Magnetic field configurations of magnetized stars

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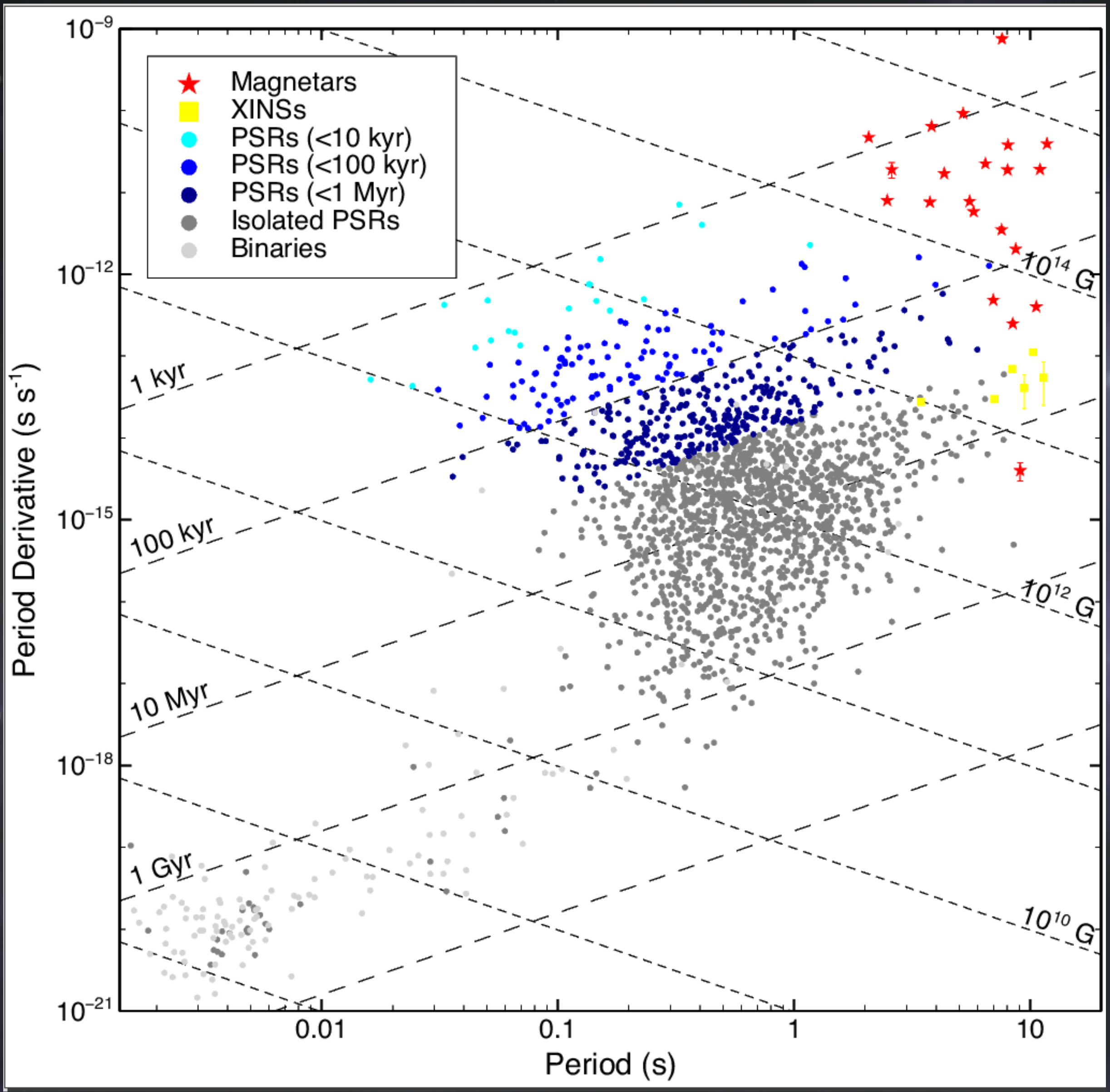
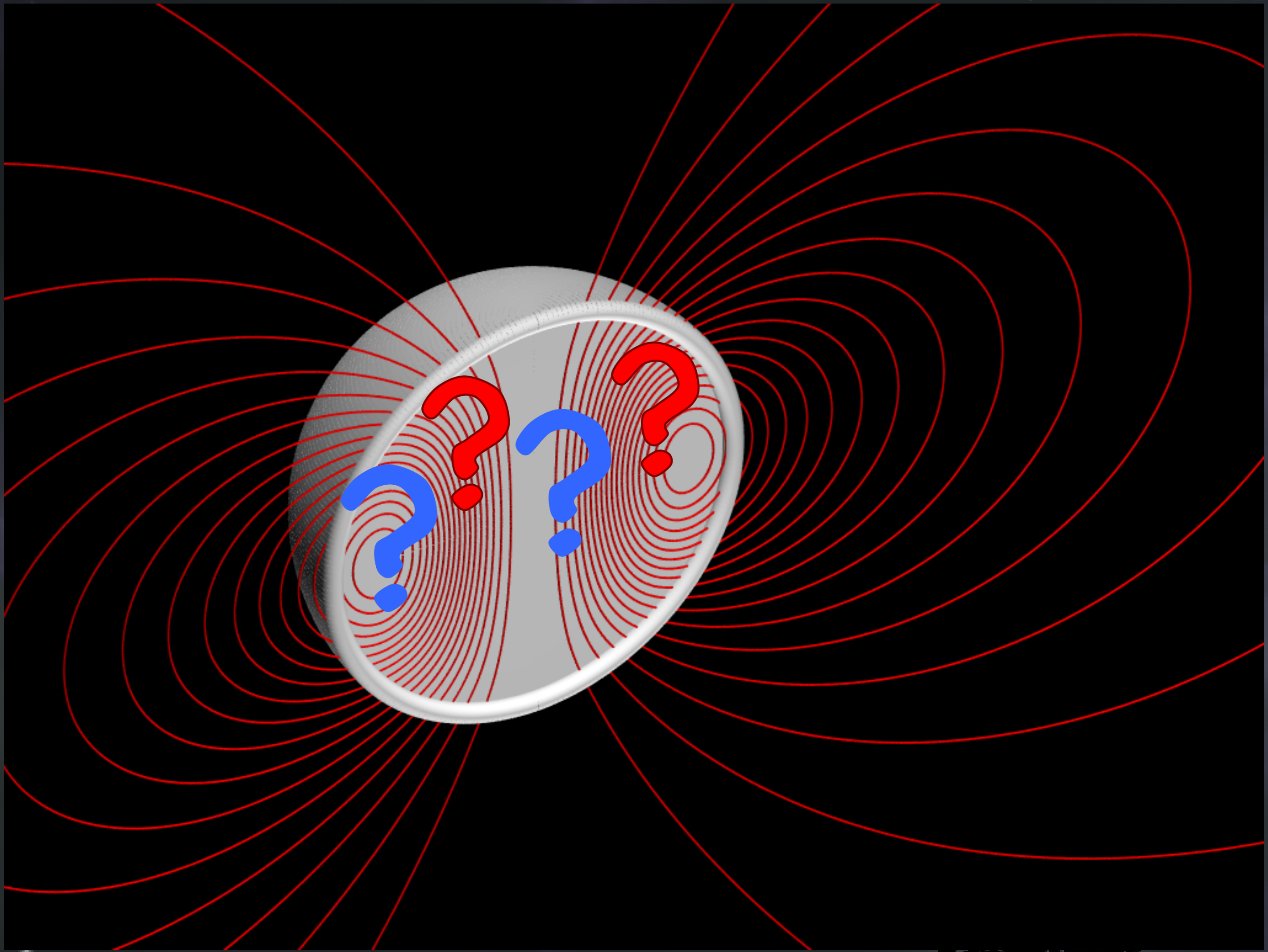
Magnetar



B ～ 10^15 G

Olausen & Kaspi (2013)

What is the internal magnetic fields?



Strong internal fields?

Basic equations

- Stationary and axisymmetric, Newtonian
- Barotropic, ideal MHD.

$$\frac{1}{\rho}\nabla p = -\nabla\phi_g + R\Omega^2 \boldsymbol{e}_R + \frac{1}{\rho}\left(\frac{\boldsymbol{j}}{c}\times\boldsymbol{B}\right),$$

$$\Delta\phi_g = 4\pi G\rho\,,$$

$$p = K_0\rho^{1+1/N}$$

$$\nabla\cdot\boldsymbol{B} = 0\,,$$

$$\nabla\times\boldsymbol{E} = 0\,,$$

$$\boldsymbol{E} = -\frac{\boldsymbol{v}}{c}\times\boldsymbol{B}\,.$$

$$\nabla\cdot\boldsymbol{E} = 4\pi\rho_e\,,$$

$$4\pi\frac{\boldsymbol{j}}{c} = \nabla\times\boldsymbol{B}$$

Tomimura & Eriguchi (2005)

What's GS eq.?

- Grad-Shavfranov eq.

$$\Delta^* \Psi = -4\pi r \sin\theta \frac{j_\varphi}{c}.$$

$$\Delta^* = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} - \frac{1}{r^2} \frac{\cos\theta}{\sin\theta} \frac{\partial}{\partial \theta} \right).$$

- Elliptic type equation.
- GS equation describes structures of stationary magnetic field.

Vitalii Dmitrievich Shafranov

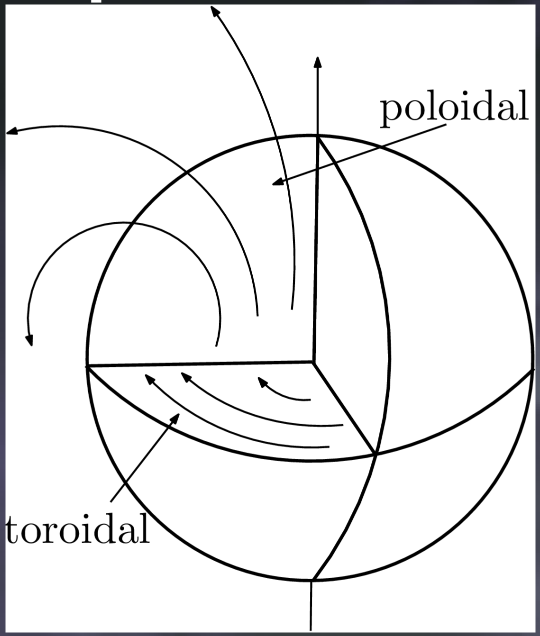
(H. Grad and H. Rubin (1958); Vitalii Dmitrievich Shafranov (1966))

How to make GS eq.

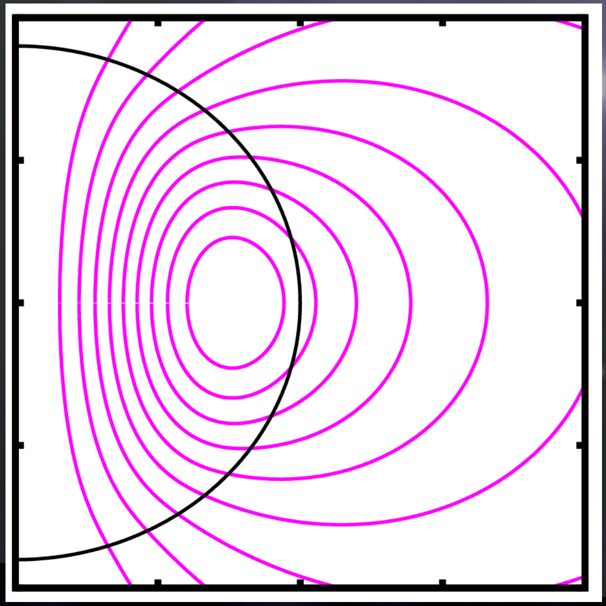
1. $$\frac{\partial}{\partial t} = 0 \qquad \frac{\partial}{\partial \varphi} = 0$$

$$\boldsymbol{B} = \frac{1}{r\sin\theta}\nabla\Psi(r,\theta)\times\boldsymbol{e}_\varphi + \frac{I(r,\theta)}{r\sin\theta}\boldsymbol{e}_\varphi$$

2.



3.



4. $$4\pi\frac{\boldsymbol{j}}{c} = \nabla\times\boldsymbol{B}.$$

$$\Delta^*\Psi = -4\pi r\sin\theta\frac{j_\varphi}{c}.$$

$$\Delta^* = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} - \frac{1}{r^2}\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\theta}\right).$$

Integrability condition and arbitrary functions

$$\frac{1}{\rho}\nabla p = -\nabla\phi_g + R\Omega^2\boldsymbol{e}_R + \frac{1}{\rho}\left(\frac{\boldsymbol{j}}{c}\times\boldsymbol{B}\right),$$

↓ ↓ ↓ ↓

$$\int\frac{dp}{\rho} = -\phi_g + \frac{1}{2}(r\sin\theta)^2\Omega_0^2 + \int F(\Psi)\,d\Psi + C,$$

Integrability condition → two arbitrary functions

$$\frac{\boldsymbol{j}}{c} = \frac{1}{4\pi}\frac{dI(\Psi)}{d\Psi}\boldsymbol{B} + \rho r\sin\theta F(\Psi)\boldsymbol{e}_\varphi$$

The functional form of jΦ

$$\frac{\boldsymbol{j}}{c} = \frac{1}{4\pi}\frac{dI(\Psi)}{d\Psi}\boldsymbol{B} + \rho r \sin\theta F(\Psi)\boldsymbol{e}_\varphi$$

- Force-free(I)　Non-Force-Free(F)

$$\boldsymbol{F} = \frac{\boldsymbol{j}}{c} \times \boldsymbol{B},$$

$$\frac{j_\varphi}{c} = \frac{1}{4\pi}\frac{dI(\Psi)}{d\Psi}\frac{I(\Psi)}{r\sin\theta} + \rho r \sin\theta F(\Psi).$$

Many studies give the functional forms of I & F as

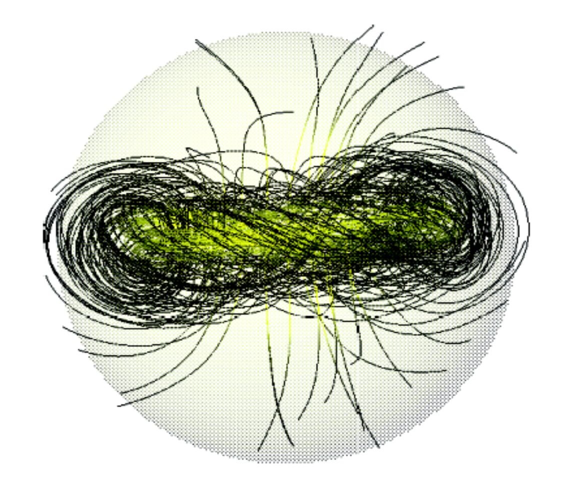
$$F(\Psi) = F_0.$$

$$I(\Psi) = \frac{I_0}{k+1}(\Psi - \Psi_{\max})^{k+1}\Theta(\Psi - \Psi_{\max})$$

Θ: Heaviside step function.

