Neutron star oscillations and equation of state

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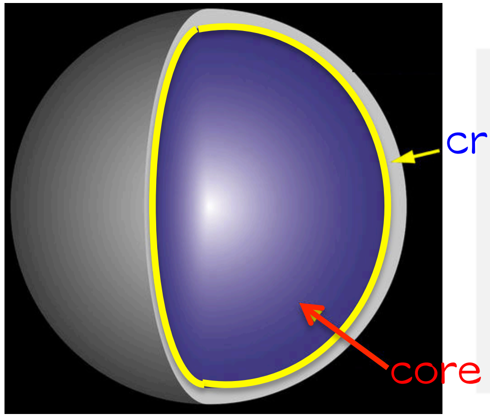
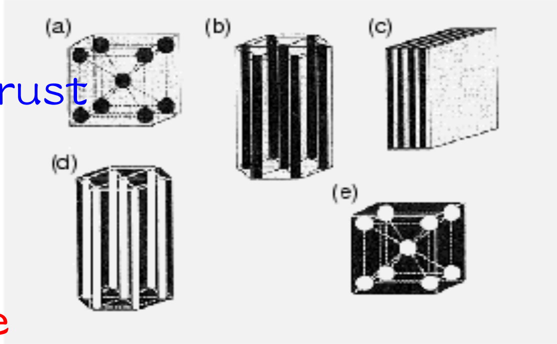
Neutron stars

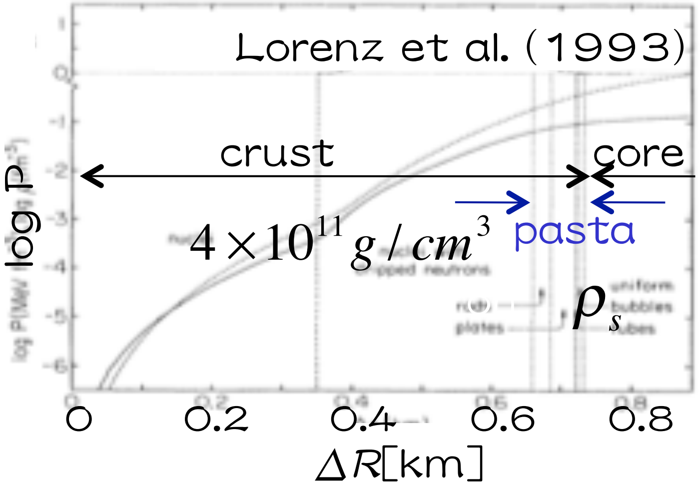
- Structure of NS
  - solid layer (crust)
  - nonuniform structure (pasta)
  - fluid core (uniform matter)
- Crust thickness ≲ 1km
- Determination of EOS for high density (core) region could be quite difficult on Earth
- Constraint on EOS via observations of neutron stars
  - stellar mass and radius
  - stellar oscillations (& emitted GWs)

“(GW) asteroseismology”





Oyamatsu (1993)



Lorenz et al. (1993)

Oscillations (QNMs) in NSs

- Quasi Normal Modes (QNMs)
  - GWs bring out the oscillation energy
  - damped oscillation → QNMs (complex frequencies)
  - Re(ω): oscillation frequency, Im(ω): damping rate
- QNMs (polar parity) in NSs
  - Fluid modes
    - ∗ fundamental mode (*f* -mode) … ~ kHz
    - ∗ pressure mode (*p*-mode) … $\gtrsim$ a few kHz
    - ∗ rotational mode (*r*-mode) … ~ rotation frequency
  - Relativistic modes
    - ∗ spacetime mode (*w*-mode) … $\gtrsim$ a few tens kHz
- QNMs (axial parity) in NSs
  - Relativistic modes; *w*-mode … $\gtrsim$ a few tens kHz
  - Fluid modes; torsional mode (*t*-mode) … $\gtrsim$ ten Hz

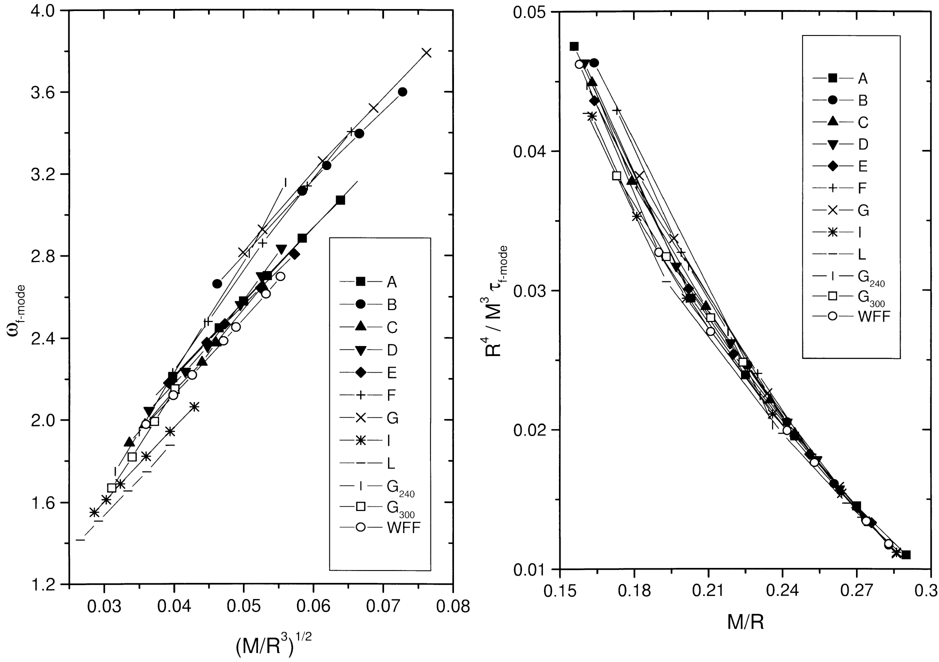
Oscillations of NSs

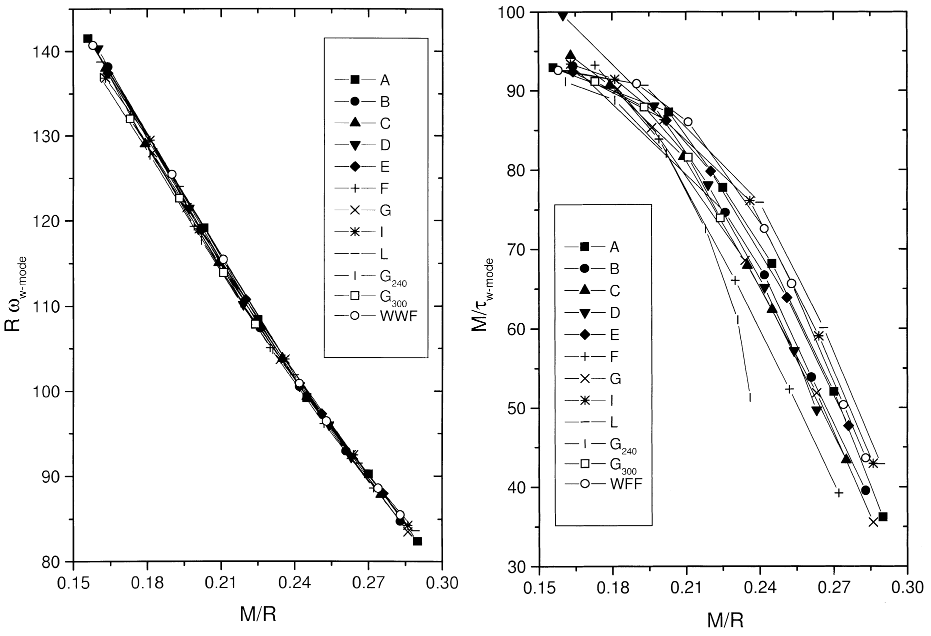
(Andersson & Kokkotas 96, 98)

f-modes

$$\boxed{\omega_f(\mathrm{kHz}) \approx 0.78 + 1.635\left(\frac{\bar{M}}{\bar{R}^3}\right)^{1/2}}$$

$$\frac{1}{\tau_f(\mathrm{s})} \approx \frac{\bar{M}^3}{\bar{R}^4}\left[22.85 - 14.65\left(\frac{\bar{M}}{\bar{R}}\right)\right]$$





w-modes

$$\boxed{\omega_w(\mathrm{kHz}) \approx \frac{1}{\bar{R}}\left[20.92 - 9.14\left(\frac{\bar{M}}{\bar{R}}\right)\right]}$$

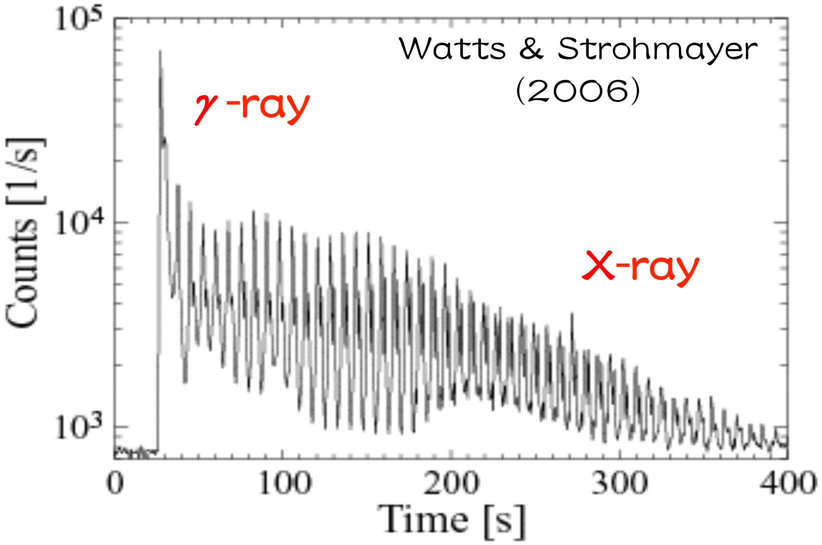
$$\frac{1}{\tau_w(\mathrm{ms})} \approx \frac{1}{\bar{M}}\left[5.74 + 103\left(\frac{\bar{M}}{\bar{R}}\right) - 67.45\left(\frac{\bar{M}}{\bar{R}}\right)^2\right]$$

(M, R) may be determined within $\sim$10% accuracy !!

