

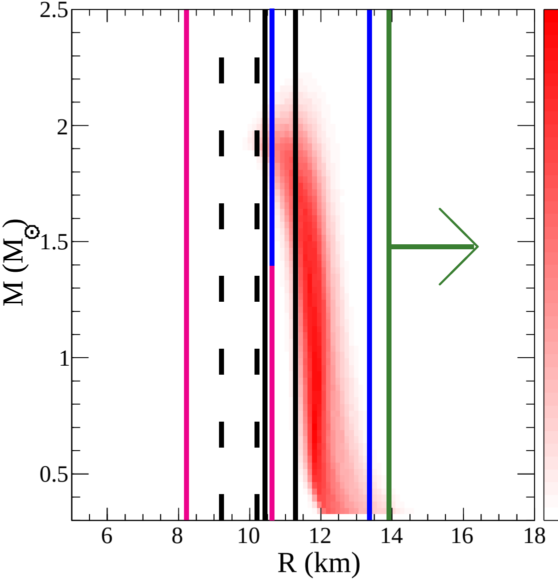
Ozel et al., XRB $z_{ph} = z$:
$R = 9.7 \pm 0.5$ km (2009-14)
XRB+QLMXB:
$R = 10.8^{+0.5}_{-0.4}$ km (2015).

Suleimanov et al., long XRB: $R_{1.4} \gtrsim 13.9$ km

Guillot & Rutledge (2014), QLMXB, equal radii stars, self N_H: $R = 9.4 \pm 1.2$ km.

Lattimer & Steiner (2013), XRB+QLMXB, TOV, crust, causality, $M_{max} > 2M_\odot$, $z_{\rm ph} \neq z$, alt N_H.

Lattimer & Lim (2013), nuclear experiments:
29 MeV $< S_v <$ 33 MeV,
40 MeV $< L <$ 65 MeV:
$R_{1.4} = 12.0 \pm 1.4$ km.



Future Astrophysical Prospects

Spin-Orbit Coupling in Relativistic Binaries

$$\dot{\mathbf{S}}_\mathbf{A} = \frac{7G}{2a^3c^2}\mathbf{L} \times \mathbf{S}_\mathbf{A}$$

$$\dot{\mathbf{L}} = \frac{7G}{2a^3c^2}\left(\mathbf{S}_\mathbf{A} - 3\mathbf{L}\frac{\mathbf{L}\cdot\mathbf{S}}{|\mathbf{L}|^2}\right)$$

- Change in inclination angle i:

$$\delta t_i = \frac{I_A}{aM_Ac}\frac{P}{P_A}\sin\theta_A\cos i \sim 0.1\ \mu\mathrm{s}$$

- Change in periastron advance (observability proportional to $\sin i$):

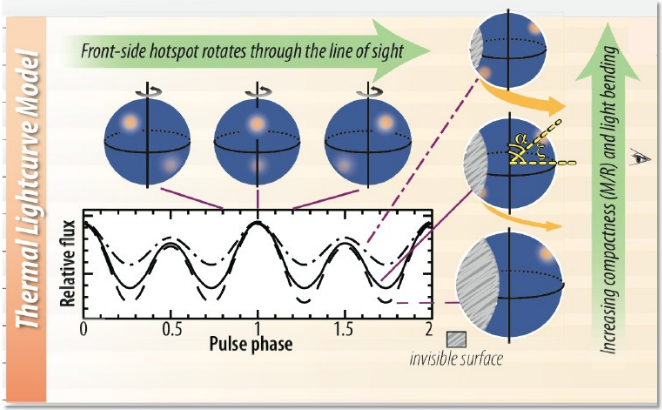
$$\frac{\delta t_{p,SO}}{\delta t_{p,2PN}} = \frac{2c^2I_A}{GM_A^2a}\frac{P}{P_A}\cos\theta_A \sim 3$$

PSR J0737-3039: $i \simeq 88^\circ \pm 0.5^\circ$, $\theta_A \simeq 13^\circ \pm 10^\circ$

Science Measurements

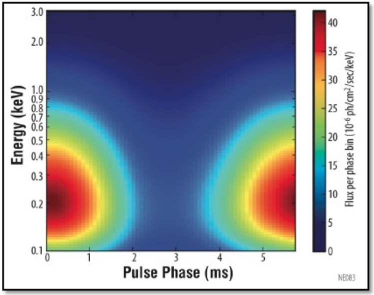
NICER

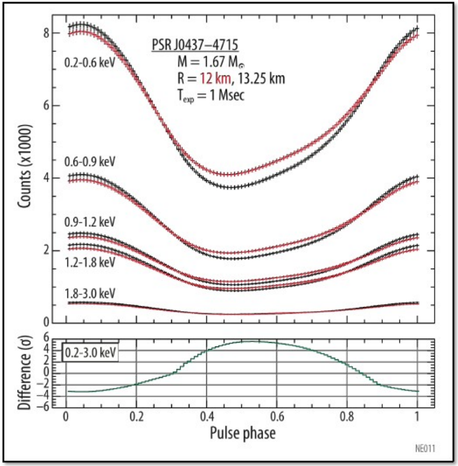
Reveal stellar structure through lightcurve modeling, long-term timing, and pulsation searches