Yoshida & Eriguchi (2006); Lander & Jones (2009);

Stability analysis of magnetic fields



Twisted-Torus field

- Linear analysis
  Purely toroidal magnetic field
  or purely poloidal magnetic field
  → unstable (Taylor 1973; Markey & Taylor 1973)
- Numerical simulations
  Twisted-Torus fields → stable
  (Braithwaite & Spruit 2004; Braithwaite 2009)

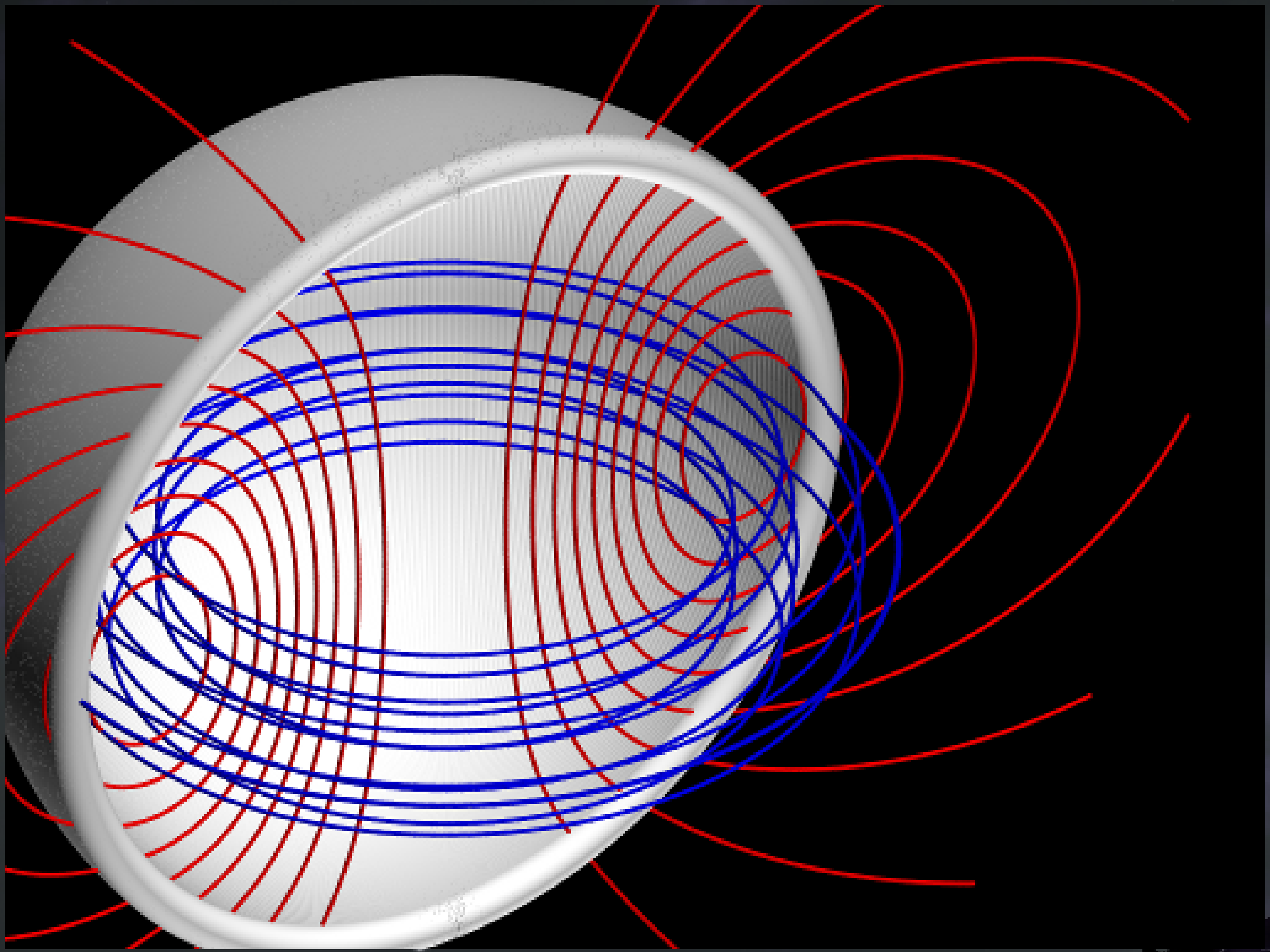
$$a \times \frac{M}{|W|} < \frac{M_p}{M} < 0.8 \rightarrow M_t > 0.25 M_p$$

M:magnetic energy; W:gravitational energy.

- Confirmed by analytic and semi-analytic studies (Akgün et al. 2013; Herbrik & Kokkotas 2015)

Twisted-torus fields with large toroidal component are favored

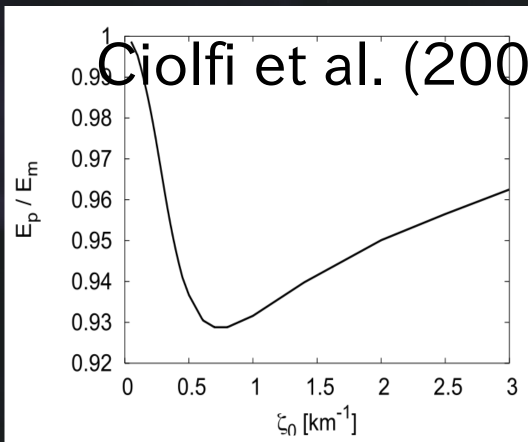
Magnetized equilibria with twisted field



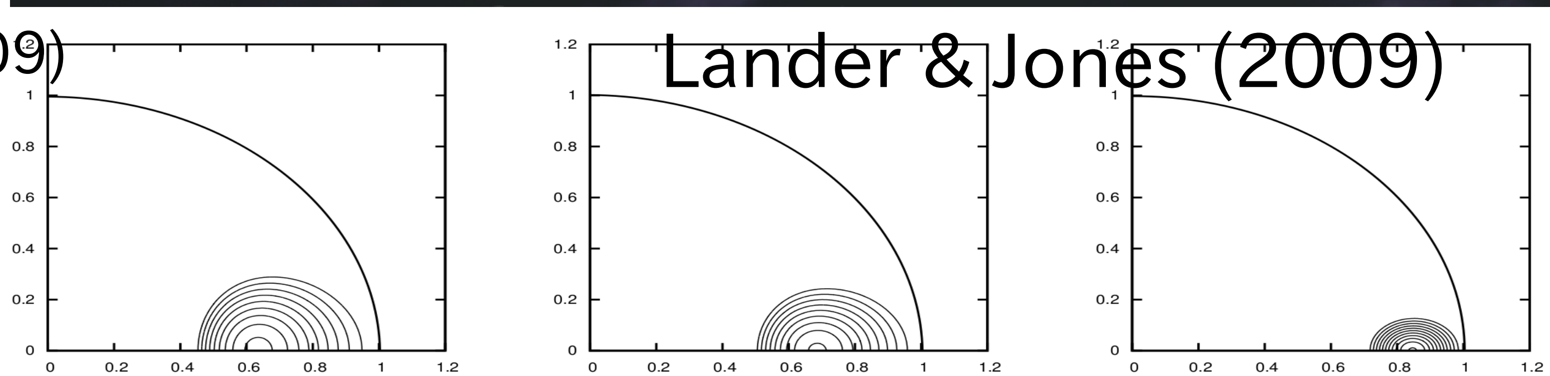
We obtain the twisted fields solutions, but ...

Fujisawa et al. (2012)

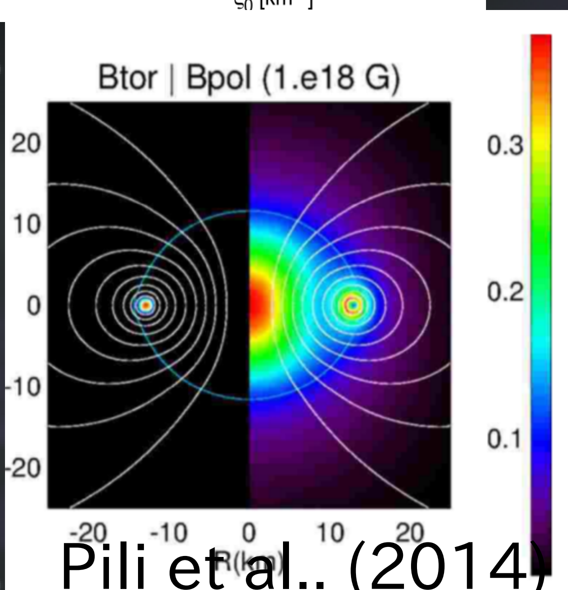
What's problem?



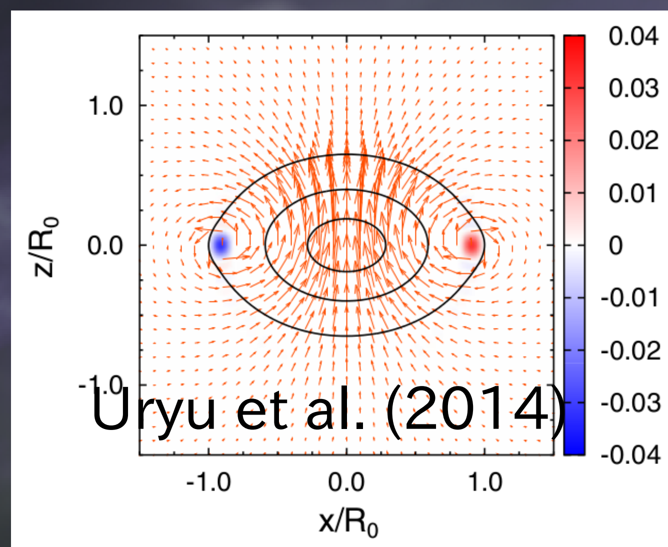
Ciolfi et al. (2009)



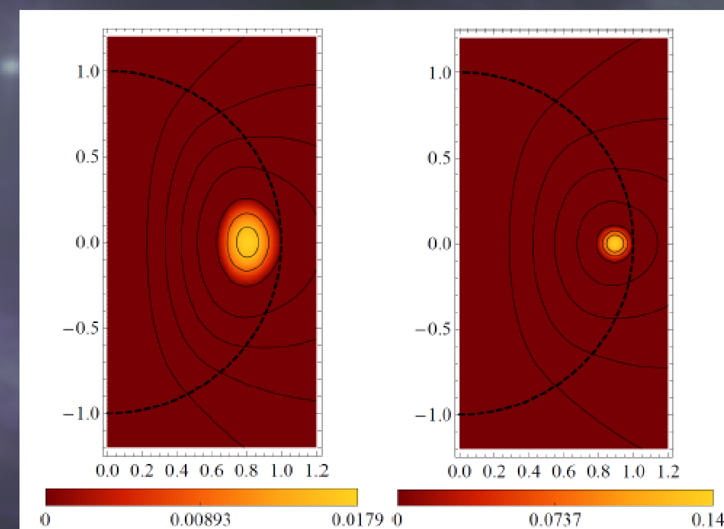
Lander & Jones (2009)



Pili et al.. (2014)



Uryu et al. (2014)

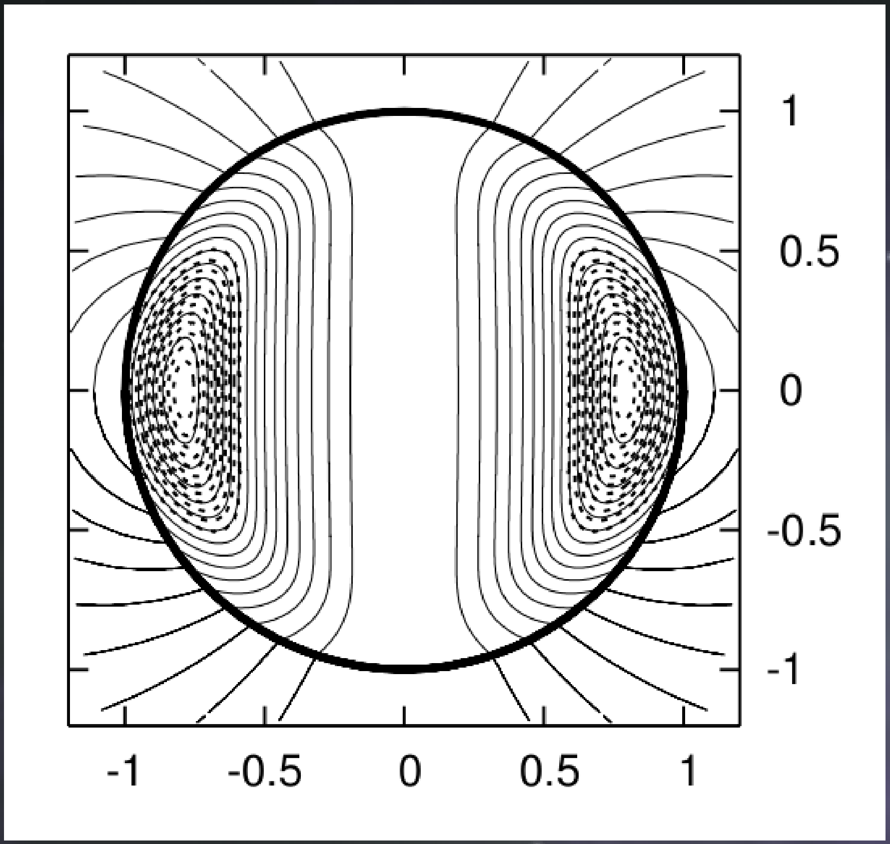


Armaza et al. (2014)

Almost all studies (self-consistent/perturbative in GR / Newtonian) obtained twisted-torus configurations, but they did not obtain the large toroidal solutions (typical values are Mt/M < 8%)...