QPOs in giant flares 1

- Magnetars : $B \gtrsim 10^{14}$ Gauss
- Candidates of magentars
  - Anomalous X-ray pulsars (AXPs)
  - Soft gamma repeaters (SGRs)

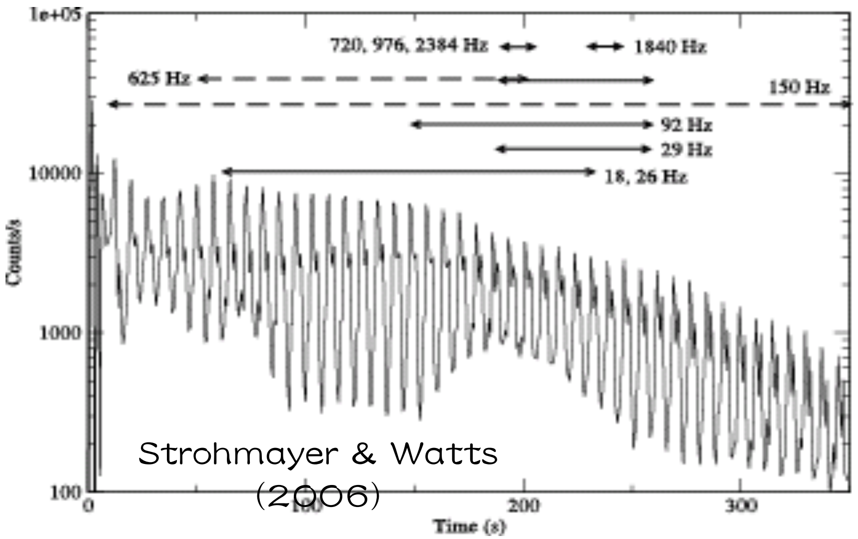
    ~ sporadic emission with X and γ-rays (~ 10^{41} erg/s)
- Giant flares from SGRs (10^{44}-10^{46} ergs/s)
  - SGR 0526-66 in March.5.1979
  - SGR 1900+14 in August.27.1998
  - SGR 1806-20 in December.27.2004



Watts & Strohmayer (2006)

γ-ray

X-ray

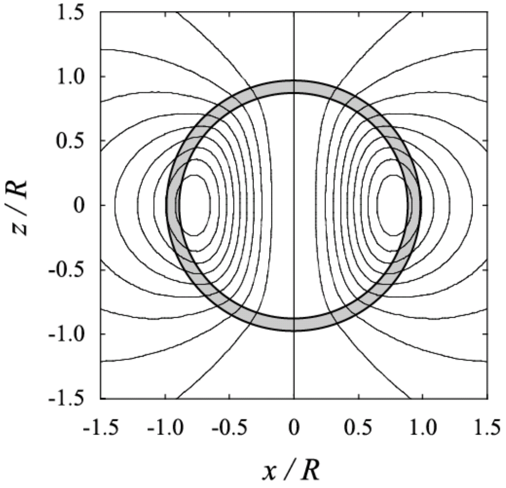


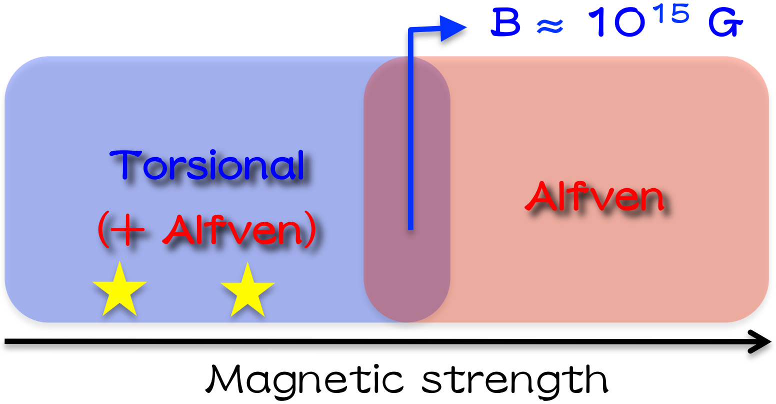
Strohmayer & Watts (2006)

QPOs in giant flares 2

- Afterglow of giant flares → quasi periodic oscillations(QPOs)

  → Barat et.al. (1983); Israel et.al. (2005);
  Watts & Strohmayer (2005, 2006)
  - SGR 0526-66 : 23ms (43Hz), B ~ 4 × 10^{14}G
  - SGR 1900+14 : B > 4 × 10^{14}G, **28, 54, 84, 155 Hz**
  - SGR 1806-20 : B ~ 8 × 10^{14}G, L ~ 10^{46} ergs/s
    18, 26, 30, 92.5, 150, 626.5, 1837 Hz + something ?
- Theoretical attempts to explain…
  - torsional oscillations in neutron star crust.
  - magnetic oscillations (Alfven oscillations)





Crustal torsional oscillations

- observed QPOs is crustal torsional oscillations?
  - In Newtonian; Hansen & Cioffi (1980), McDermott et al. (1998), Carroll et al. (1986), Storhmayer (1991), ...

→ without magnetic field

$$\boxed{{}_{\ell}t_0 \sim \frac{\sqrt{\ell(\ell+1)\mu/\rho}}{2\pi R} \sim 16\sqrt{\ell(\ell+1)}\ \text{Hz} \qquad {}_{\ell}t_n \sim \frac{\sqrt{\mu/\rho}}{2\Delta r} \sim 500 \times n\ \text{Hz}}$$

→ ${}_{2}t_0 = 39,\ {}_{3}t_0 = 55,\ {}_{4}t_0 = 72,\ {}_{5}t_0 = 88,\ {}_{6}t_0 = 104,\ \cdots,\ {}_{\ell}t_1 = 500,\ \cdots$

- relativistic models; Schumaker & Thone (1983), Leins (1994), Samuelsson & Andersson (2007)

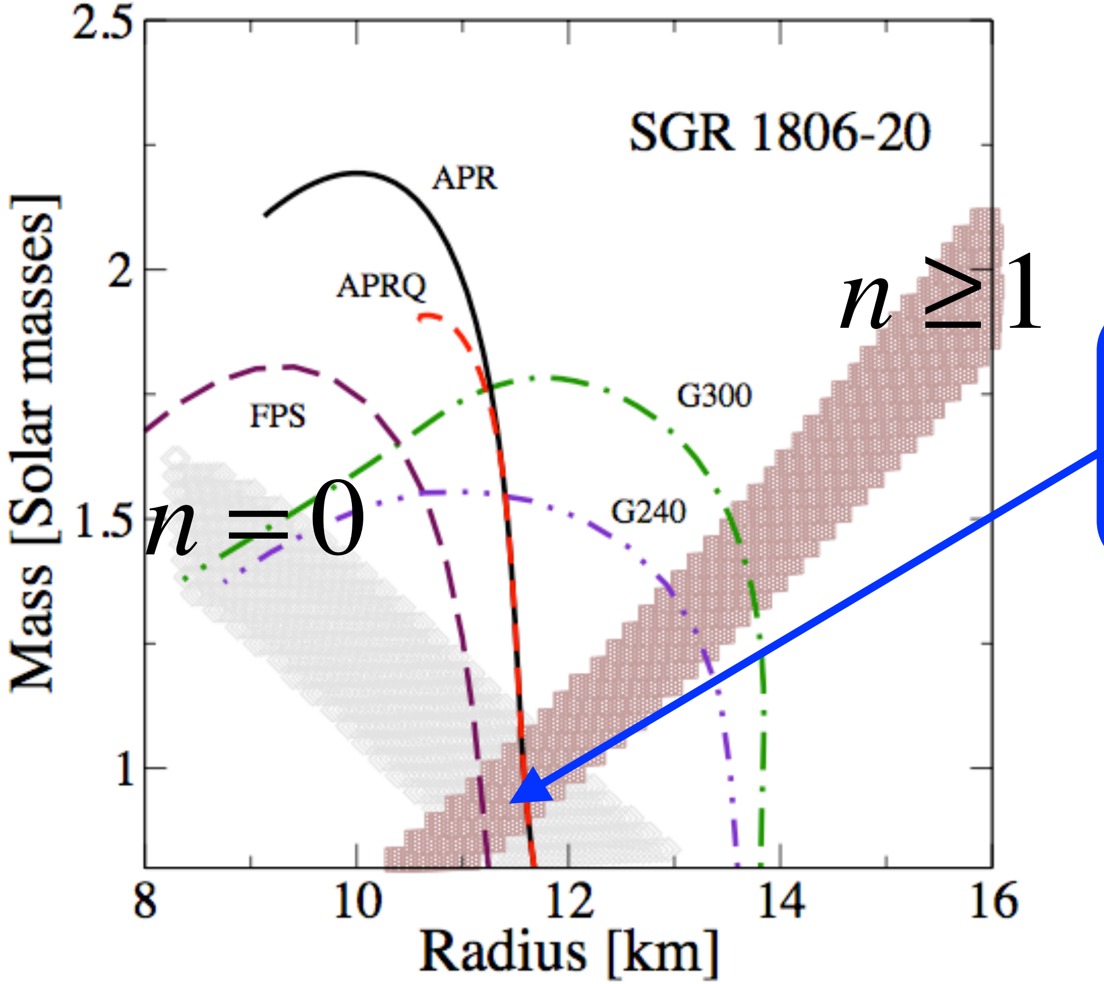
SGR 1806-20

QPOs	18	26	30	92.5	150	626.5	1837
n	?	?	0	0	0	1	3
l	?	?	2	6	10	---	---

Samuelsson & Andersson (2007)

Constraint on NS model

Samuelsson & Andersson (2007)



