Pion Production via Proton Synchrotron Radiation in Strong Magnetic Fields in Relativistic Quantum Approach

Particle Productions in TeV Energy Region

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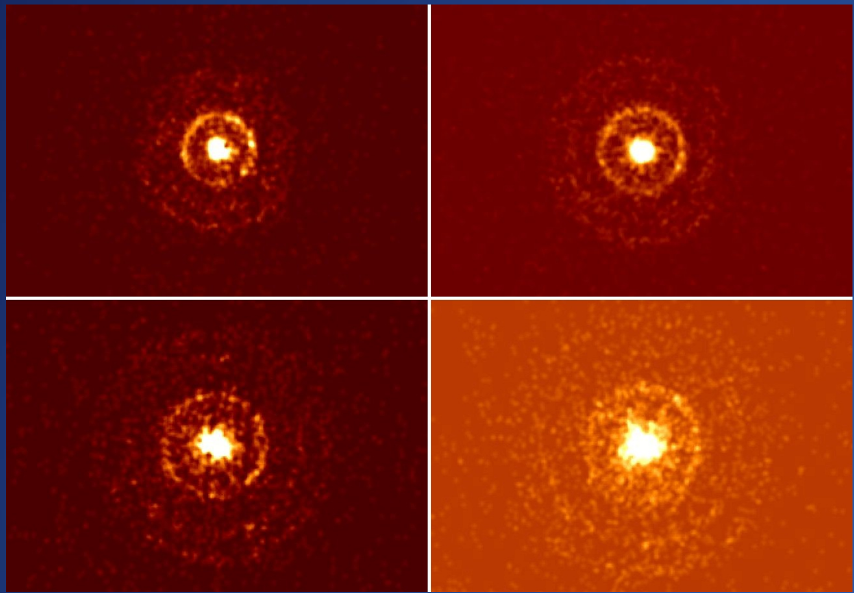
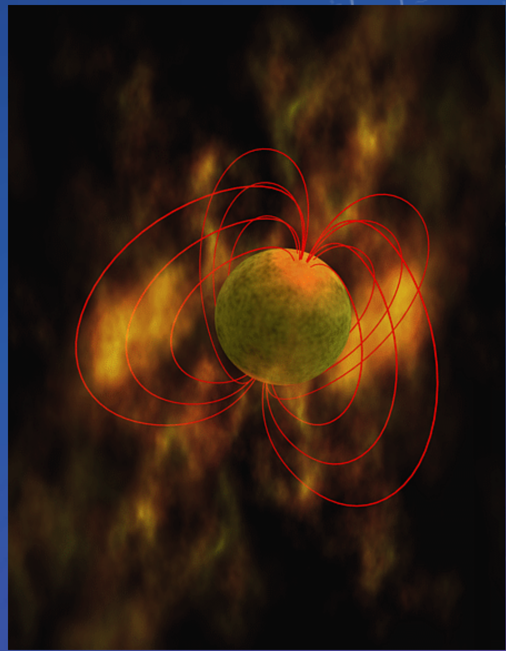
T.Maruyama et al., Phys. Rev. D91, 123007 (2015).
Phys. Lett.. B757, 125 (2016).

§ 1　Introduction

Soft Gamma Repeater (SGR) , Anomalous Xray pulsar (AXP)





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⇒　**Magnetar　10^{15}G in surface　$10^{17\text{-}19}$G inside**

B.C.Duncan & C.Thompson ApJL 392, L9 (1992)

S.Merghetti, A&AR 15, 225 (2008)