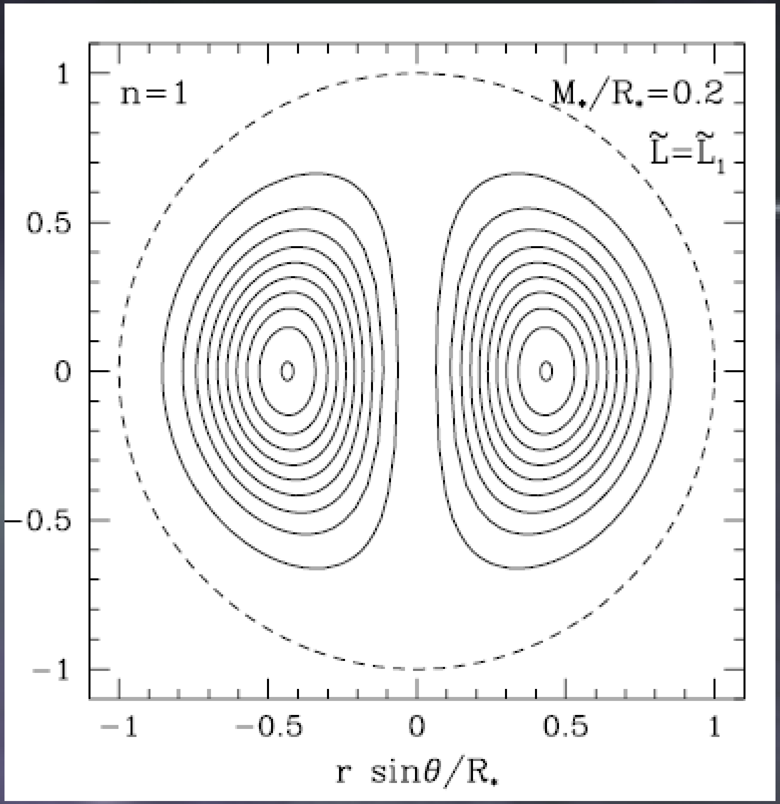
What's problem ?

Mt/M ～ 25%



Surface current makes
the toroidal magnetic field
energy large.
(Glampedakis et al. 2012)

Mt/M ～ 66%

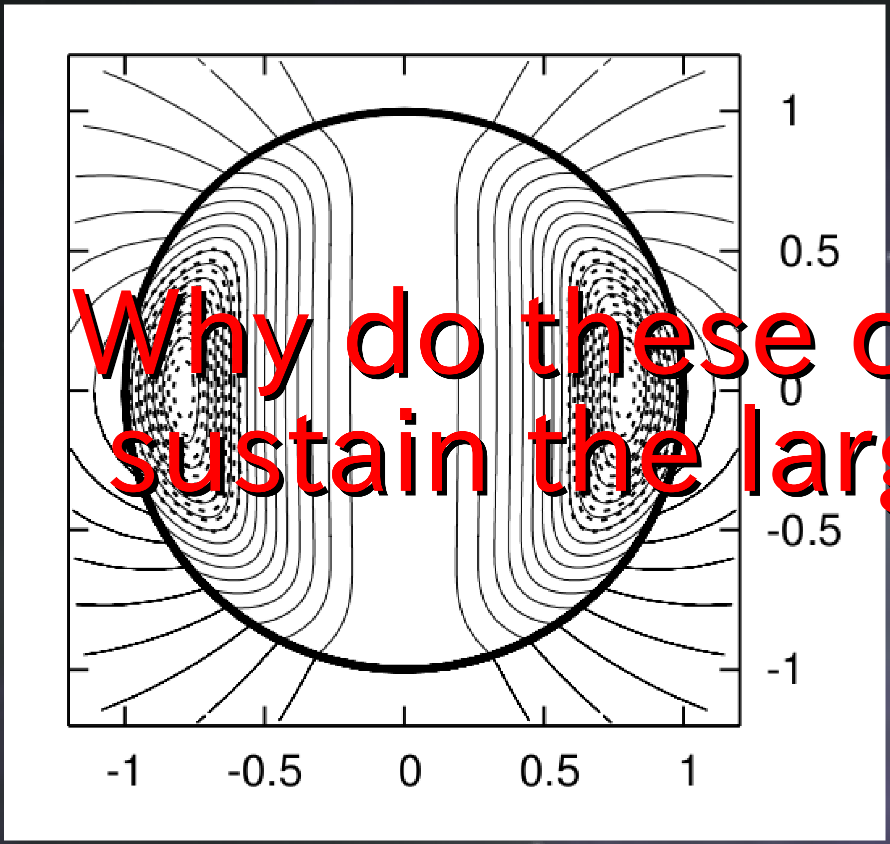


The closed fields sustain
the large toroidal field.

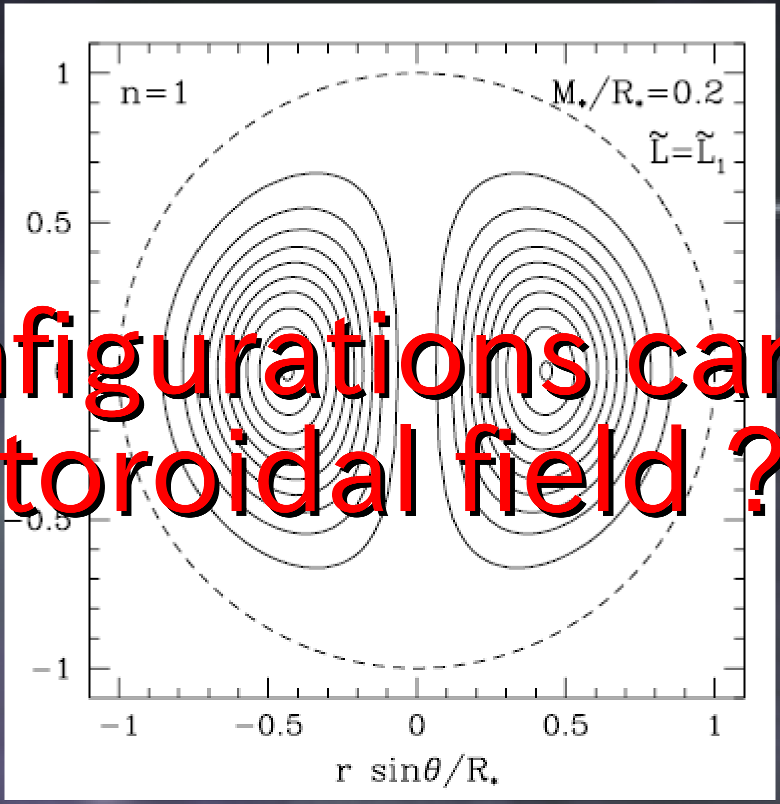
(Ioka & Sasaki (2004).
Duez & Mathis 2010;
Yoshida et al. 2012)

What's problem ?

Mt/M ～ 25%



Mt/M ～ 66%



Why do these configurations can sustain the large toroidal field ?

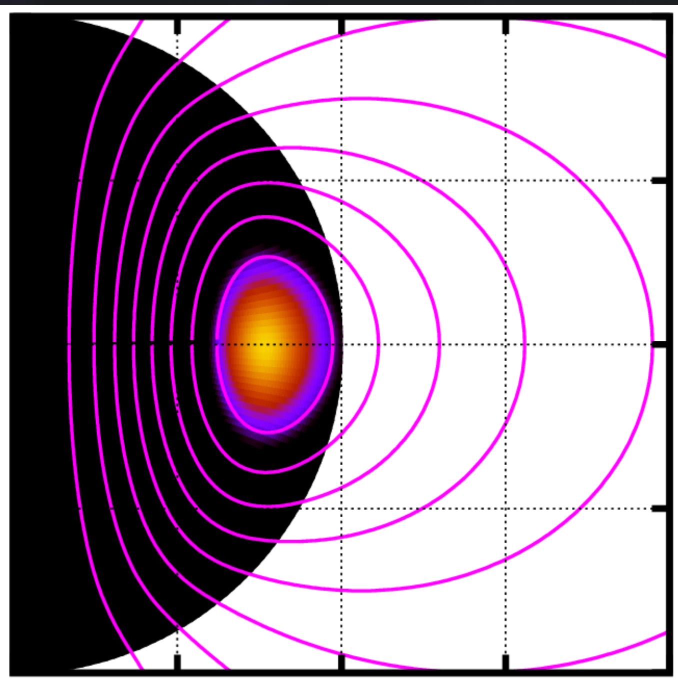
Surface current makes the toroidal magnetic field energy large.
(Glampedakis et al. 2012)

The closed fields sustain the large toroidal field.

(Ioka & Sasaki (2004).
Duez & Mathis 2010;
Yoshida et al. 2012)

TE solution

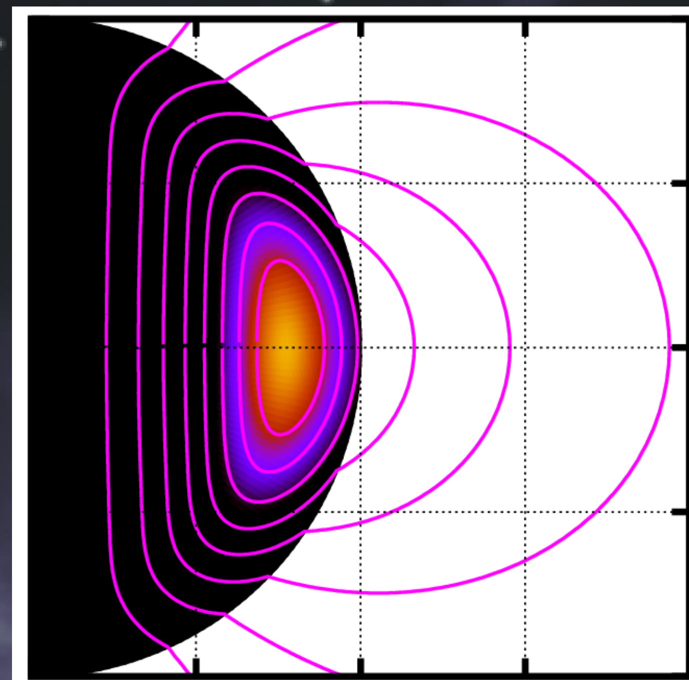
Mt/ M ～ 1%



Tomimura & Eriguchi (2005)
Yoshida & Eriguchi (2006)
Yoshida et al.(2006)
Ciolfi et al.(2009)
Lander & Jones (2009)
Fujisawa et al. (2012)
Gourgouliatos et al. (2013)
Uryu et al. (2014)
Armaza et al. (2015)

GAL solution

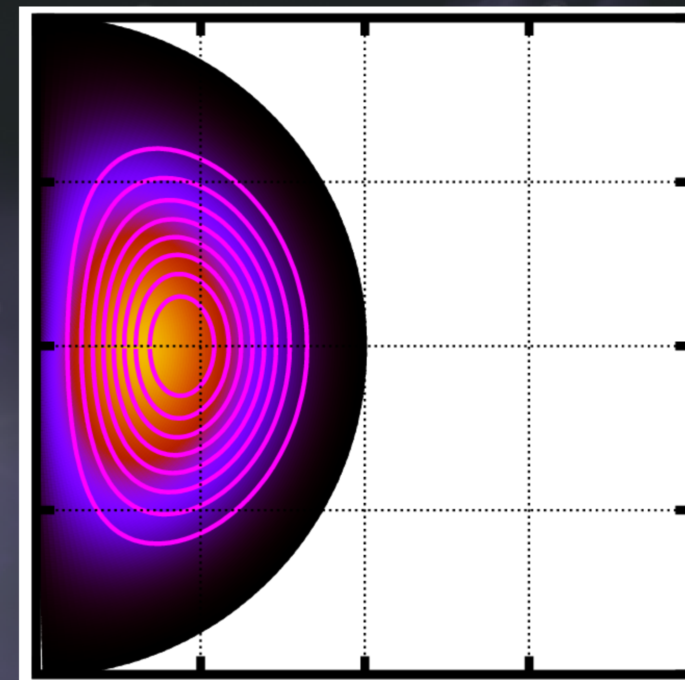
Mt/M ～ 25%



Glampedakis et.al(2012)
Colaiuda et al.(2008)