$M = 0.96 M_{\text{sun}}$

$R = 11.4\text{km}$

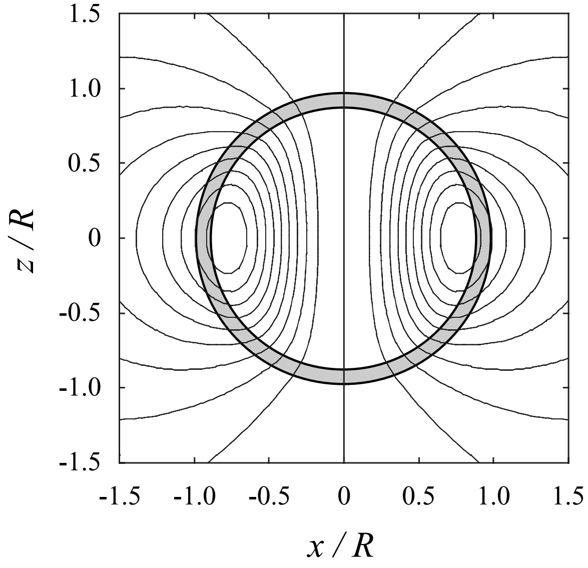
$\Delta r = 1.9\text{km}$

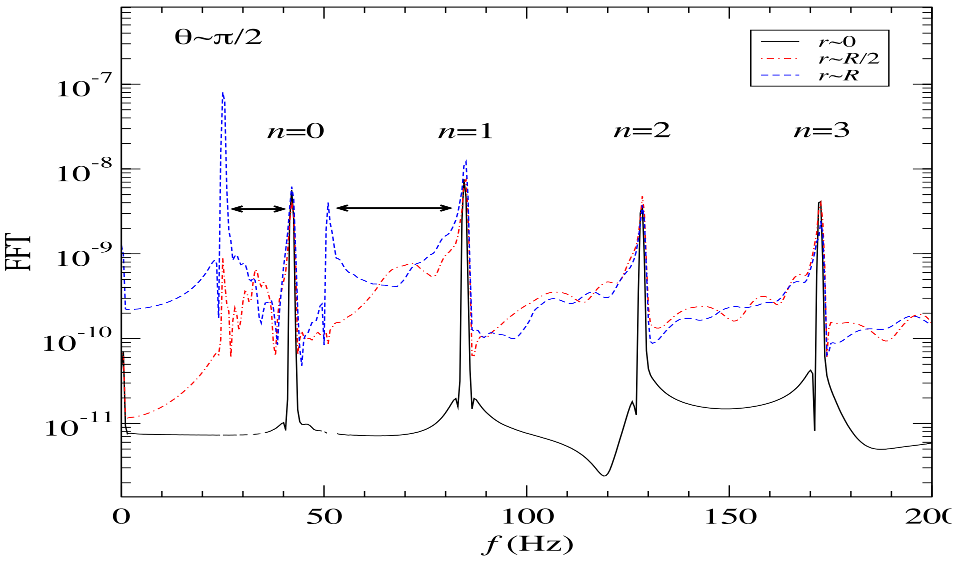
Axial Alfven oscillations

(HS+2008a)

two families in Alfven oscillations

- continuum spectrum
- upper & lower QPOs
- $f_{\mathrm{L}n} \cong (n+1)\, f_{\mathrm{L}0}$, $f_{\mathrm{U}n} \cong (n+1)\, f_{\mathrm{U}0}$
- $f_{\mathrm{L}n} / f_{\mathrm{U}n} \cong 0.6$ independently of the stellar model
- $f_{\mathrm{L}n}$ & $f_{\mathrm{U}n}$ $\propto$ the magnetic field strength

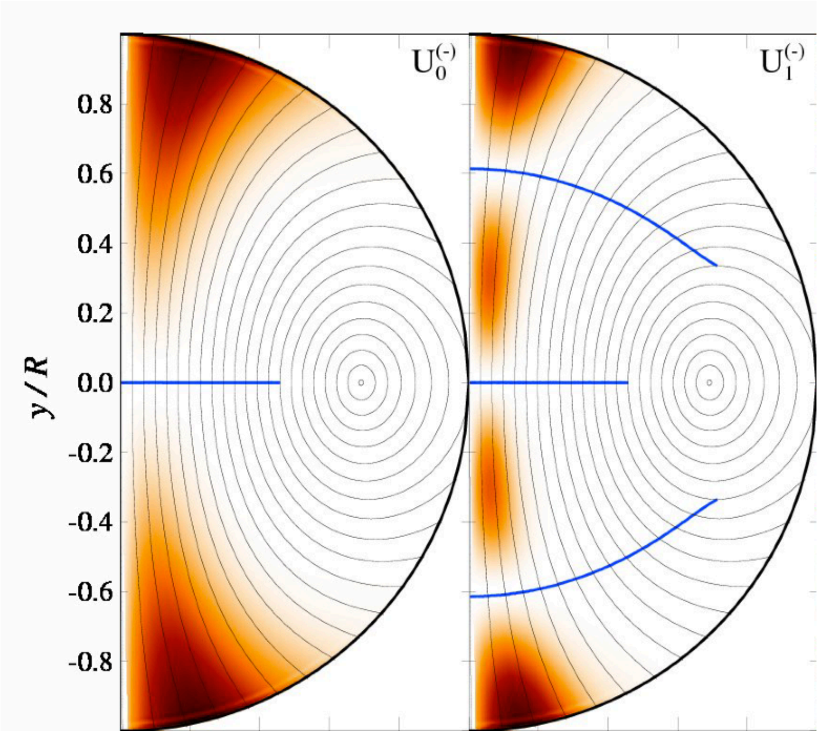


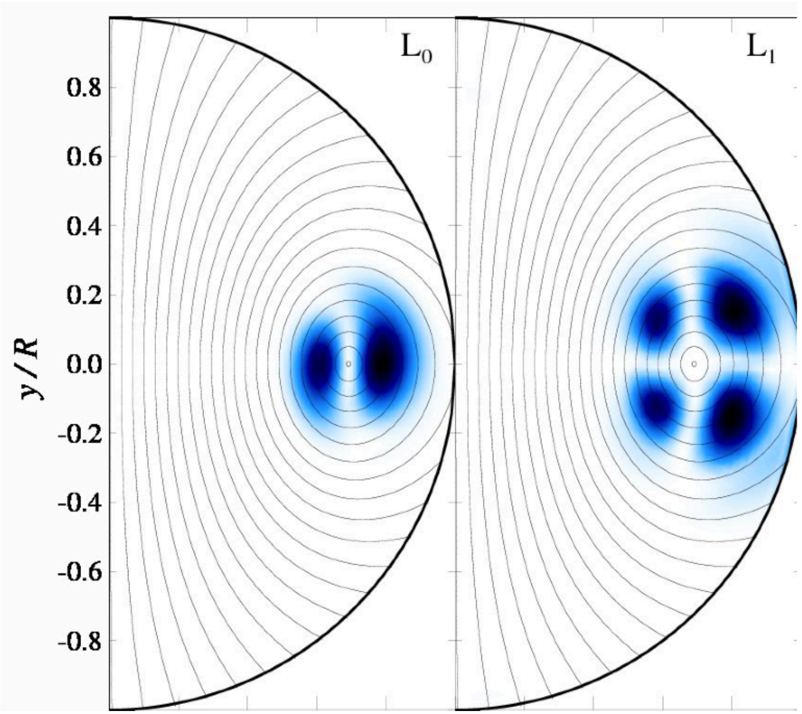


Effective amplitude

(Cerda-Dulan+2010)

- Upper QPOs are associated with the open field liens
- Lower QPOs are associated with the closed field liens

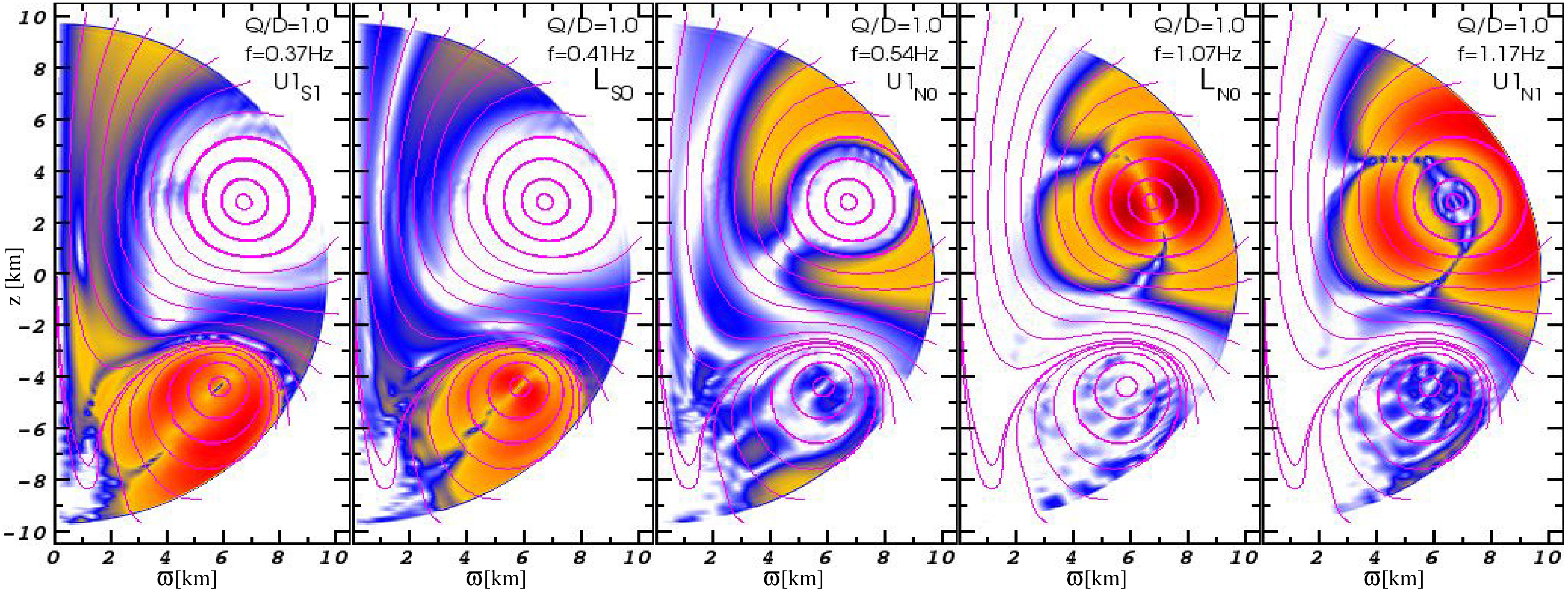




Different type of magnetic distribution

(Gabler+2012)