Observation of γ-ray　→　Study od Magnetar Structure

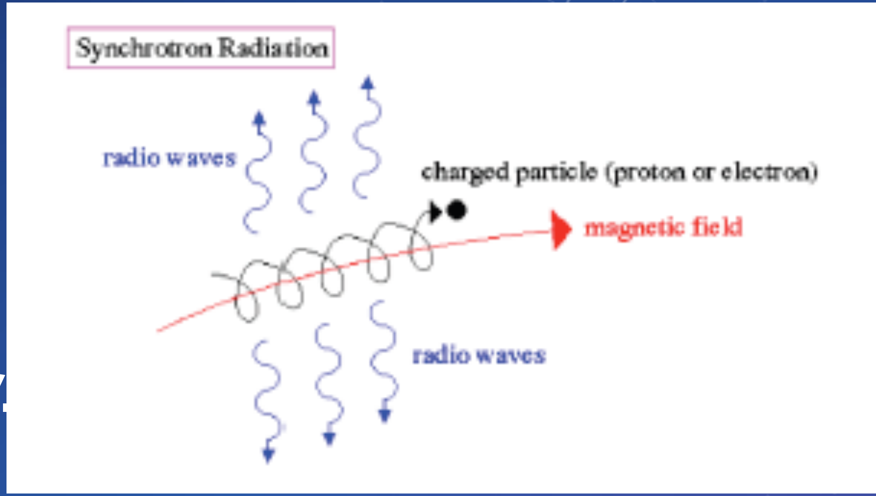
γ-ray Radiation

Proton is accelerate

up to 1GeV～1TeV

⇒ Synchtotron Radiation

･･･ Meson Prod (*Str. > El.Mag.*)



All Theories are Semi-Classical

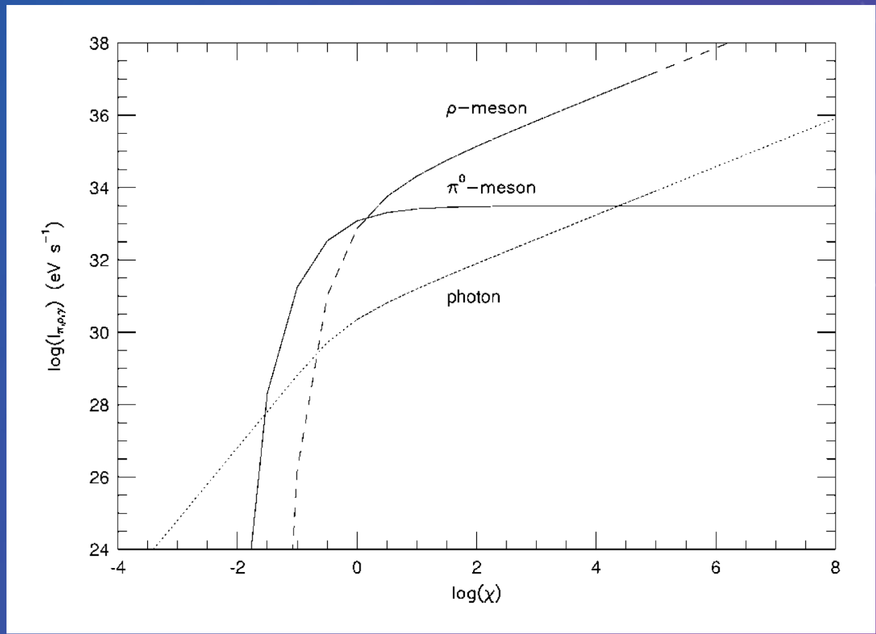
V.L.Ginzburg et al., UsFiN 87, 65, ARA&A 3, 297 (65)

G.F. Zharkov, Sov. J. Nucl. Phys., 1, 17314 (65)

V. Berezinsky, et al., Phys. Lett. B 351, 261 (95)

A. Tokushita and T. Kajino, ApJ. 525, L117 (99).

T.Kajino et al., ApJ 782, 70 (2014)



Many Assumption and Approxs.

Mom.-Dist. cannot be calculated

Quantum Calulations.

⇒ Exact Imformation

§ 2 Formulation in Relativistic Quantum Approach

Magnetic Field : $\vec{B} = B\hat{z}.$ $\vec{A} = (0, xB, 0)$

Dirac Equation

$$\left\{\vec{\alpha}(-i\vec{\nabla}_r - e\vec{A}) + \beta m_N + \frac{e\kappa}{2m_N} B\beta\Sigma_z\right\}\tilde{\psi}(\boldsymbol{r}) = \varepsilon\tilde{\psi}(\boldsymbol{r})$$

$$\Sigma_z = \begin{pmatrix} \sigma_z & 0 \\ 0 & -\sigma_z \end{pmatrix} = -\sigma_{12} = \frac{i}{2}[\gamma_1, \gamma_2]$$