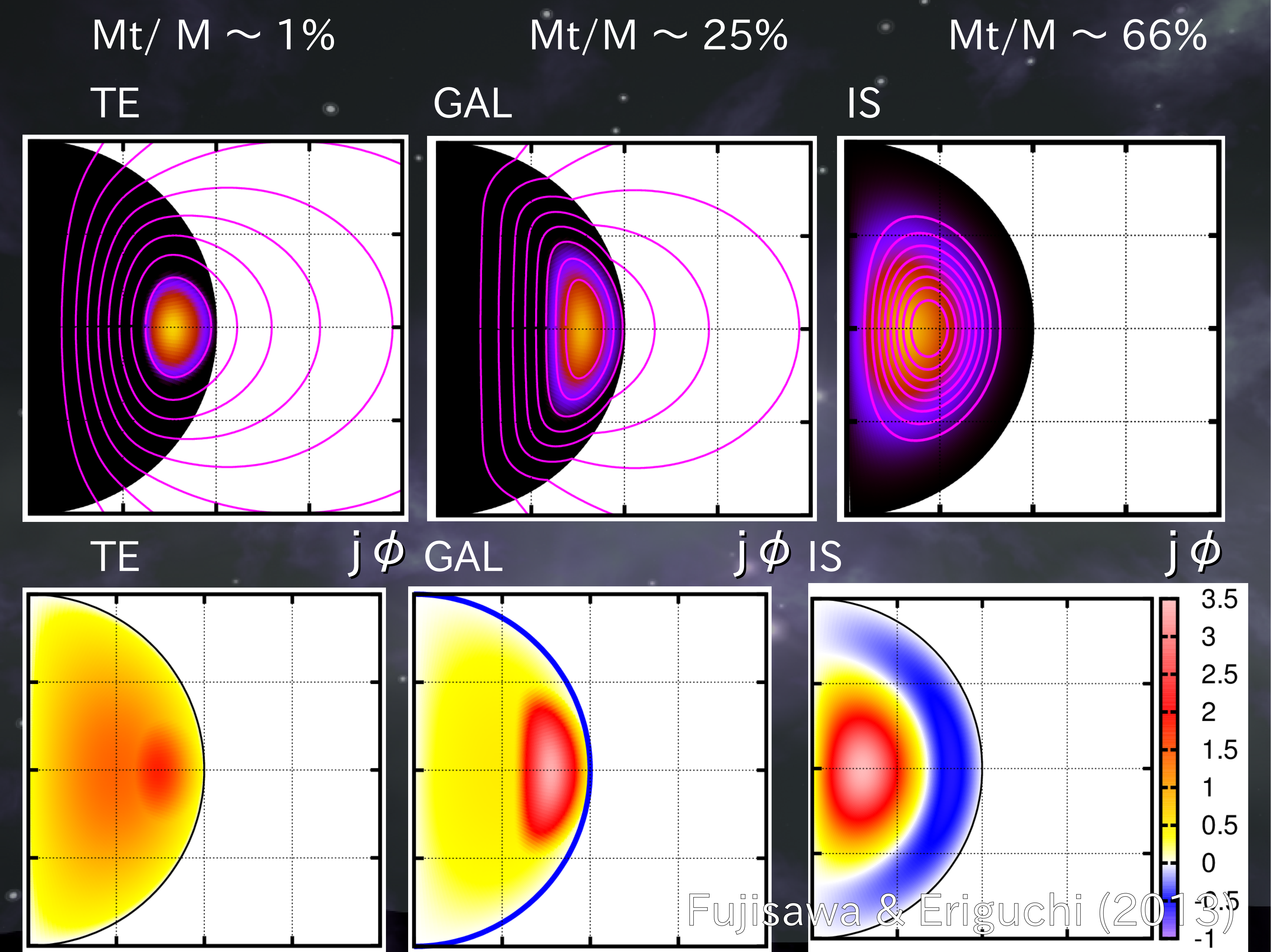
IS solution

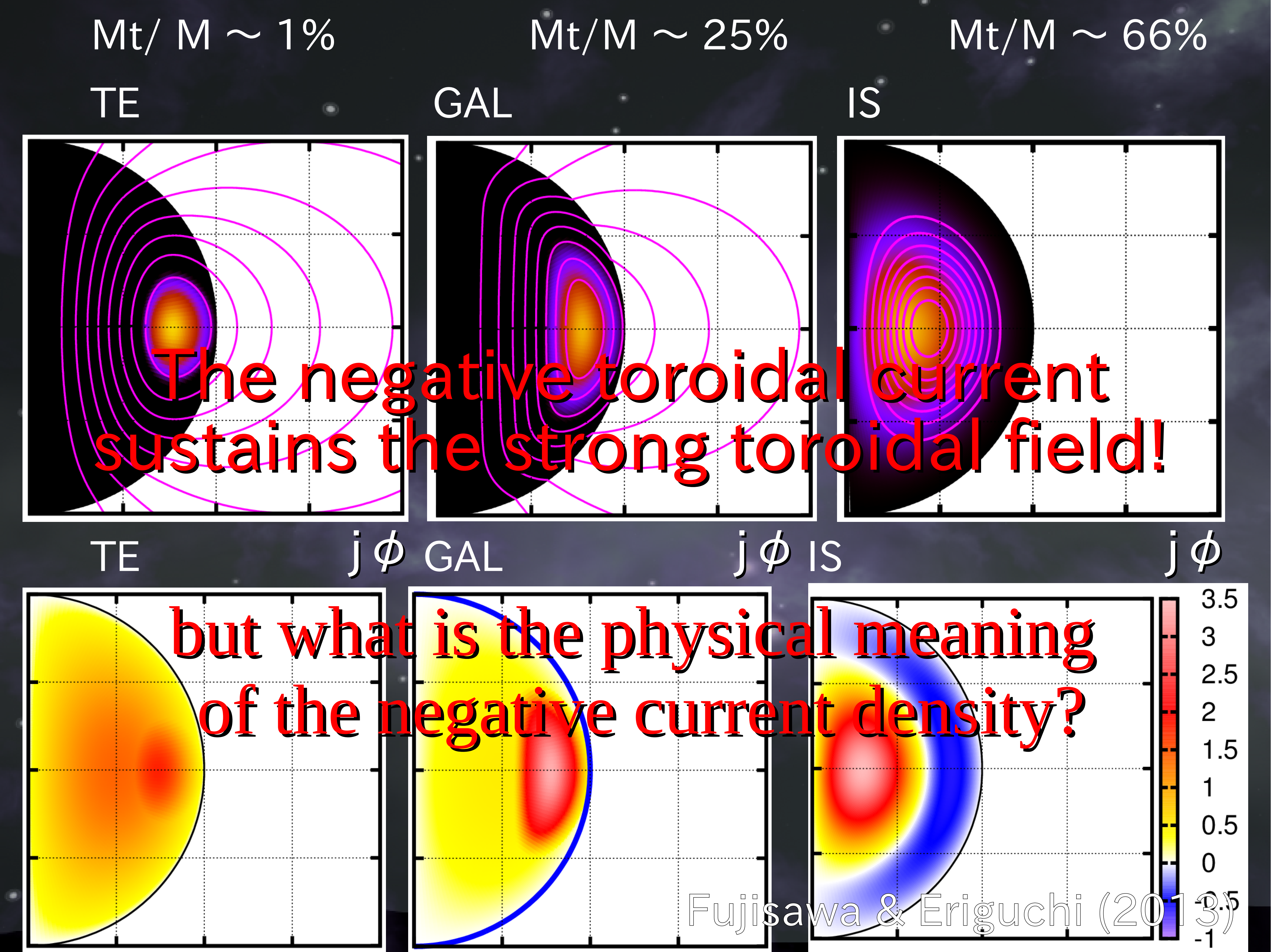
Mt/M ～ 66%

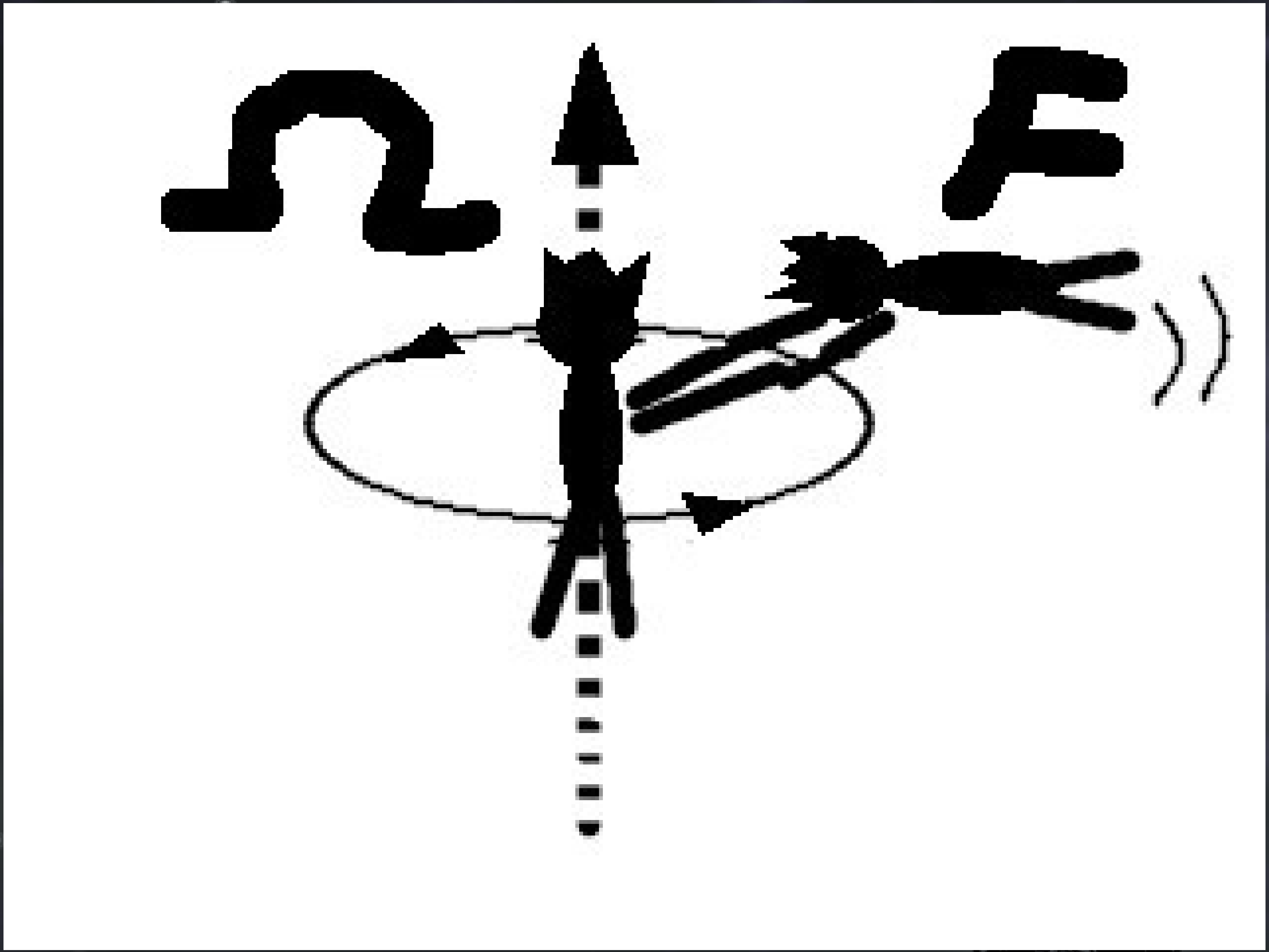


Ioka & Sasaki (2004)
Duez & Mathis (2010)
Yoshida et al. (2012)

Fujisawa & Eriguchi (2013)

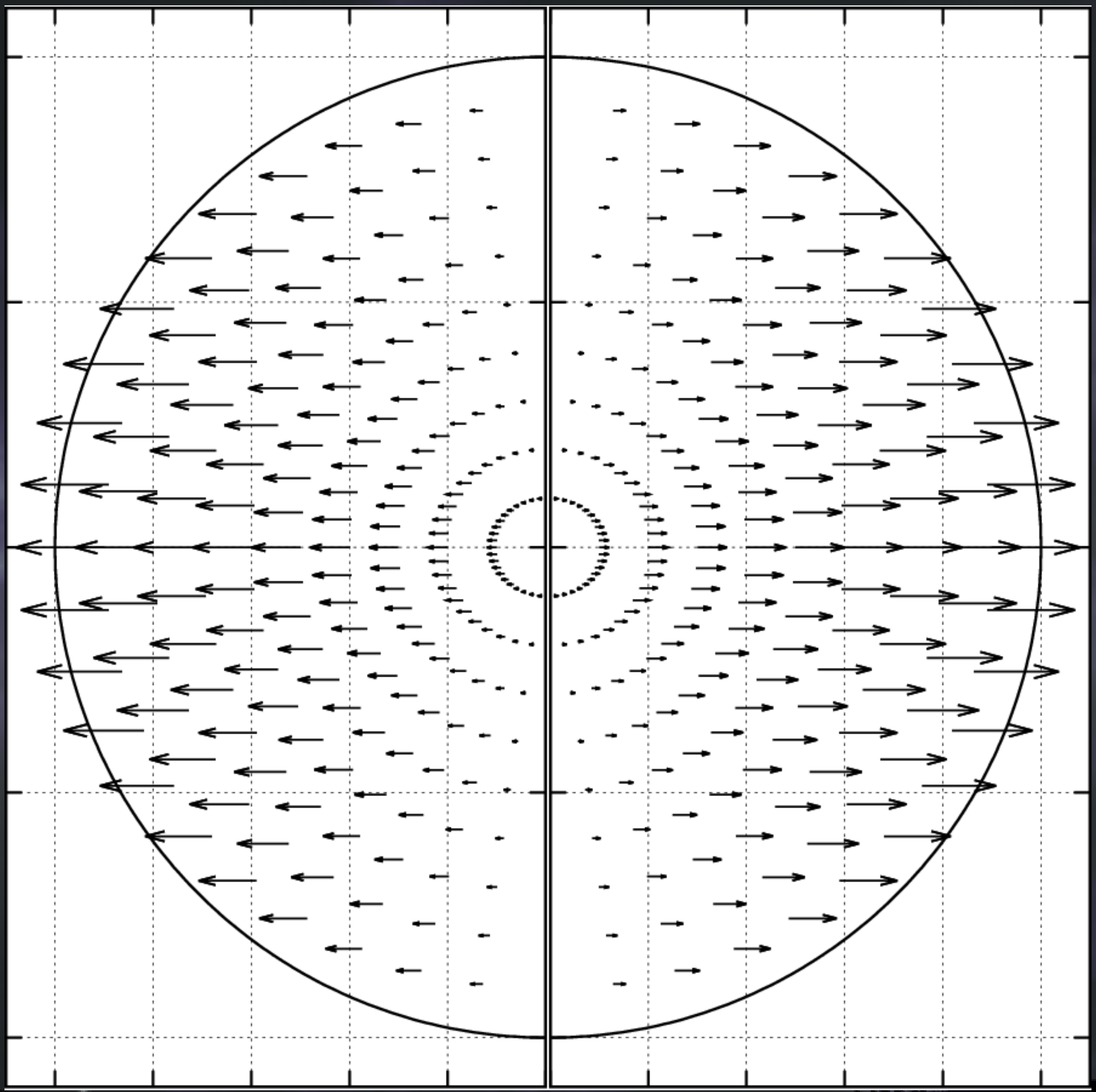

Mt/ M ～ 1%
Mt/M ～ 25%
Mt/M ～ 66%
TE
GAL
IS
TE
j φ
GAL
j φ
IS
j φ
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
Fujisawa & Eriguchi (2013)


Mt/ M ～ 1%
Mt/M ～ 25%
Mt/M ～ 66%
TE
GAL
IS
The negative toroidal current
sustains the strong toroidal field!
TE
jφ
GAL
jφ
IS
jφ
but what is the physical meaning
of the negative current density?
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
Fujisawa & Eriguchi (2013)