Taking into account the quadruple component as well as dipole component, the Alfven oscillations are examined…

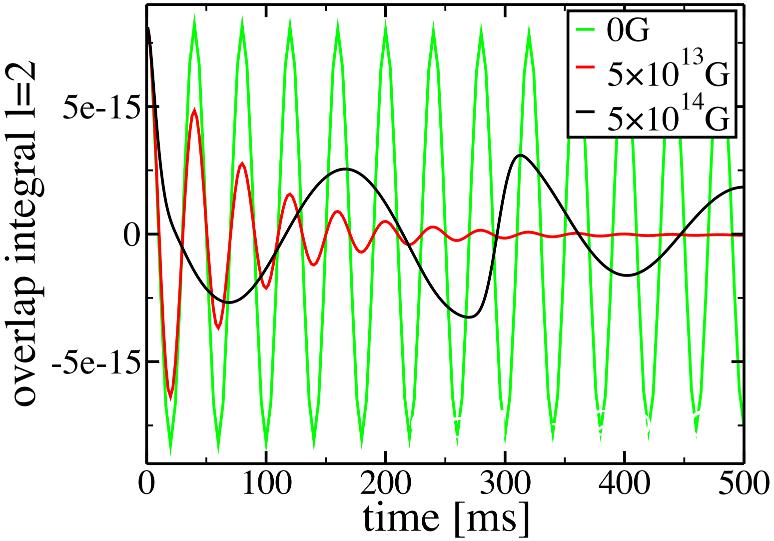


Magnetic oscillations strongly depend on the magnetic configuration !

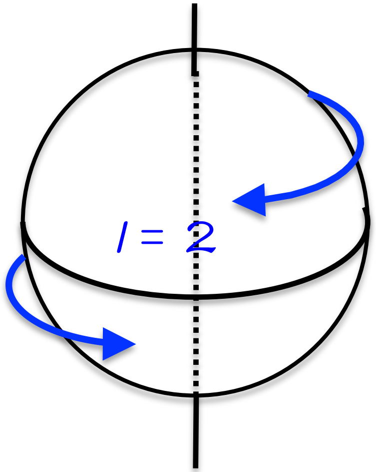
Crust effect

(Colaiuda+2011, Gabler+2010, 2012)

- Strong magnetic field
  - no crust torsional oscillations
- Weak magnetic field
  - Alfven oscillations are confined in core region
  - surface oscillations are crust torsional oscillations



Axial Alfven oscillations



Continuum spectrum

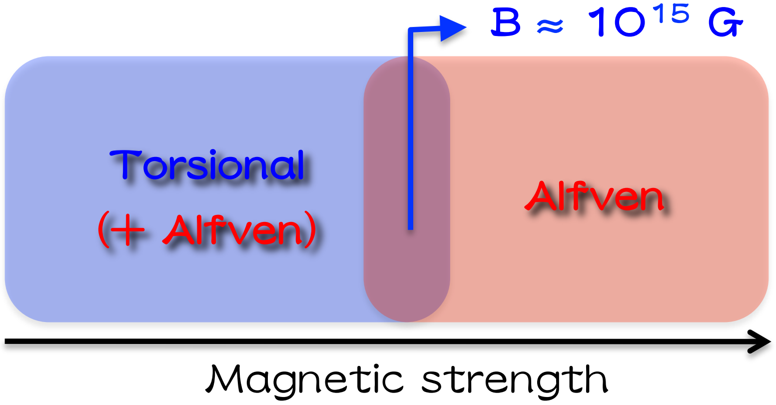
- upper & lower QPOs

Stronger magnetic field than $\sim 10^{15}$ G

- only Alfven oscillations can be excited

Weaker magnetic field than $\sim 10^{15}$ G

- crust torsional oscillations can be excited near surface
- Alfven oscillations are confined in the core region



Constraint on magnetic configuration

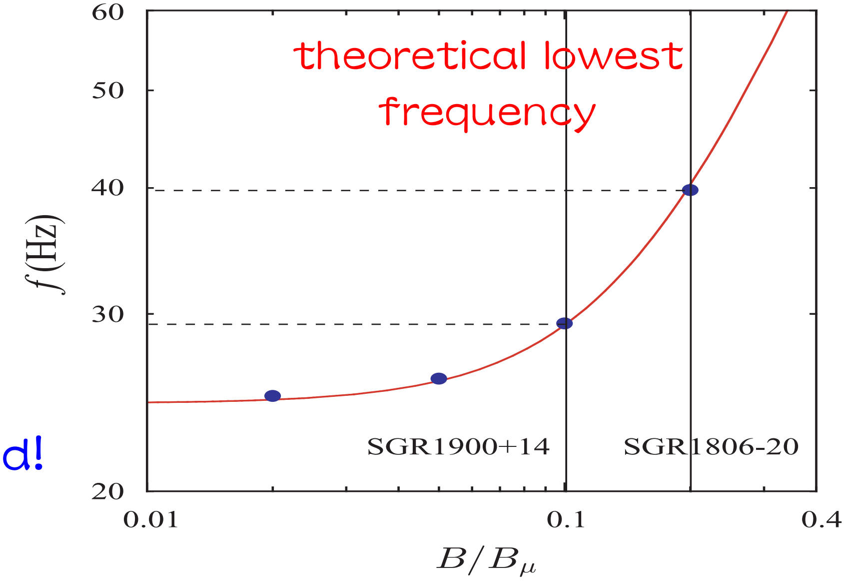
(HS+2008b)

Observed magnetic field strength

- SGR 1900+14: $B \gtrsim 4 \times 10^{14}$ G (Hurley+1999)
- SGR 1806-20: $B \sim 8 \times 10^{14}$G (Kouveliotou+1998)

If magnetic field is confined in crust…

- type I super conductor
- oscillation is confined in crust.
- we have no way to explain the lower frequencies
- magnetic field should permeate the whole star
- type II super-conductor is favored!



remarks

- magnetic configuration inside NSs are still unknown.
- EOS for core region is unfixed yet.

- **to avoid such uncertainties, we focus on the crustal torsional oscillations without magnetic field effects**
  - fluid core; zero shear modulus ---> No torsional oscillations
  - torsional oscillations localize only in crust region.

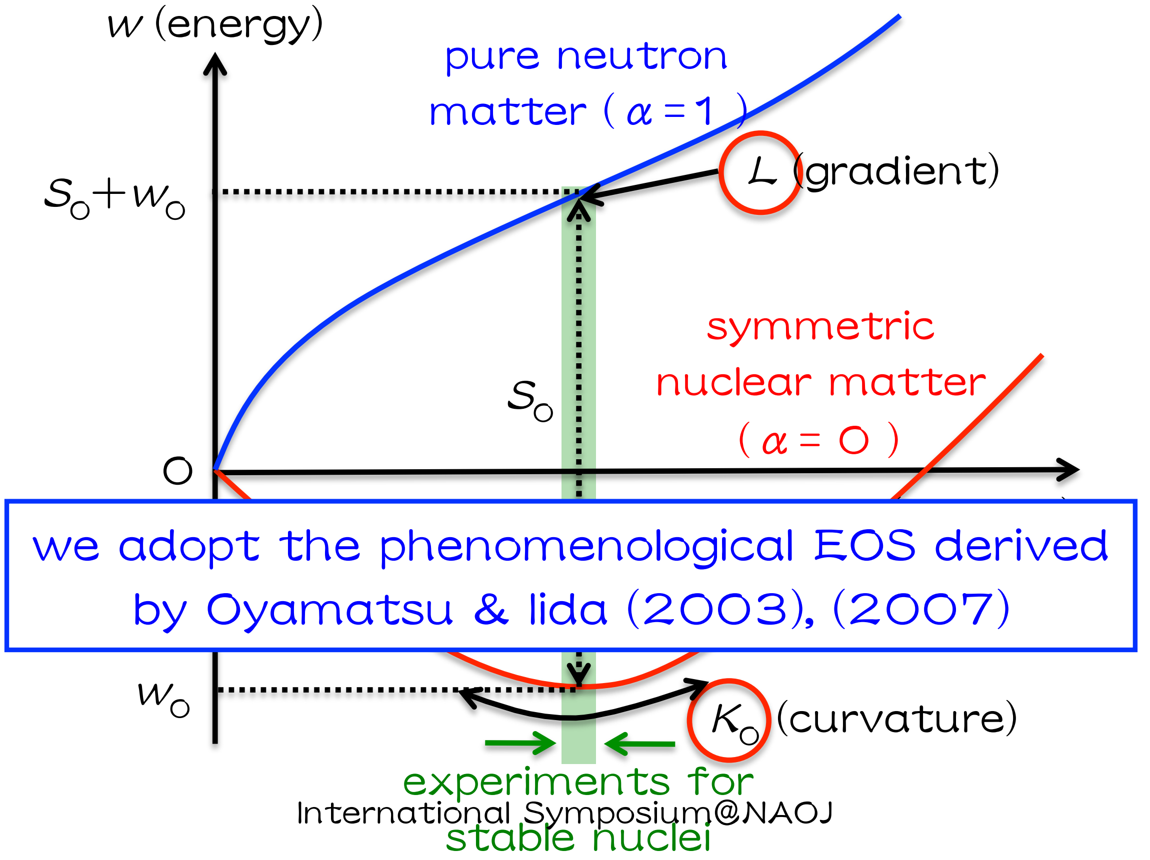
EOS near the saturation point

- Bulk energy per nucleon near the saturation point of symmetric nuclear matter at zero temperature;