Lightcurve modeling constrains the compactness (M/R) and viewing geometry of a non-accreting millisecond pulsar through the depth of modulation and harmonic content of emission from rotating hot-spots, thanks to gravitational light-bending…

Science Measurements (cont.) NICER



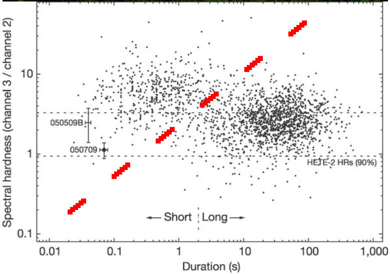


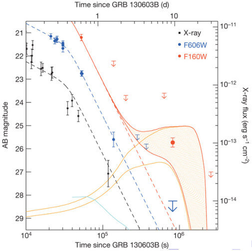
... while phase-resolved spectroscopy promises a direct constraint of radius *R*.

Compact Binary Mergers

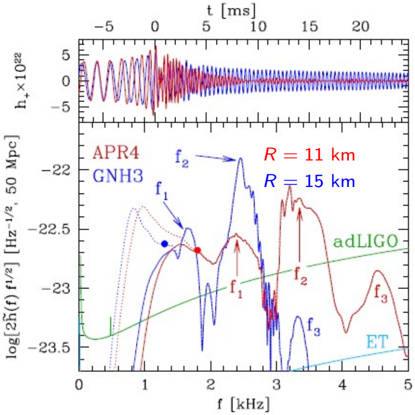
Neutron star–neutron star or black hole–neutron star mergers

- Strongest sources of gravitational waves observable with aLIGO
- Leading candidates for short-hard gamma-ray bursts (10% of all gamma-ray bursts)
- Eject 0.001–0.01 $M_\odot$ of neutron-rich matter
    - R-process nucleosynthesis
    - Infrared afterglows observed 10-15 days after short gamma-ray bursts, due to thermalization of r-process γ-rays in high-opacity actinides (GRBs 130603B and 060614)

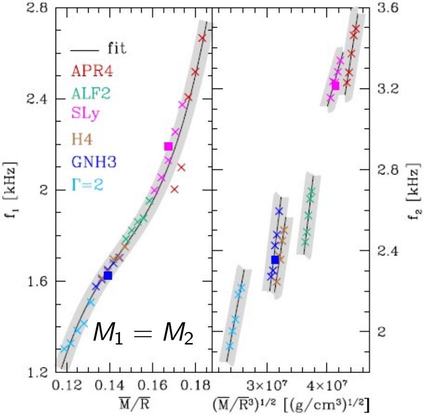




Constraints from Observations of Gravitational Radiation



Takami, Rezzolla and Baiotti (2014)

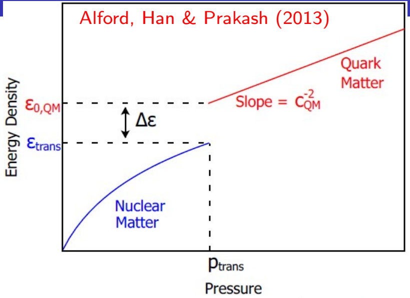


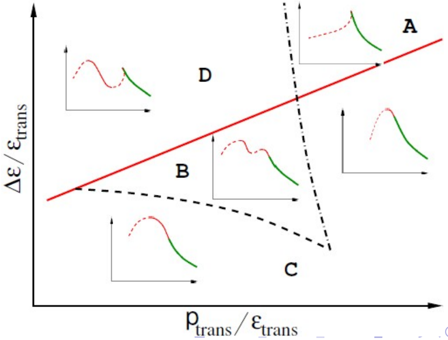
- Masses and tidal deformability measurable during inspiral.
- Frequency peaks are tightly correlated with compactness.
- Mass determinations from prompt and delayed black hole formation.
- In neutron star-black hole mergers, disc mass depends on a/M_{BH} and on $M_{NS}M_{BH}/R^2$.
- R-mode instabilities in rotating neutron stars.