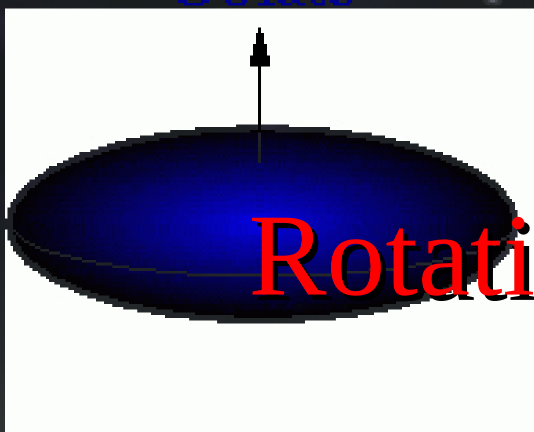


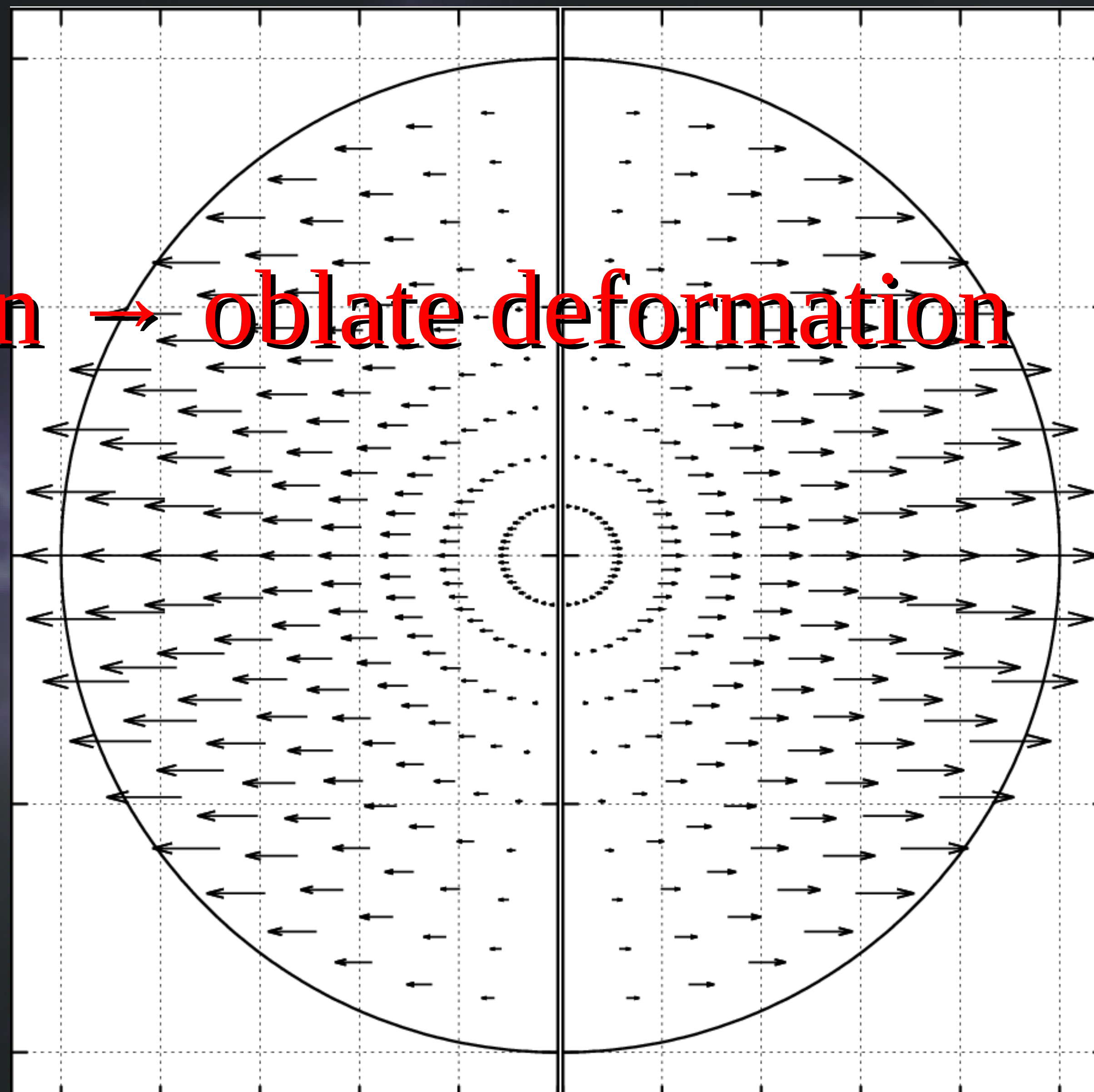
From http://d.hatena.ne.jp/naruto_AG23/20080730/p1

Deformation of rotating star



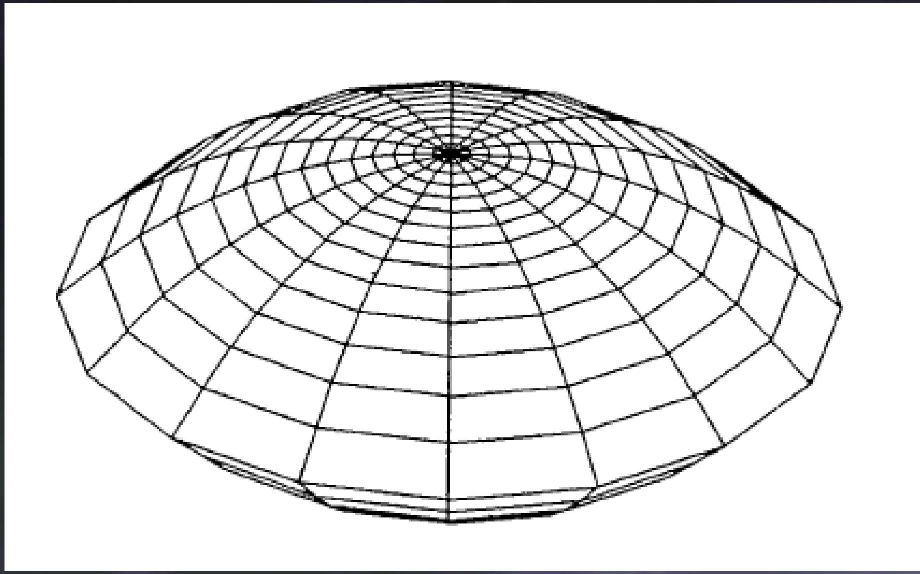
Deformation of rotating star

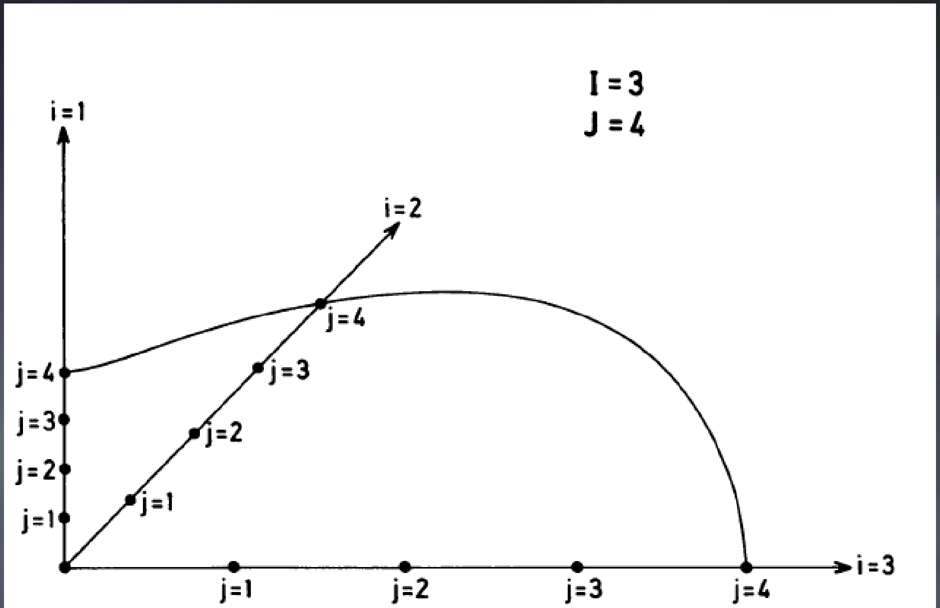




Rotation → oblate deformation

Deformation of rotating star



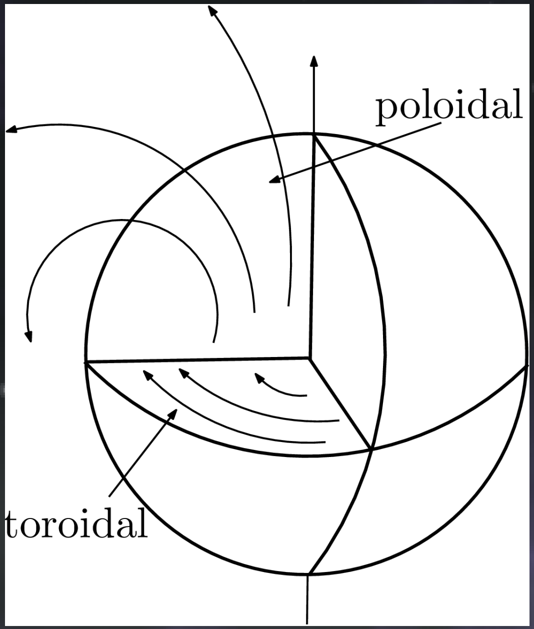


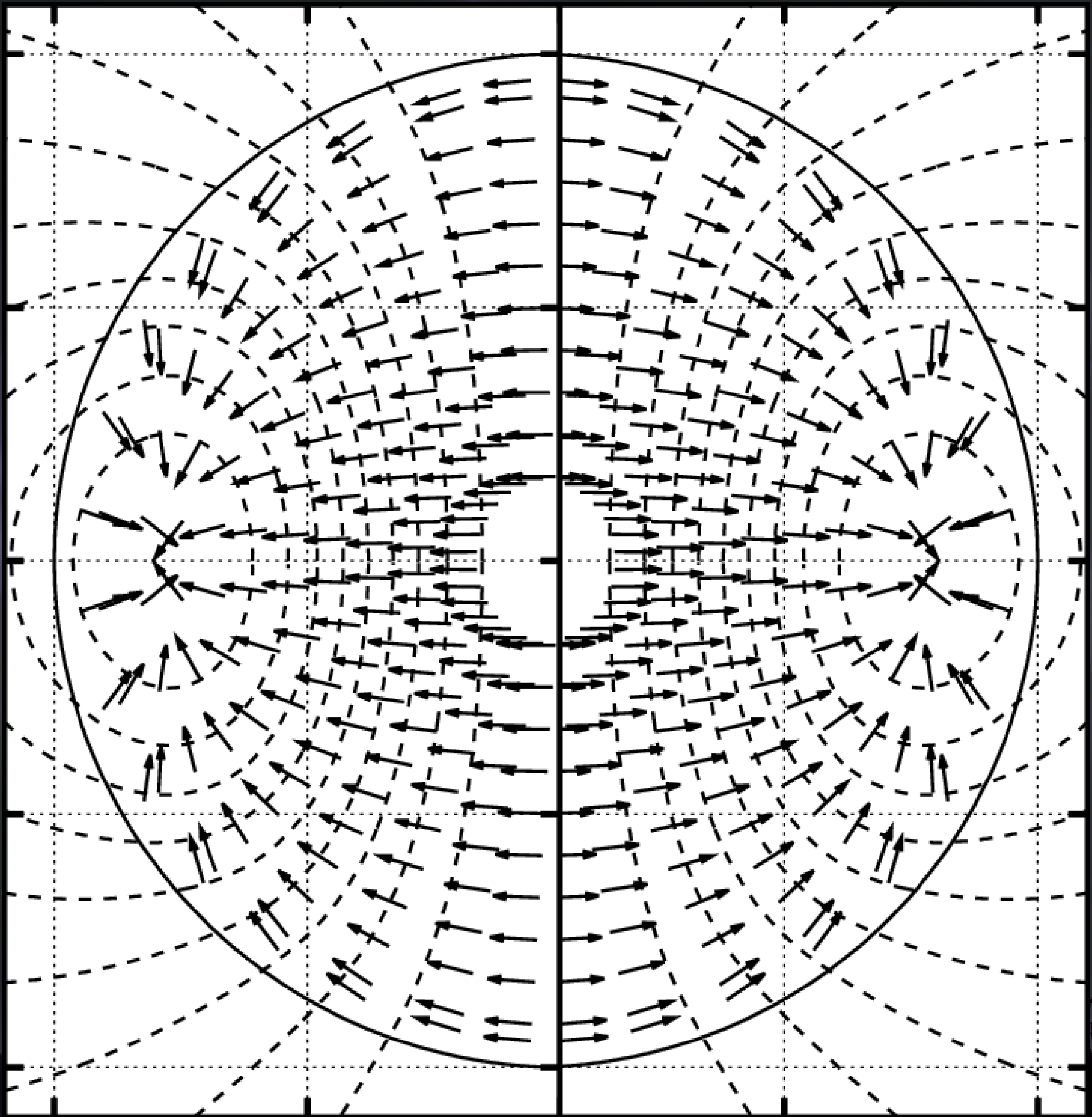
Eriguchi & Mueller (1985); Hachisu (1986)

Rapidly rotation = Large oblate deformation

Lorentz force due to purely poloidal field

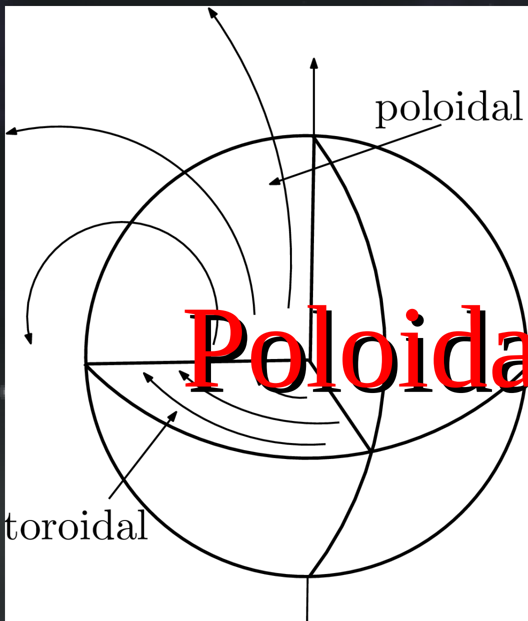
Ferraro (1954)