incompressibility

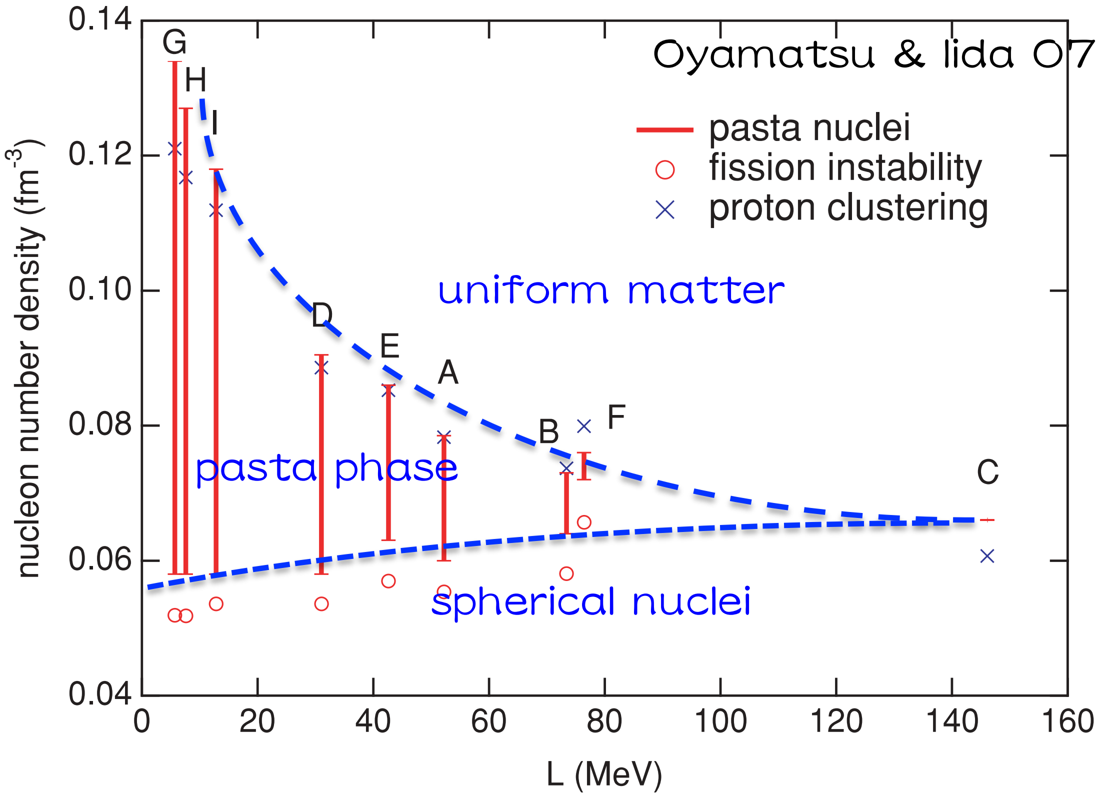
symmetry parameter

$$w = w_0 + \frac{K_0}{18n_0^2}(n - n_0)^2 + \left[S_0 + \frac{L}{3n_0}(n - n_0)\right]\alpha^2$$



we adopt the phenomenological EOS derived by Oyamatsu & Iida (2003), (2007)

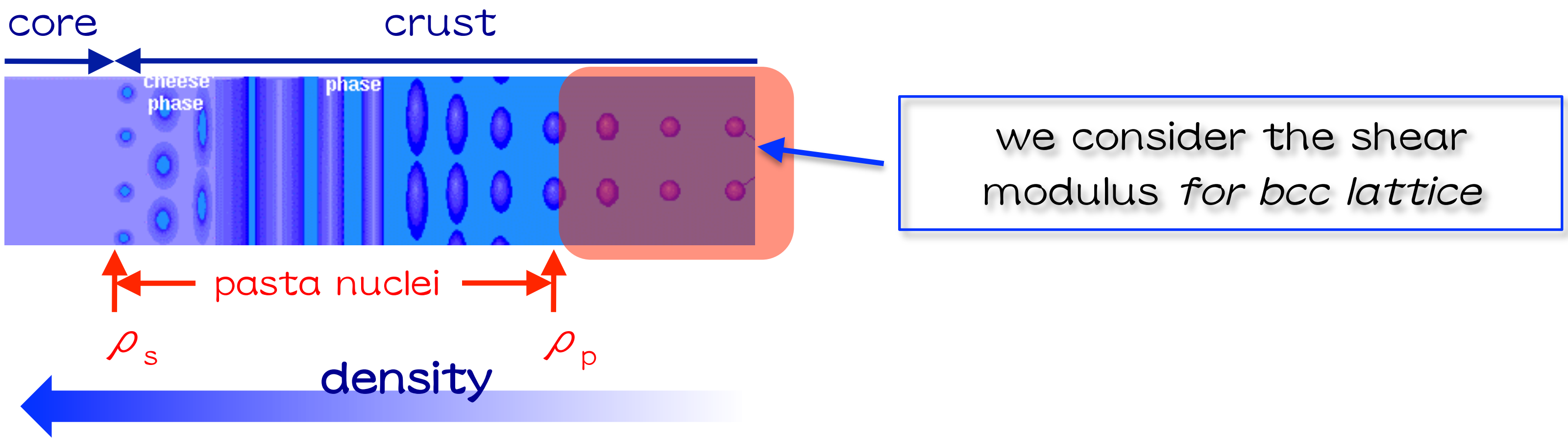
pasta phase



- Whether pasta phase exists or not depends strongly on L.
- For $L \gtrsim$ 100MeV, pasta structure almost disappears.

What we do

- EOS for core region is still uncertain.
- To prepare the crust region, we integrate from $r=R$.
  - M, R : parameters for stellar models
  - L, K_0 : parameters for curst EOS (Oyamatsu & Iida (2003), (2007))
    - → For $L \gtrsim 100$MeV, pasta structure almost disappears
- In crust region, torsional oscillations are calculated.
  - considering the shear only in spherical nuclei.
  - frequency of fundamental oscillation $\propto v_s$ ($v_s^2 \sim \mu/H$)
  - calculated frequencies could be lower limit



we consider the shear modulus *for bcc lattice*

Effect of neutron superfluidity

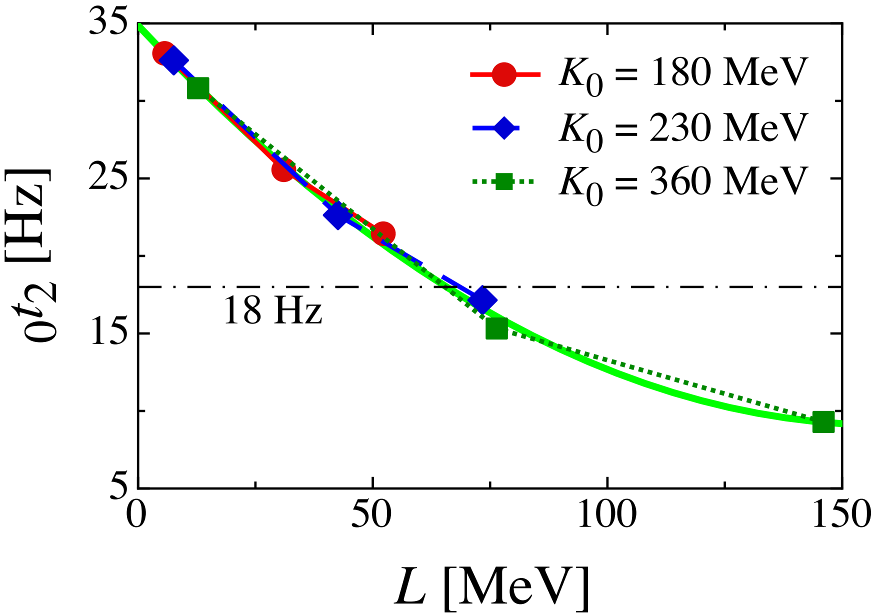
- For $\rho \gtrsim 4 \times 10^{11}$ g cm^{-3}, neutron could drip from nuclei
- Some of dripped neutron play a role as superfluid
- Effective enthalpy affecting on the shear oscillations could be reduced
  - shear speed ($v_s^2 \sim \mu/H$) increases due to the effect of superfluidity

$$\mathcal{Y}'' + \left[\left(\frac{4}{r} + \Phi' - \Lambda'\right) + \frac{\mu'}{\mu}\right]\mathcal{Y}' + \left[\frac{\epsilon + p}{\mu}\omega^2 e^{-2\Phi} - \frac{(\ell+2)(\ell-1)}{r^2}\right] e^{2\Lambda}\mathcal{Y} = 0.$$

- ${}_0t_l$ could also increase due to the effect of superfluidity
- While, the fraction of superfluid neutron in dripped neutron is still unknown…
  - Chamel (2012): superfluid neutron are not so much (~10-30%?)

$l = 2$ fundamental oscillations (${}_0t_2$)

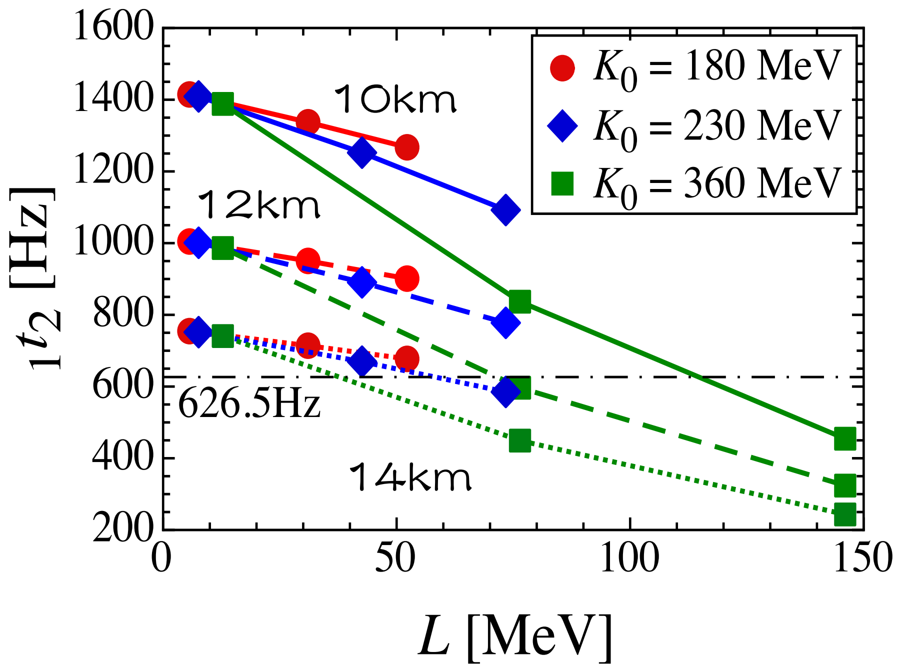
- For $M = 1.4M_\odot$ & $R = 12$km, calculated frequencies ${}_0t_2$
- ${}_0t_2$ is almost independent of the value of K_0
- For $R = 10\sim14$ km and $M/M_\odot = 1.4\sim1.8$, similar dependence on K_0
- One can write fitting line
- Focus on L dependence of ${}_0t_2$