Conclusions

- Measured neutron star masses imply lower limits to radii of typical neutron stars.
- Symmetry energy determines typical neutron star radii.
- Nuclear experiments set reasonably tight constraints on symmetry energy parameters.
- Theoretical calculations of pure neutron matter predict very similar symmetry constraints.
- These constraints predict neutron star radii $R_{1.4} = 12.0 \pm 1.4$ km.
- Combined astronomical observations of photospheric radius expansion X-ray bursts and quiescent sources in globular clusters suggest $R_{1.4} \sim 12.1 \pm 0.6$ km.
- The properties of a high-density phase, such as quark matter, are tightly constrained by current mass measurements.
- A mass measurement above $2.4 M_{\odot}$ may be incompatible with other constraints, assuming GR is correct.

First Order Phase Transition in Neutron Stars



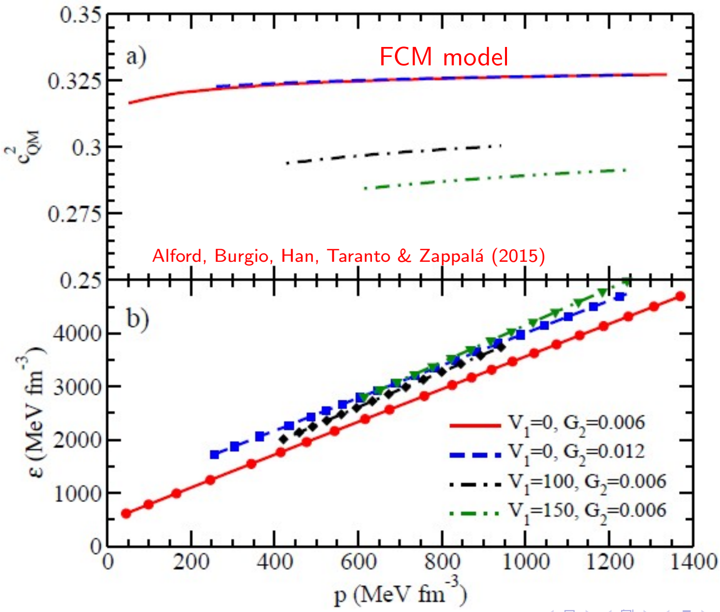


- Generic first order phase transition with 3 parameters: ε_t, P_t, and $\Delta\varepsilon$.
- Make 2 dimensionless parameter combinations: $\Delta\varepsilon/\varepsilon_t$ and P_t/ε_t.
- Critical condition for an appearance of a stable hybrid core connected to the normal branch (**B, C**):

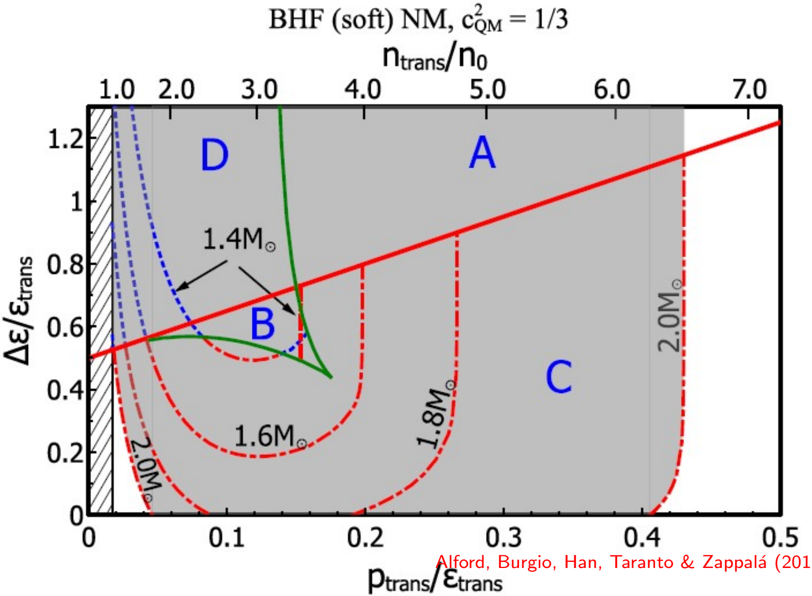
$$\frac{\Delta\varepsilon}{\varepsilon_t} \leq \frac{1}{2} + \frac{3}{2}\frac{P_t}{\varepsilon_t}.$$

- It is also possible to have a stable hybrid core disconnected from the normal branch (**B, D**).
- Approximate high-density phase with a constant sound speed $c_{\rm QM}^2 = dp/d\varepsilon \sim 1/3$.

Sound Speed in Quark Matter



Mass Constraint



Alford, Burgio, Han, Taranto & Zappalá (2015)