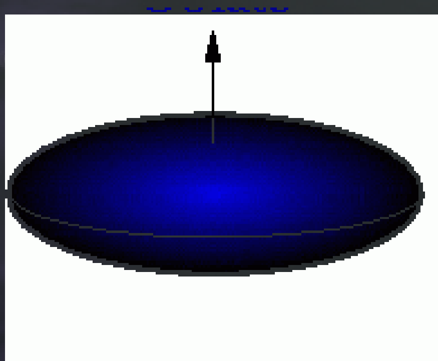


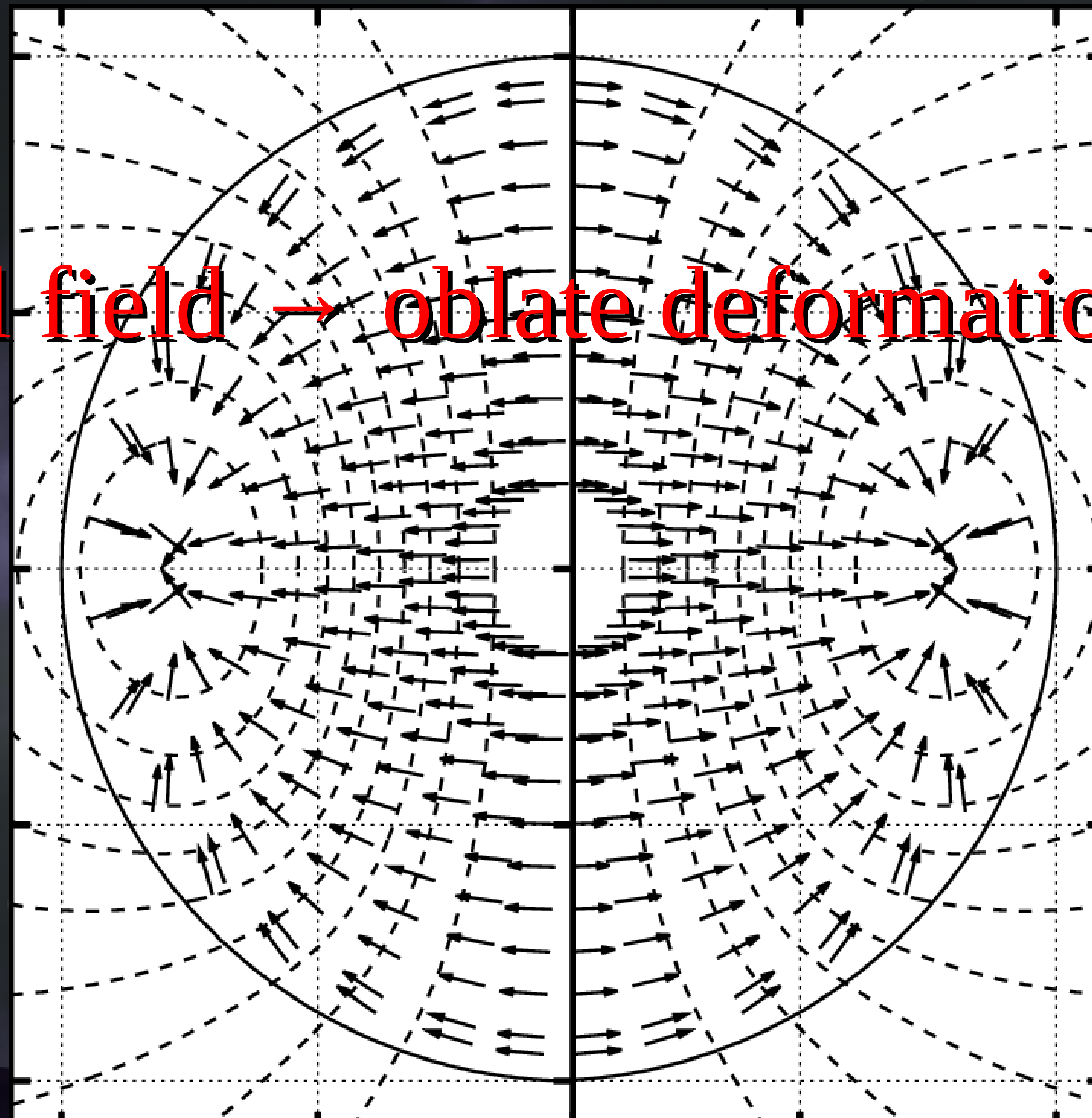
Lorentz force due to purely poloidal field

Ferraro (1954)

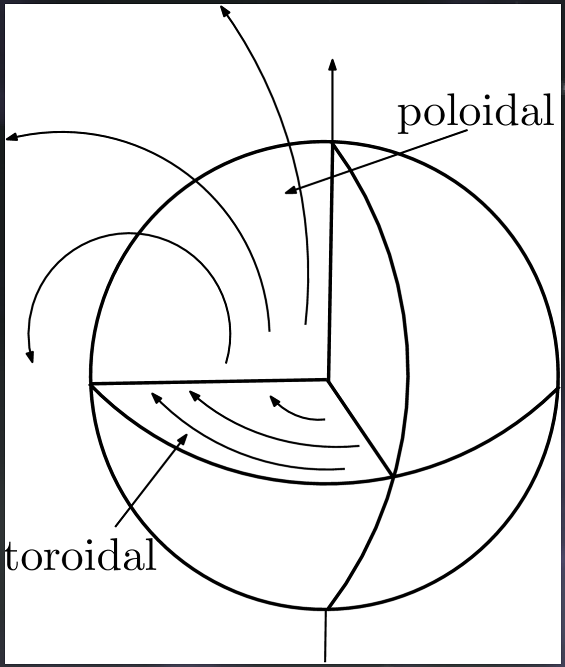


Poloidal field → oblate deformation



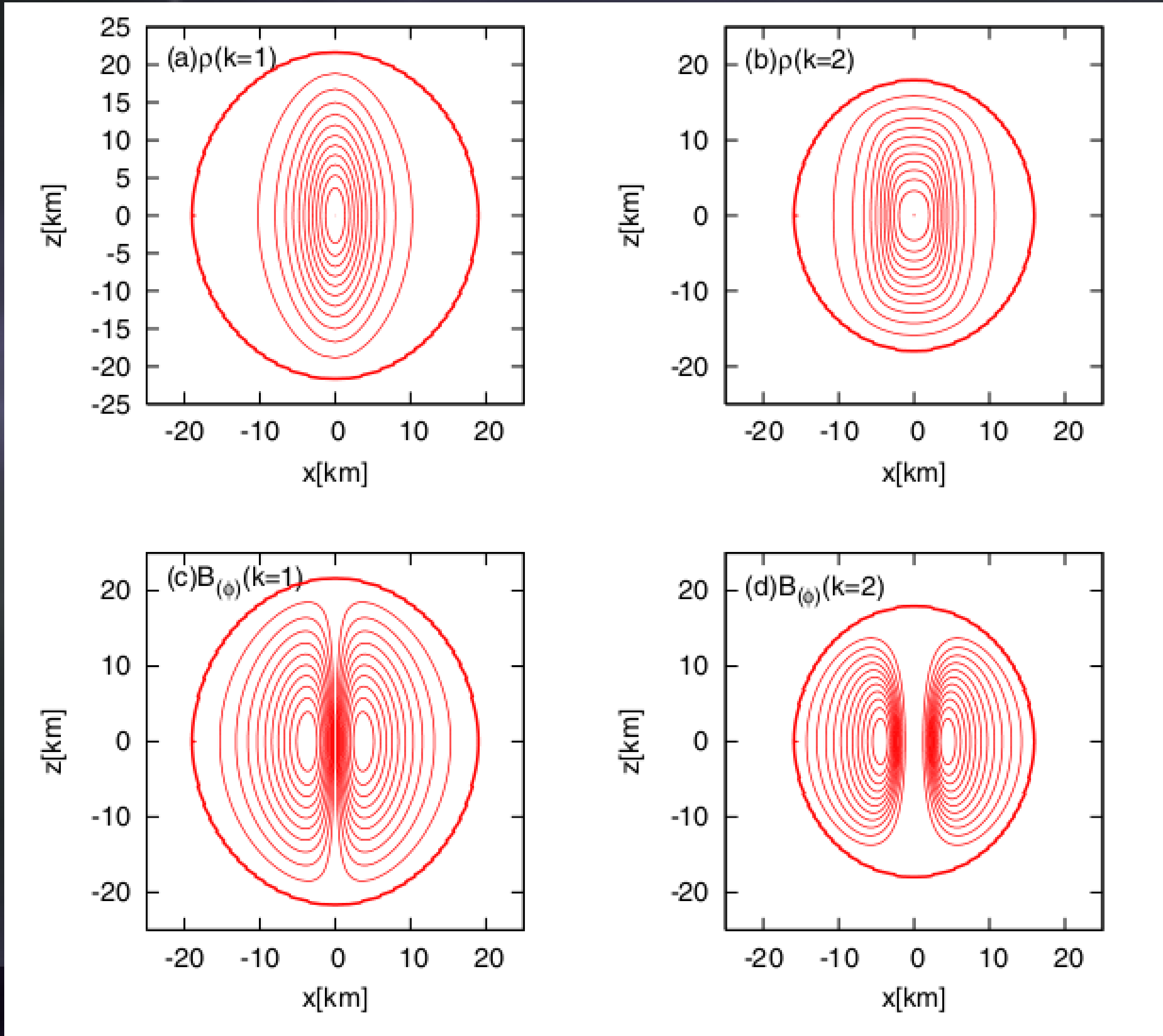


Deformation due to toroidal field



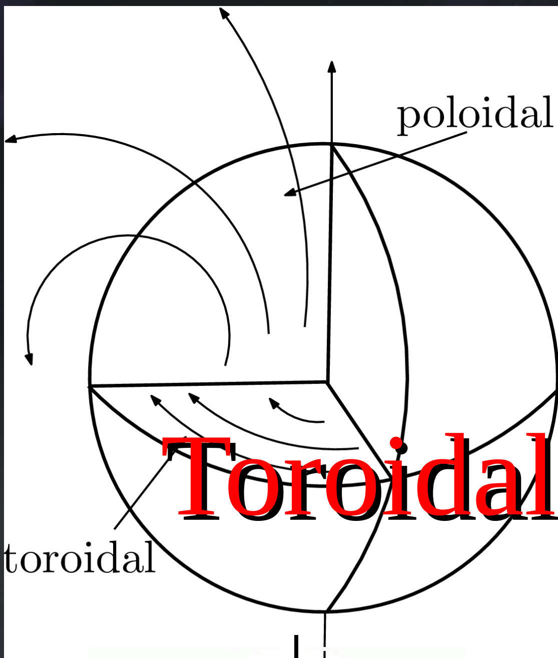
Miketinac (1973)

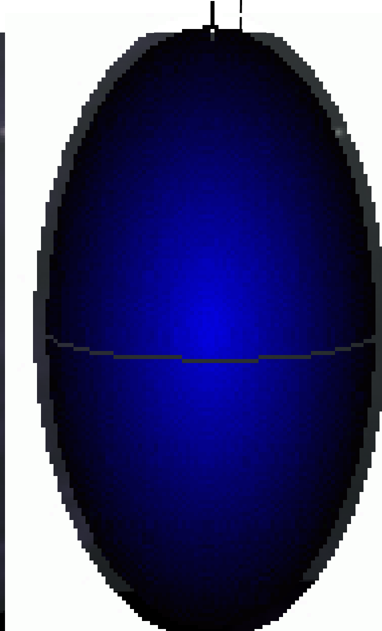
Kiuchi & Yoshida (2008)