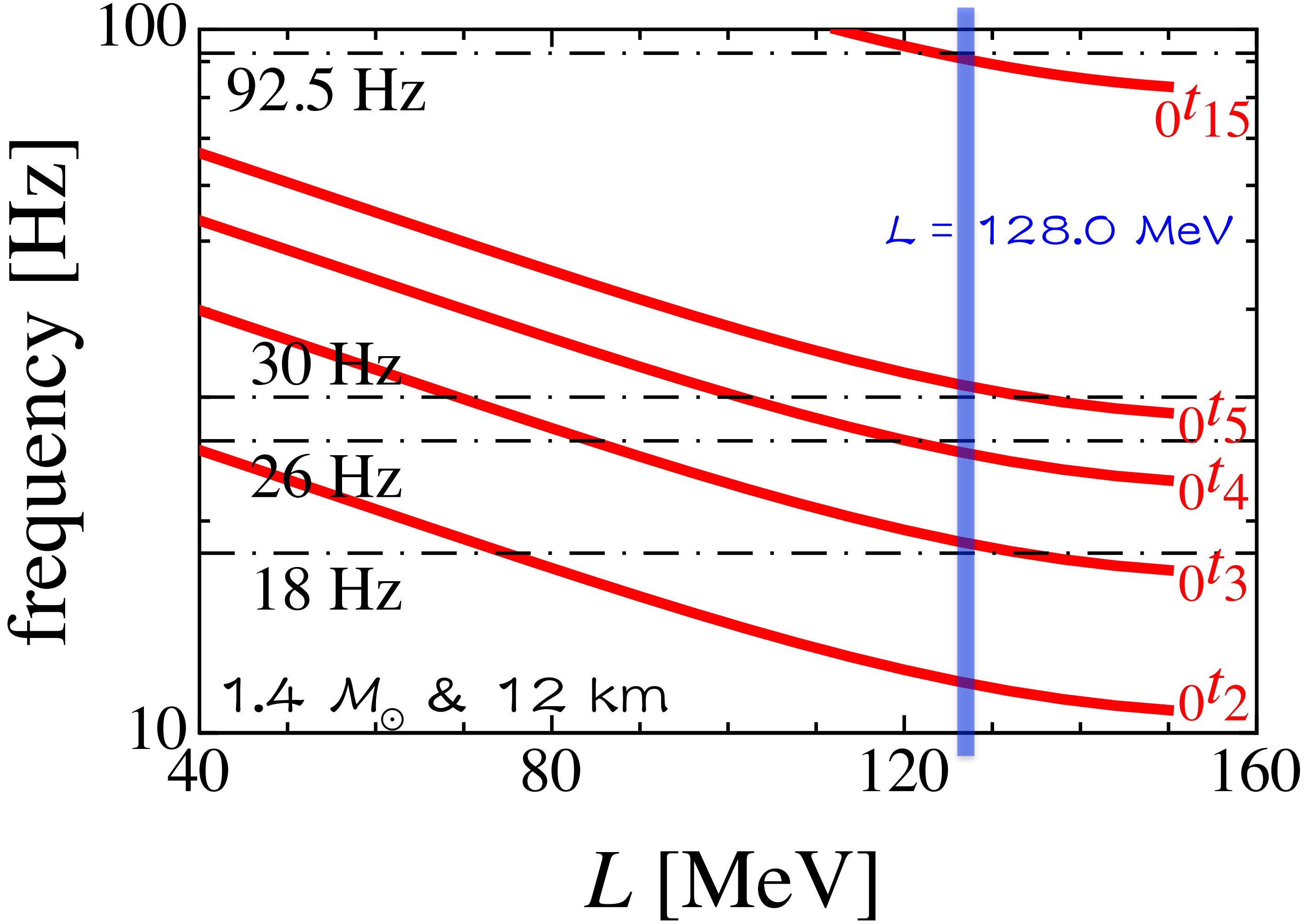
1^{st} overtone (${}_1t_2$)

(HS+ 2012)

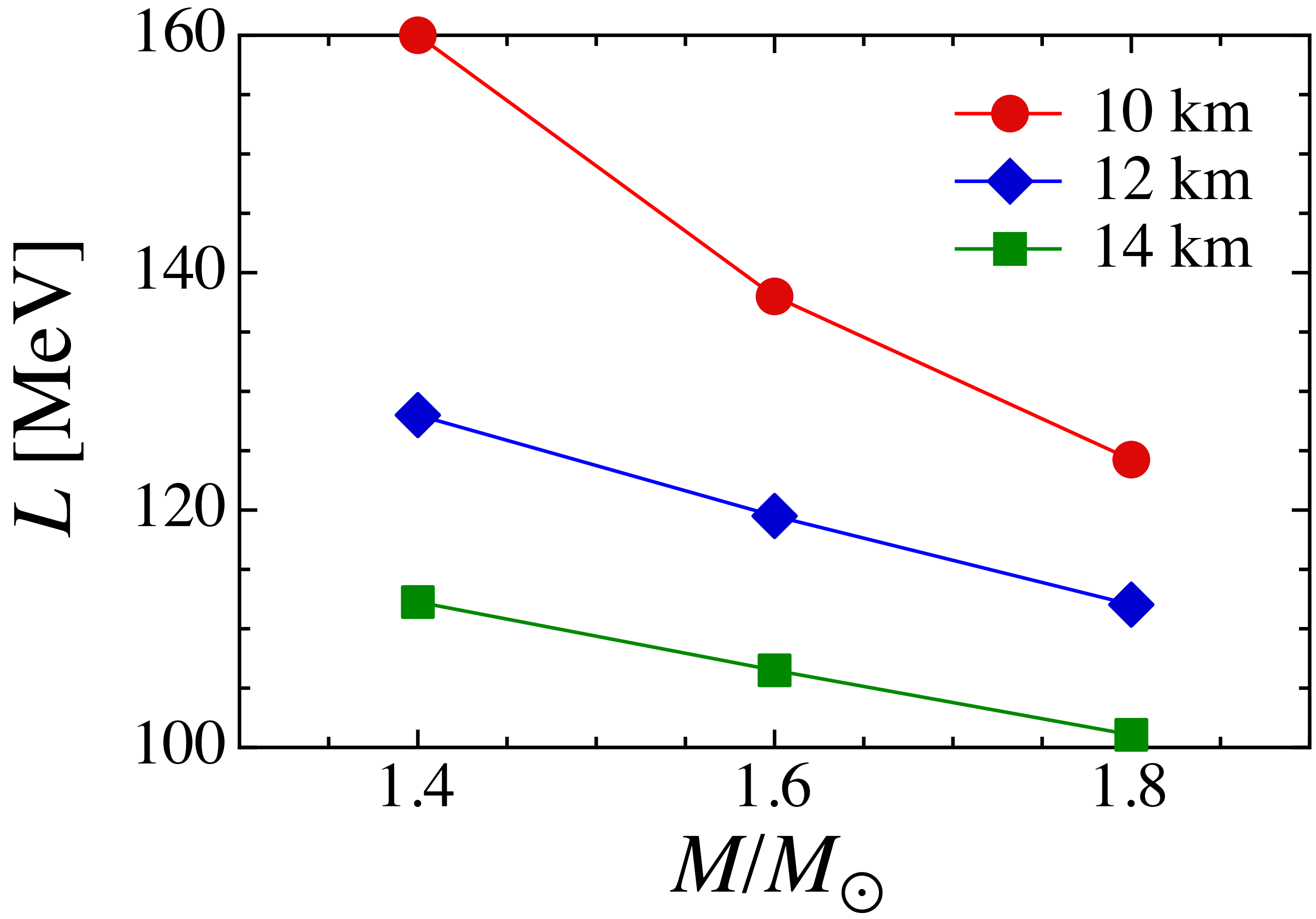
- for the stellar models with $M = 1.4 M_\odot$ & $R = 10$, 12, 14 km
- Unlike ${}_0t_2$, ${}_1t_2$ depends not only L but also K_0
- dependence of K_0 :
  - pasta phase becomes crucial ?