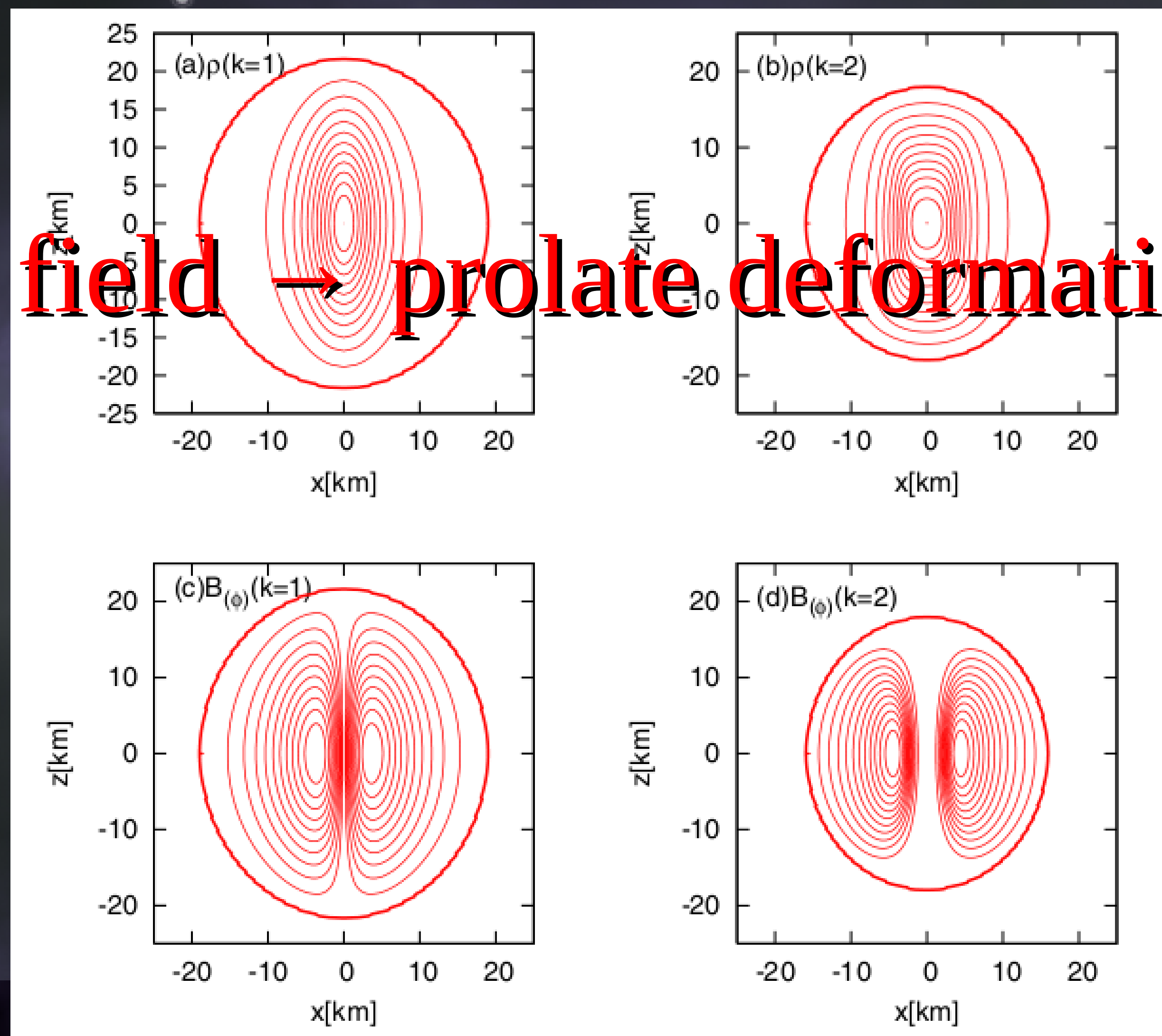


Deformation due to toroidal field

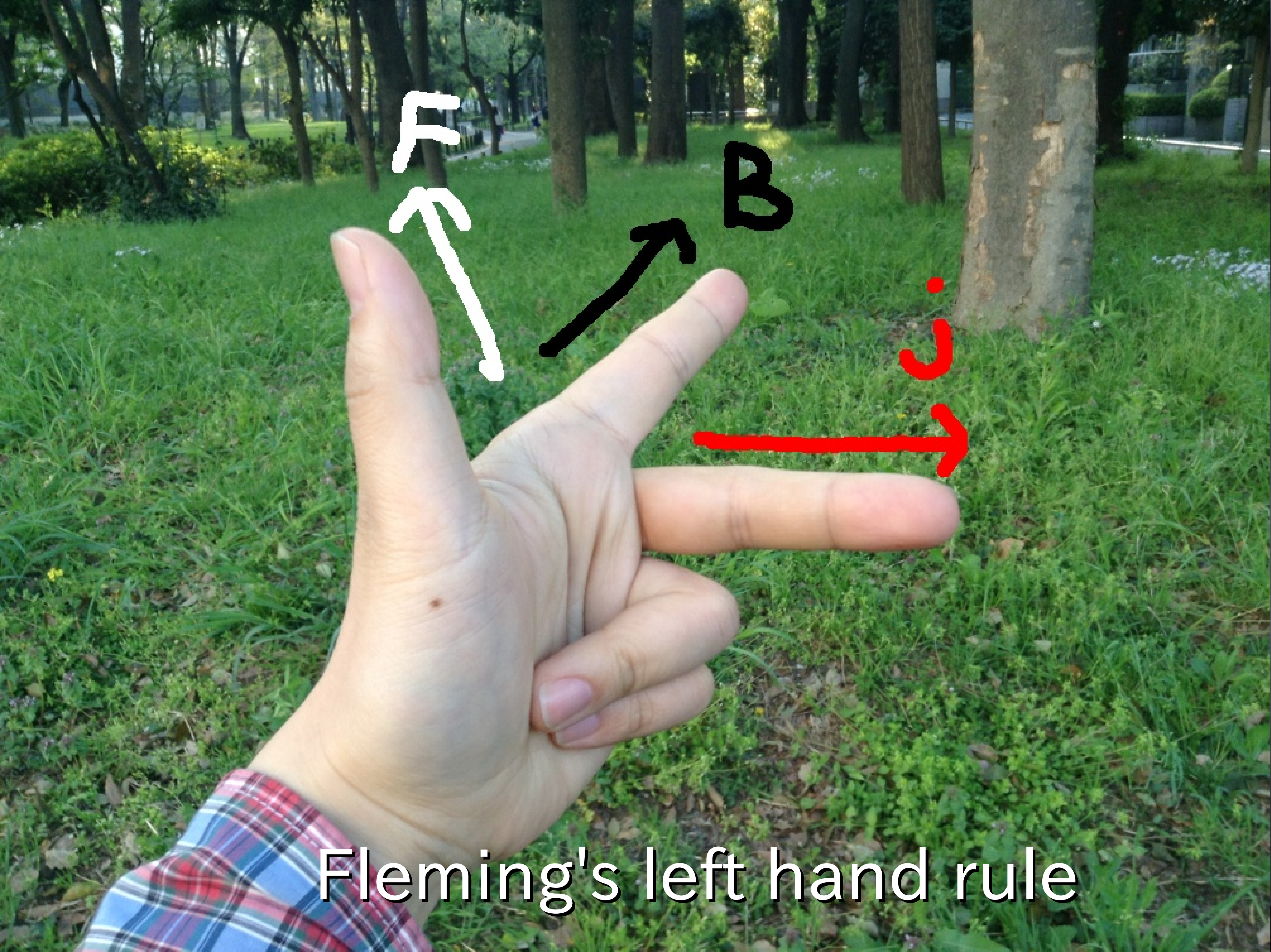
Miketinac (1973)
Kiuchi & Yoshida (2008)







Toroidal field → prolate deformation

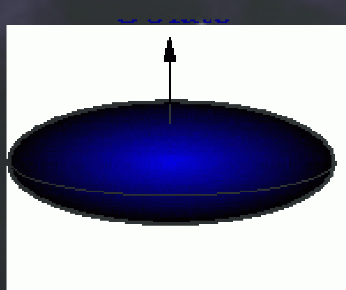

F
B
j
Fleming's left hand rule

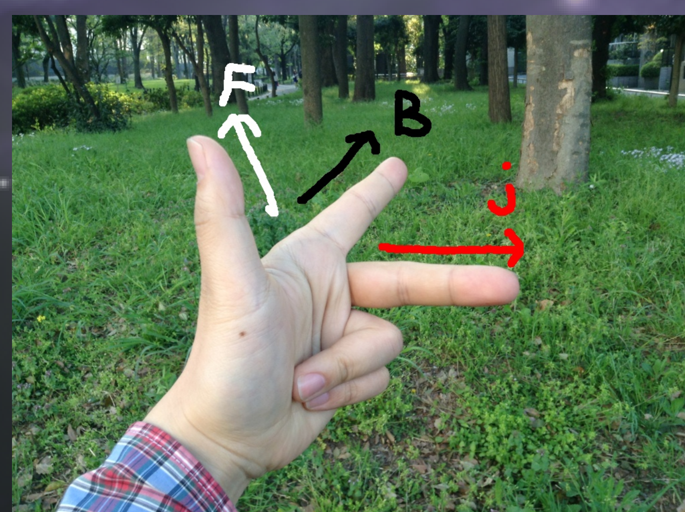
Relations among current, toroidal fields and deformation

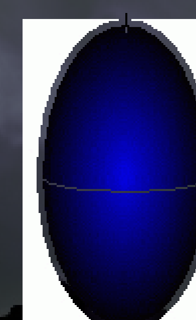
small ~ B_Φ ~ large

- positive ~ current ~ negative

- oblate ~ deformation ~ prolate







(Fujisawa & Eriguchi 2015)

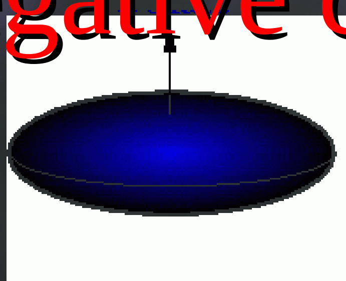
Relations among current, toroidal fields and deformation

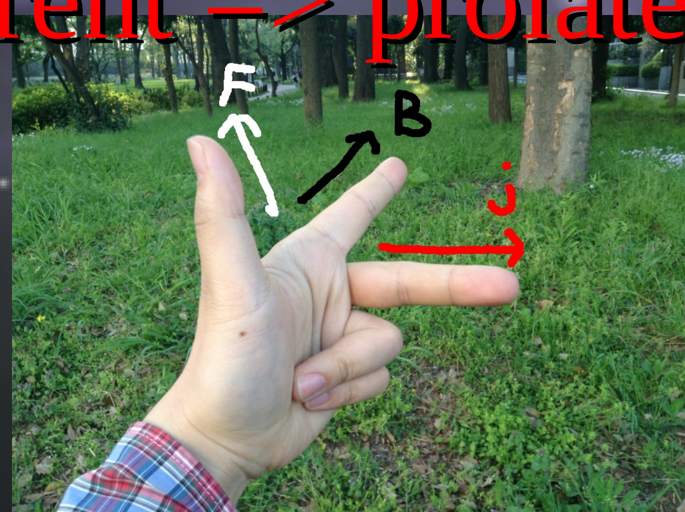
small ～ B_Φ ～ large

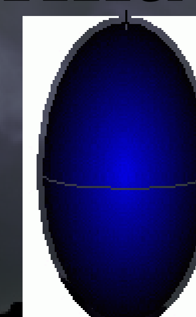
- positive ～ current ～ negative

oblate ～ deformation ～ prolate

Negative current => prolate deformation







(Fujisawa & Eriguchi 2015)

Strategies for large toroidal field

1. Make arbitrary functions which give the negative current density

2. Multi-components (core-crust) models with negative current density

3. Non-barotropic model

Strategy 1

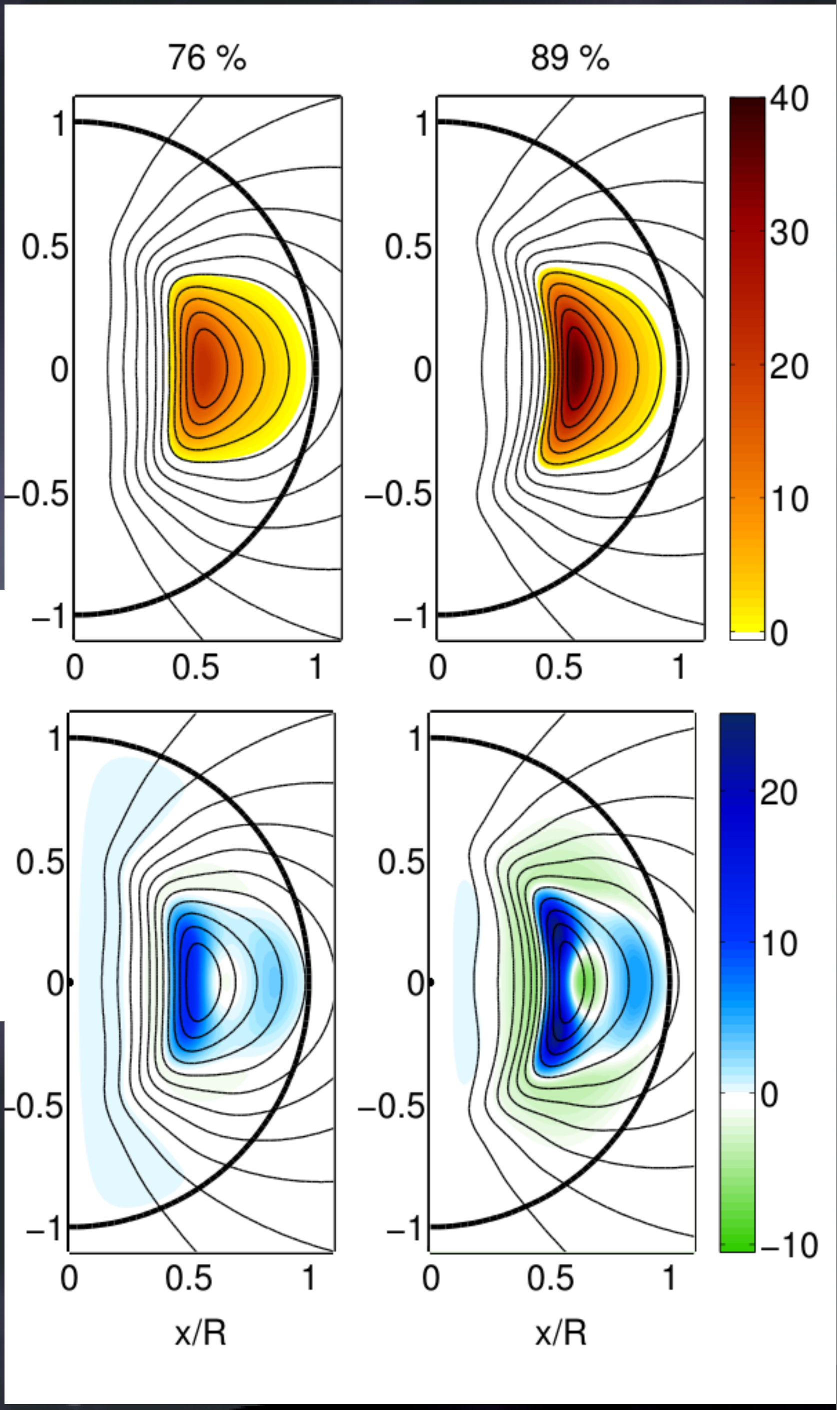
Mt/M = 76%, 89%

Ciolfi & Rezzolla (2013)

- Perturbative method in GR
- Very complicated functional form

limit would give a poloidal field entirely confined in the star (see also Fujisawa et al. 2012). If toroidal fields are included, a larger closed-line region would still undergo a contraction, but we expect the maximum toroidal-field energy to be considerably larger. To produce a larger region of closed field lines we extend (2) as

$$F(\psi) = c_0 \left[(1 - |\psi/\bar{\psi}|)^4 \Theta(1 - |\psi/\bar{\psi}|) - \bar{k} \right] , \qquad (3)$$



Strategy 2

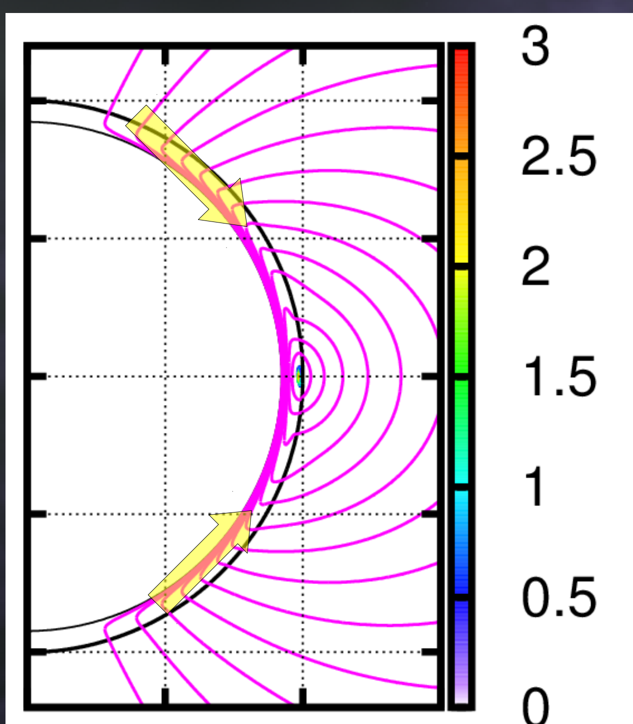
Core-crust model

Fujisawa & Kisaka (2014)

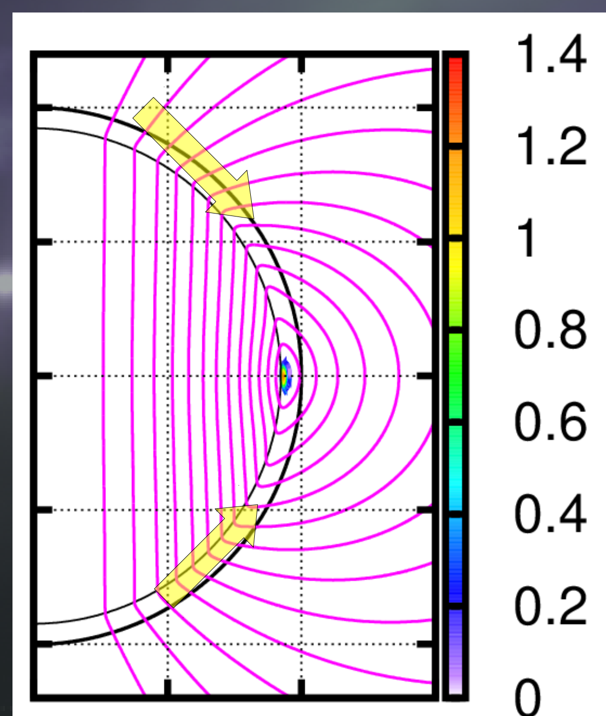
Mass ~ 1.4 solar mass; Radius ~ 1.3 km

(SLy. EOS Douchin & Haensel 2001)