Identification of SGR 1806-20

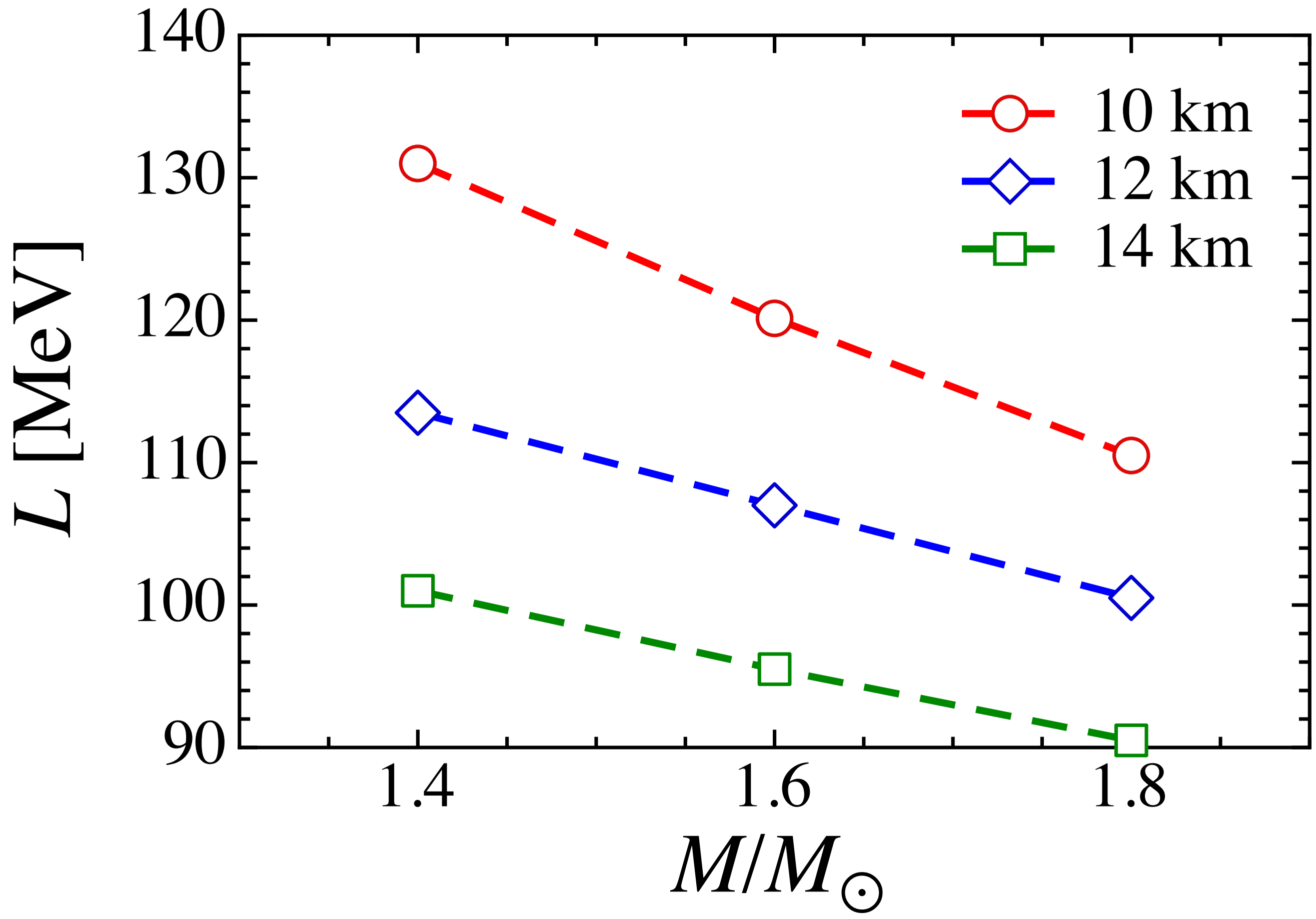


Constraint on L via SGR 1806-20



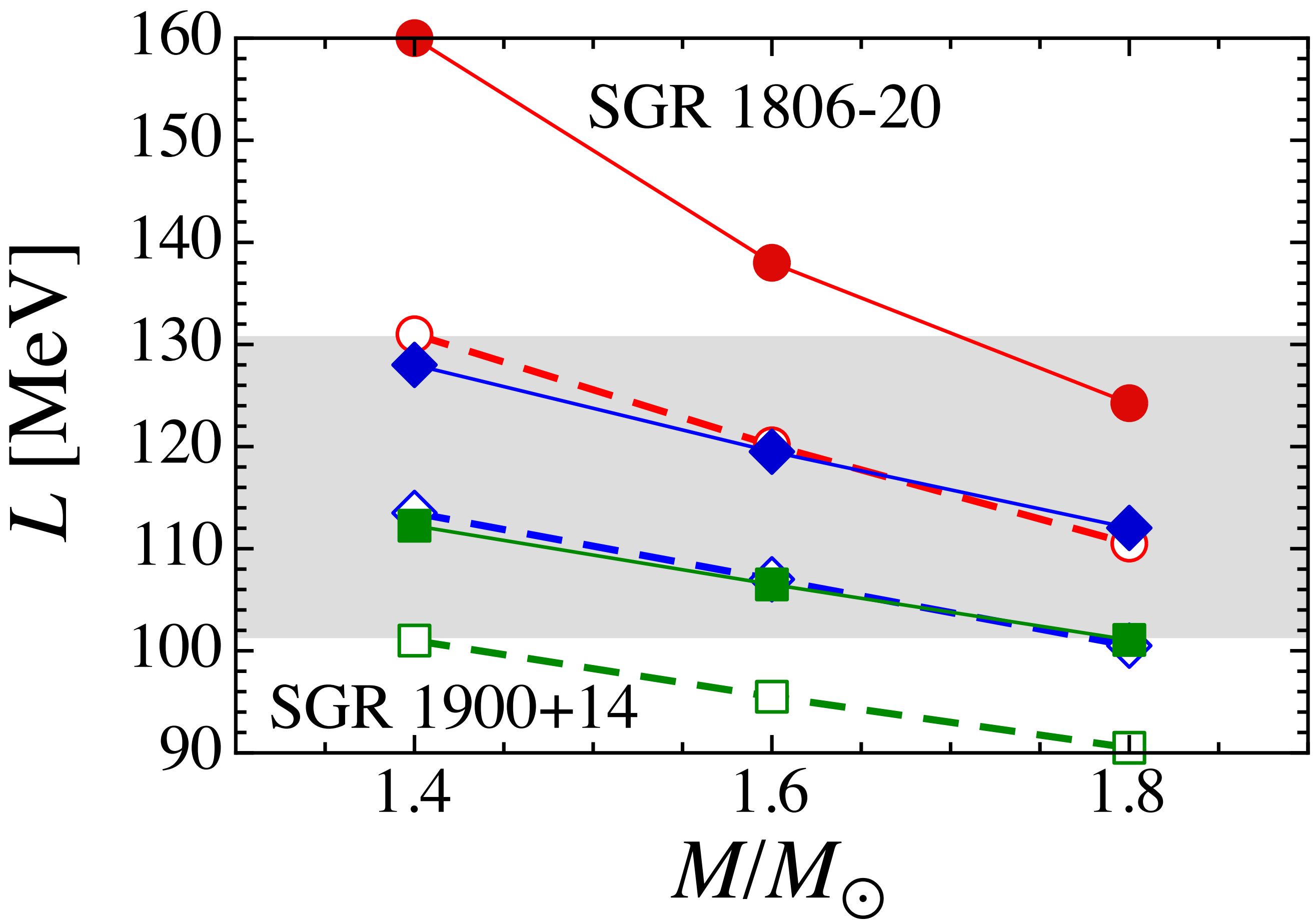
➡ $101 \leq L \leq 160$ MeV

Constraint on L via SGR 1900+14



➡ $91 \leq L \leq 131$ MeV

Allowed region for L



➡ $101.1 \leq L \leq 131.0$ MeV

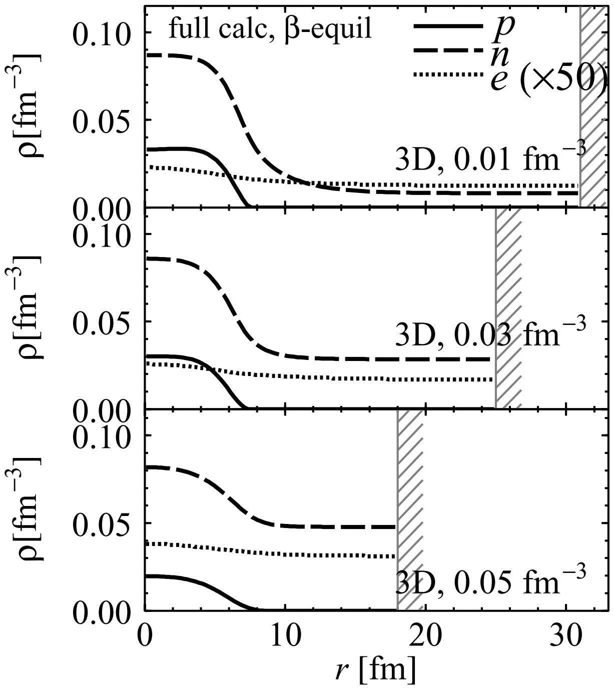
Effect of electron screening

- crust configuration is almost independent from such effect
- shear modulus can be modified
  - contribution due to Coulomb interaction

    Ogata, Ichimaru 1990; Strohmayer+ 1991

$$\mu = 0.1194 \times \frac{n_i(Ze)^2}{a}$$

n_i : number density of quark droplet
Z : charge of quark droplet
a : Wigner-Seitz radius



Maruyama+ (2005)

  - including effect of electron screening

    Horowitz & Hughto 2008 : 10% reduction

    Kobyakov & Pethick 2013

$$\mu = 0.1194\left[1 \boxed{-\,0.010Z^{2/3}}\right]\frac{n_i(Ze)^2}{a}$$

effect of electron screening

~11.7% reduction for $Z = 40$

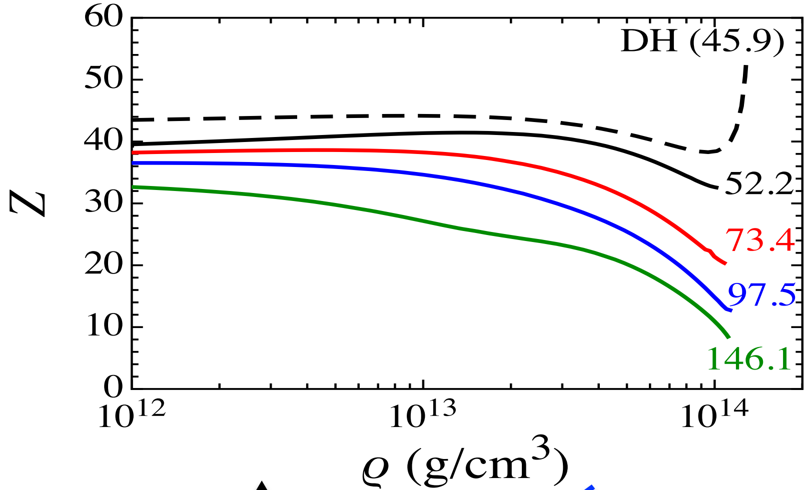
  - frequency ∝ shear speed

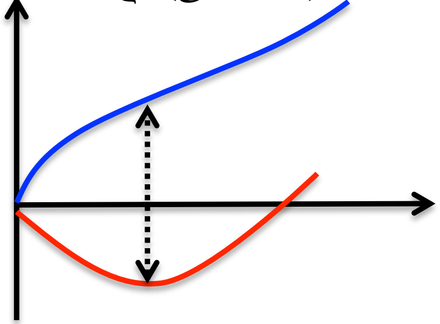
    frequency reduces due to electron screening effect (HS 2014)