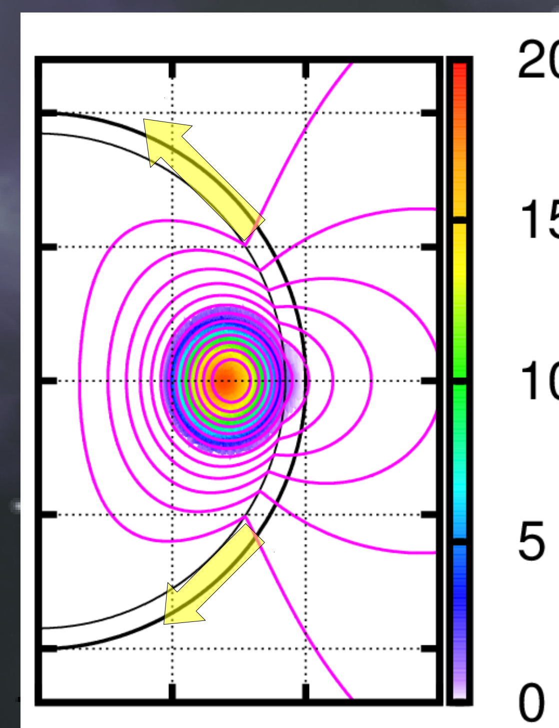
Mt/M ~ 0.01%



Mt/M ~ 1%

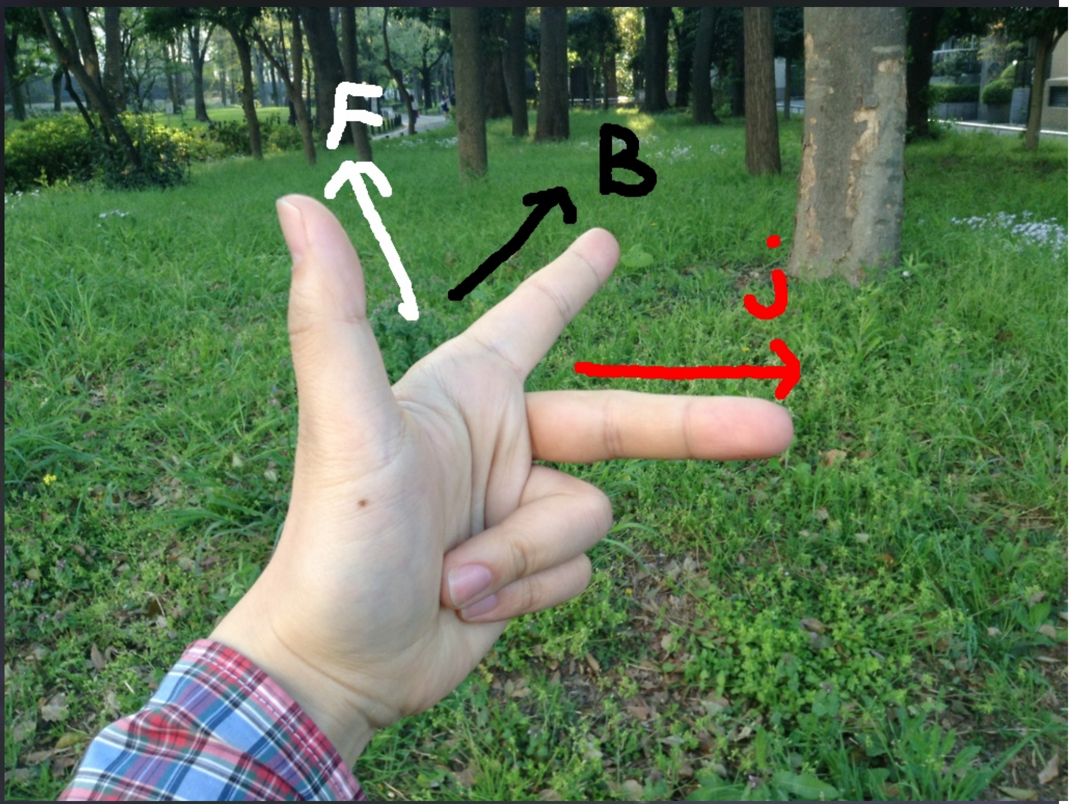


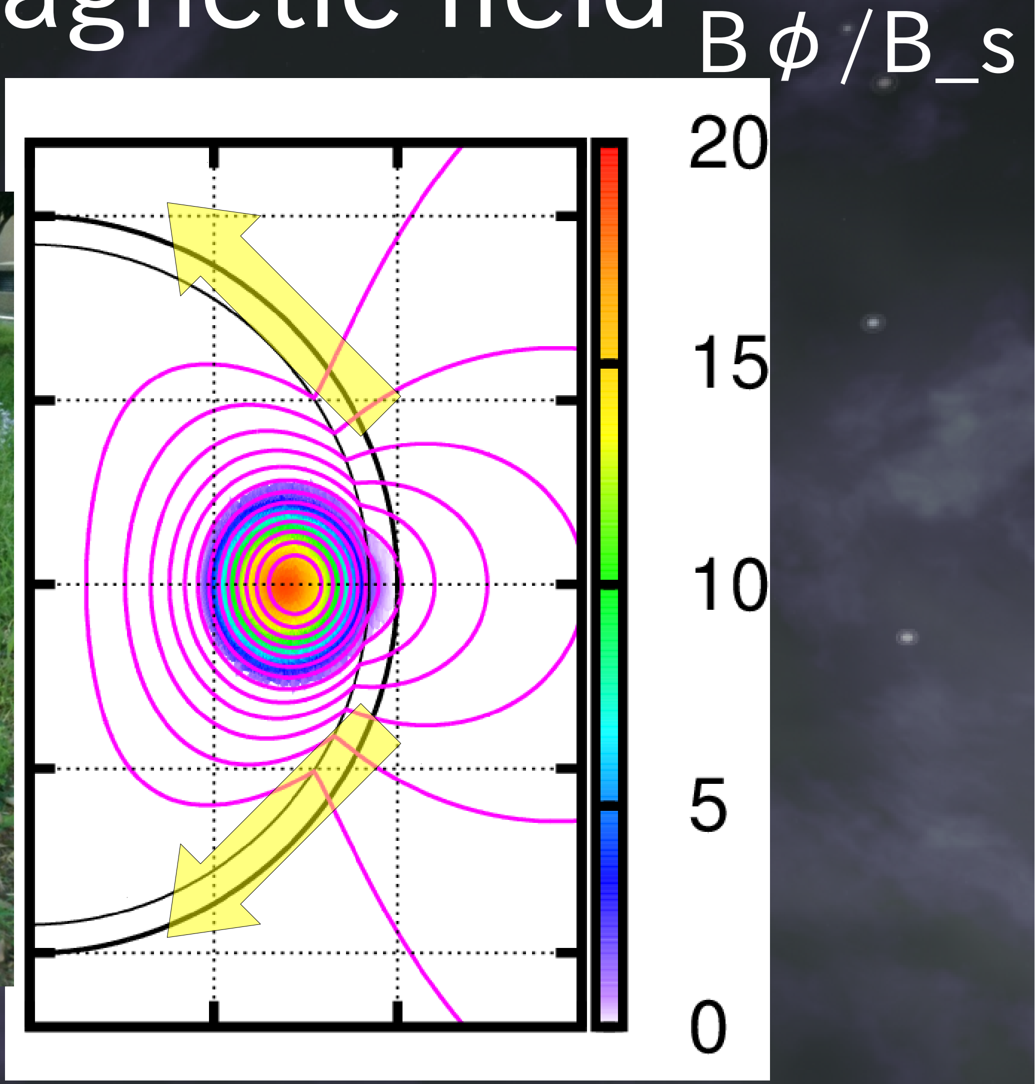
Mt/M ~ 33%



Strategy 2

Core-Crust magnetic field

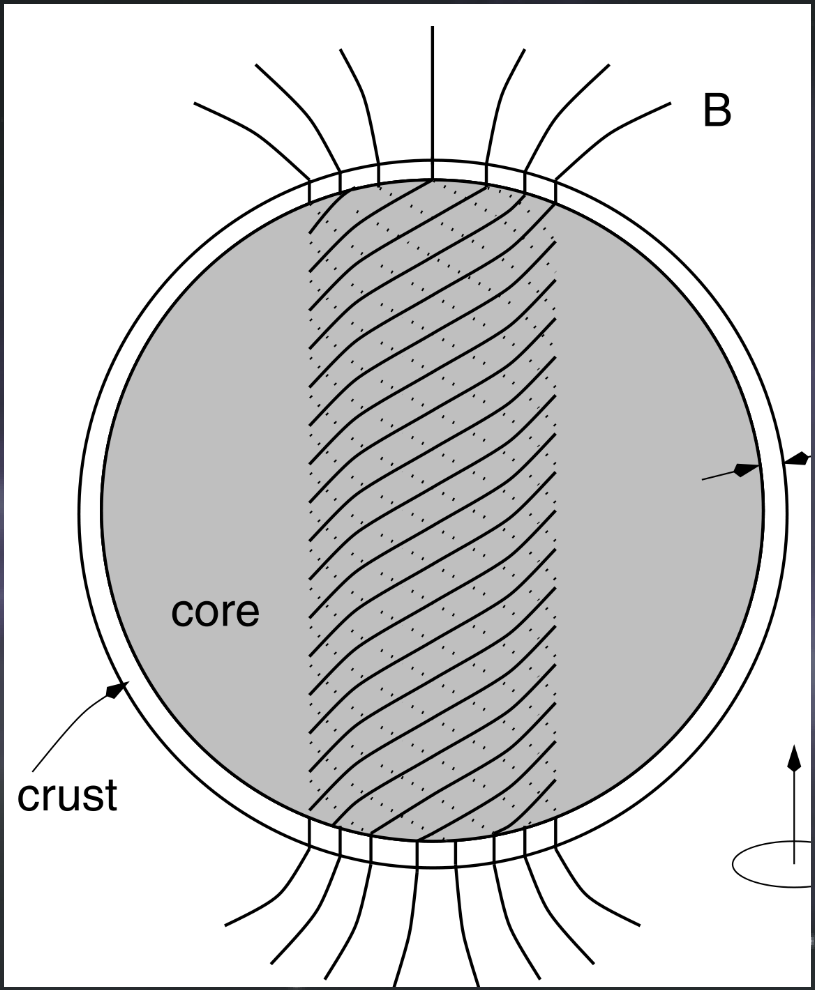




The negative current means
the magnetic stress on the crust.

Strategy 2

Negative current
→ core-crust boundary!



Thompson & Duncan (2001)

Strategy 3

Baroclinic

.p is not a function of ρ

$$p \neq p(\rho)$$

⇔ barotropic

.p is a function of ρ

$$p = p(\rho)$$

Baroclinic increases degrees of freedom of the solutions!

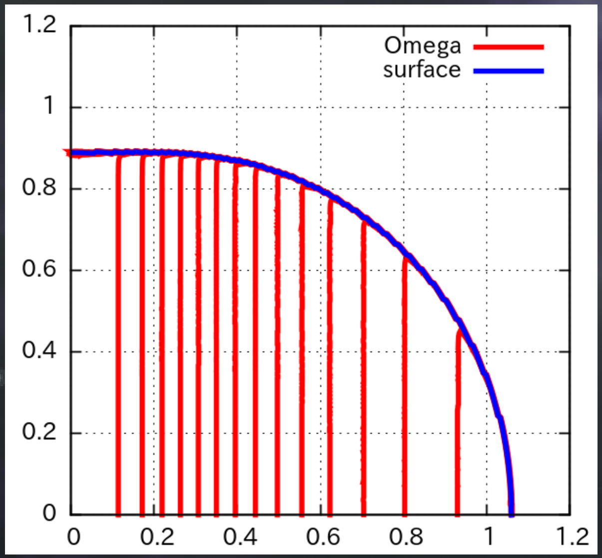
Strategy 3

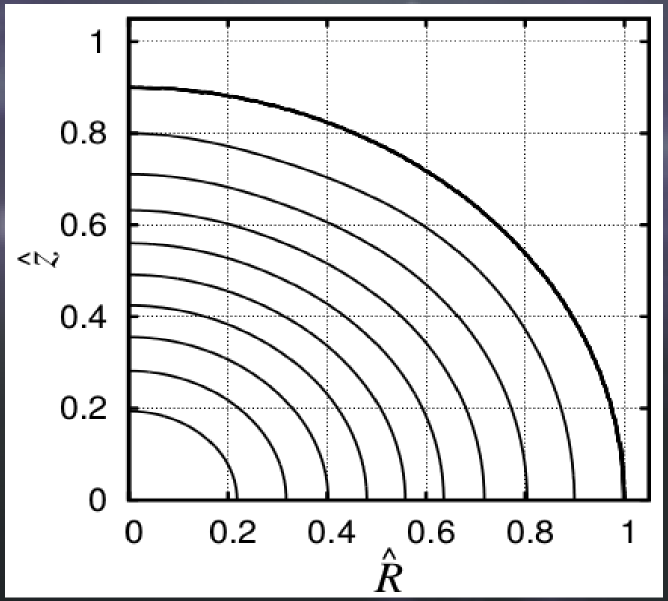
Baroclinic rotating star

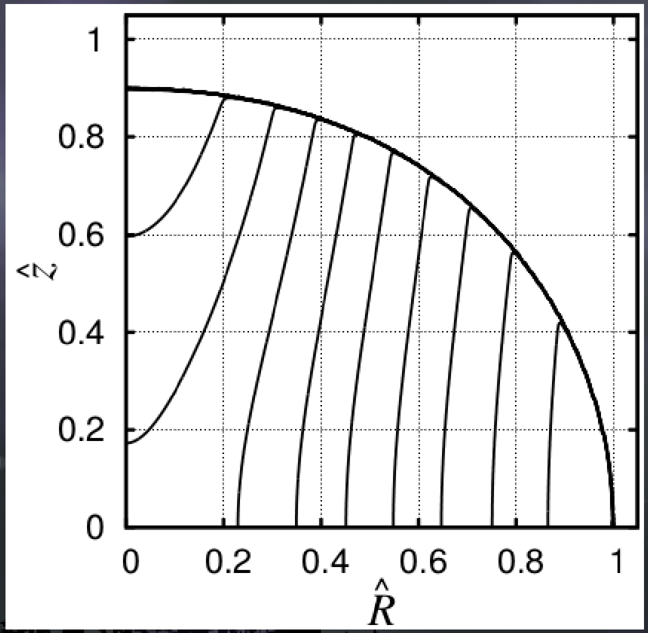
Fujisawa (2015b)

Angular velocity (Ω) distributions

- barotropic
- barotroclinic







c.f. von Zeipel (1924)

Summary

- The distribution of current density is important to consider the magnetic field configurations.
  →Negative current density can sustain the strong toroidal magnetic field and induce the prolate deformation.
- Negative current density is realized by
  special functional form
  core – crust model
  non-barotropic model (baroclinic)
- These magnetic fields are useful for the stability analysis and mode analysis of magnetars.

Magnetic fields!