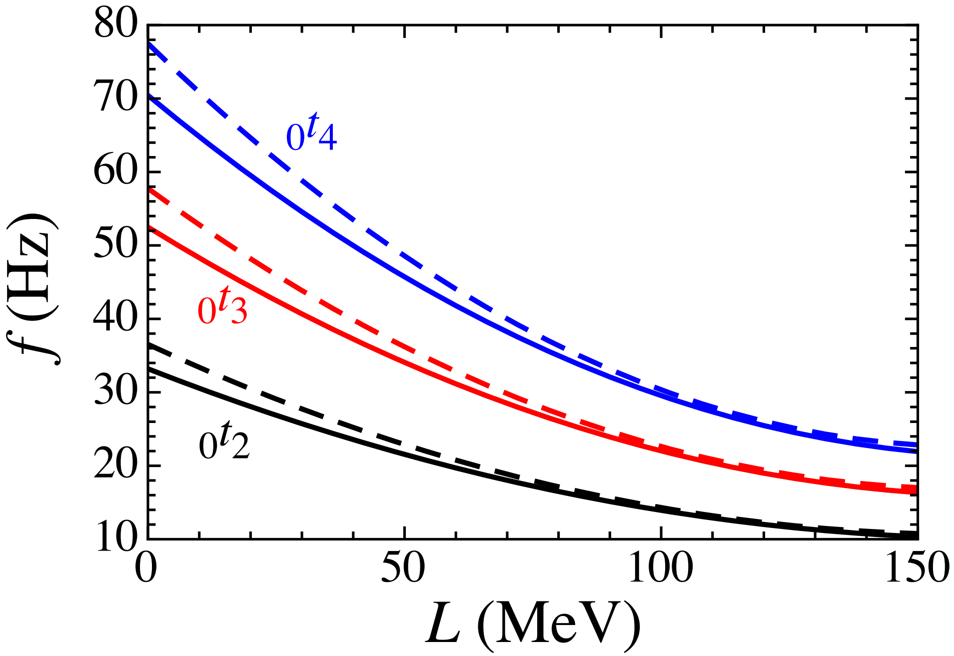
Shift of frequencies due to the electron screening effect

charge number generally reduces with *L*

- the shift of frequencies due to electron screening effect, becomes small with large *L*.

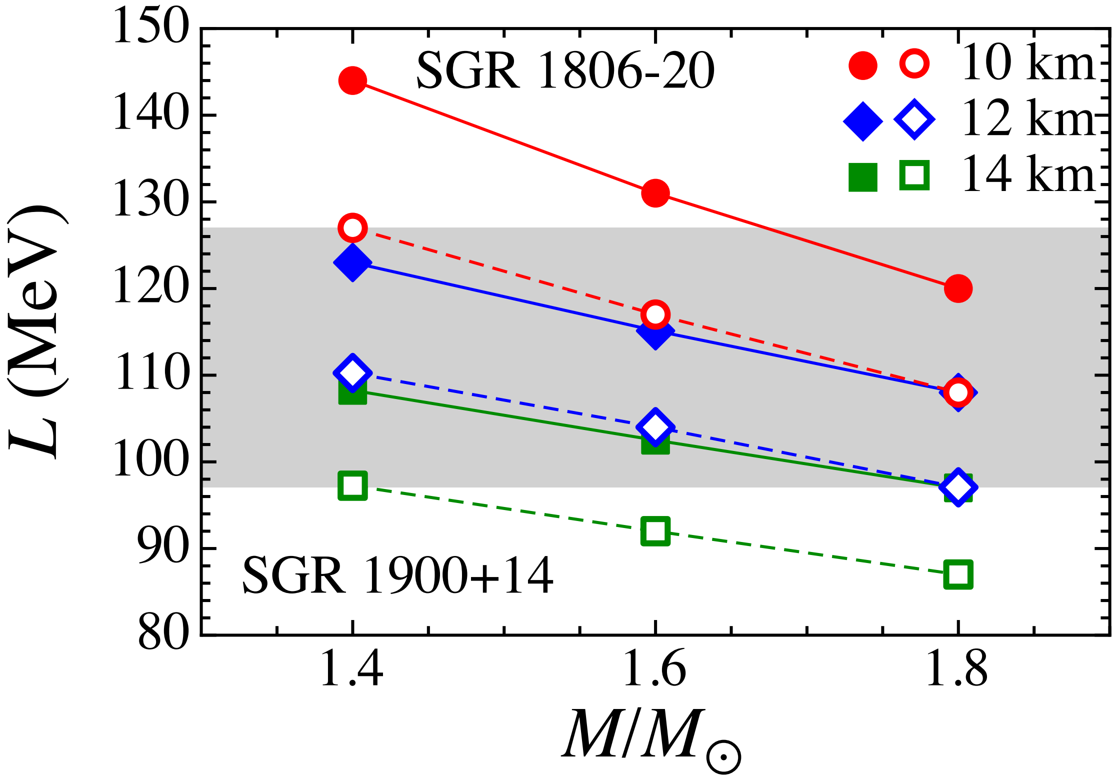






Constraint on L

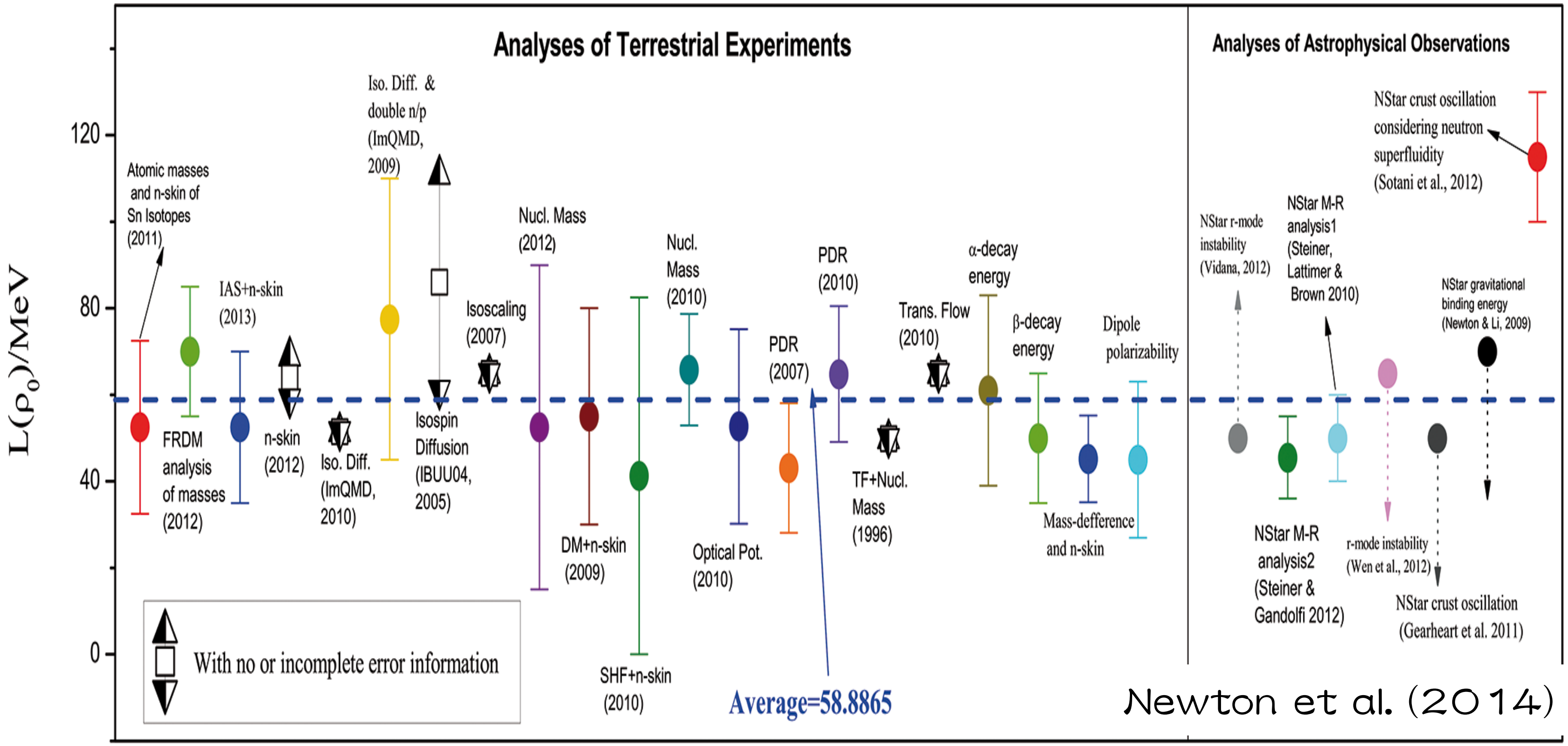
Constraint on L for explaining the both observations in SGR 1806-20 and SGR 1900+14;



→ **$97 \leqq L \leqq 127$MeV**

without effect
$101 \leqslant L \leqslant 131$ MeV

other constraints on L



Newton et al. (2014)

most of constraints on L predict around $40 \lesssim L \lesssim 80$MeV

Conclusion

NS oscillations are good candidates to extract the interior information.

- QPOs in SGRs are good evidences for adopting the asteroseismology
  - magnetic and/or crustal torsional oscillations?
- identifying the observed QPO frequencies in SGRs with the crustal torsional oscillations, we make a constraint on $\mathcal{L}$
  - $101 \leqslant \mathcal{L} \leqslant 131$ MeV without electron screening effect
  - $97 \leqq \mathcal{L} \leqq 127$MeV with electron screening effect.
- still, a little larger than the predictions from nuclear experiments, which is around $40 \lesssim \mathcal{L} \lesssim 80$MeV.

We should take into account the additional missing effects

- magnetic fields, even though the magnetic configuration is unknown
- more realistic shear modulus, including the size effect and in pasta structure
- maybe shell effect
- examination of overtones