Anomalous Mag. Moment
Tensor-Type Mean-Field

Scale Transformation : $M_N = m_N/\sqrt{eB},\ P_i \equiv p/\sqrt{eB},\ X_i = \sqrt{eB}x_i.$

Def: $U_T = \kappa\sqrt{eB}/2m_N = \kappa/2M_N.$

$$\boxed{E_T = \sqrt{P_z^2 + \left(\sqrt{2n+1-s+M_N^2} - s\,\kappa_p B/M_N\right)^2}}$$

Decay Width of p to $p + \pi^0$

πN interaction $\mathcal{L} = \frac{i f_\pi}{m_\pi} \psi \gamma_5 \gamma_\mu \tau_a \psi \partial^\mu \phi_a$

PV coupling

$$\frac{d^3\Gamma_{p\pi}}{dQ^3} = \frac{1}{8\pi^2 E_\pi} \left(\frac{f_\pi}{M_\pi}\right)^2 \sum_{n_f, s_f} \frac{\delta(E_f + E_\pi - E_i)}{4E_i E_f} R_E$$

$$R_E = 4E_i E_f \mathrm{Tr}\left\{\mathcal{O}_\pi \rho_M^{(+)}(n_f, s_f, P_z - Q_z) \mathcal{O}_\pi^\dagger \rho_M^{(+)}(n_i, s_i, P_z)\right\},$$

$$\begin{aligned}
\mathcal{O}_\pi &= \int dX \tilde{F}(n_i, s_i, X + Q_T/2) \gamma_5 \not{Q} \tilde{F}(n_f, s_f, X - Q_T/2). \\
&= \gamma_5 \left\{ \left[M\left(n_i + \frac{1-s_i}{2}, n_f + \frac{1-s_f}{2}\right) \frac{1+\Sigma_z}{2} \right.\right. \\
&\qquad \left. + M\left(n_i - \frac{1+s_i}{2}, n_f - \frac{1+s_f}{2}\right) \frac{1-\Sigma_z}{2} \right] [\gamma_0 Q_0 - \gamma^3 Q_z] \\
&\quad - \left[M\left(n_i + \frac{1-s_1}{2}, n_f - \frac{1+s_f}{2}\right) \frac{1+\Sigma_z}{2} \right. \\
&\qquad \left.\left. + M\left(n_i - \frac{1+s_i}{2}, n_f + \frac{1-s_f}{2}\right) \frac{1-\Sigma_z}{2} \right] \gamma^2 Q_y \right\}.
\end{aligned}$$

$$\boldsymbol{Q} = (0, Q_T, Q_z) = \boldsymbol{q} / \sqrt{eB}$$

$$\begin{aligned}
M(n_1, n_2) &= \int dx f_{n_1}\left(x + \frac{Q_y}{2}\right) f_{n_2}\left(x - \frac{Q_y}{2}\right). \\
&= (2^{n_1+n_2} \pi n_1! n_2!)^{-1/2} e^{-Q_T^2/4} \int dx e^{-x^2} H_{n_1}\left(x + \frac{Q_T}{2}\right) H_{n_2}\left(x - \frac{Q_T}{2}\right) \\
&= \sqrt{\frac{n_2!}{n_1!}} \left(-\frac{Q_T^2}{\sqrt{2}}\right)^{n_1-n_2} e^{-\frac{Q_T^2}{4}} L_{n_2}^{n_1-n_2}\left(\frac{Q_T^2}{2}\right) \qquad (n_1 \le n_2) \\
&= \sqrt{\frac{n_1!}{n_2!}} \left(\frac{Q_T^2}{\sqrt{2}}\right)^{n_2-n_1} e^{-\frac{Q_T^2}{4}} L_{n_1}^{n_2-n_1}\left(\frac{Q_T^2}{2}\right) \qquad (n_1 \ge n_2)
\end{aligned}$$

$H_n(x)$: Hermit Polynomial

$L_n^m(x)$: Associated Laguerre Polynomlal

§3 Results of π^0 Production

$$E_i = 1\,\mathrm{GeV},\ \ B = 5\times 10^{18}\ \mathrm{G}$$

$$\chi = eBe_p / m_N^3 = 0.069$$

$$\chi \approx 0.01 - 1 \quad \pi\text{ - Prod. Dominant}$$

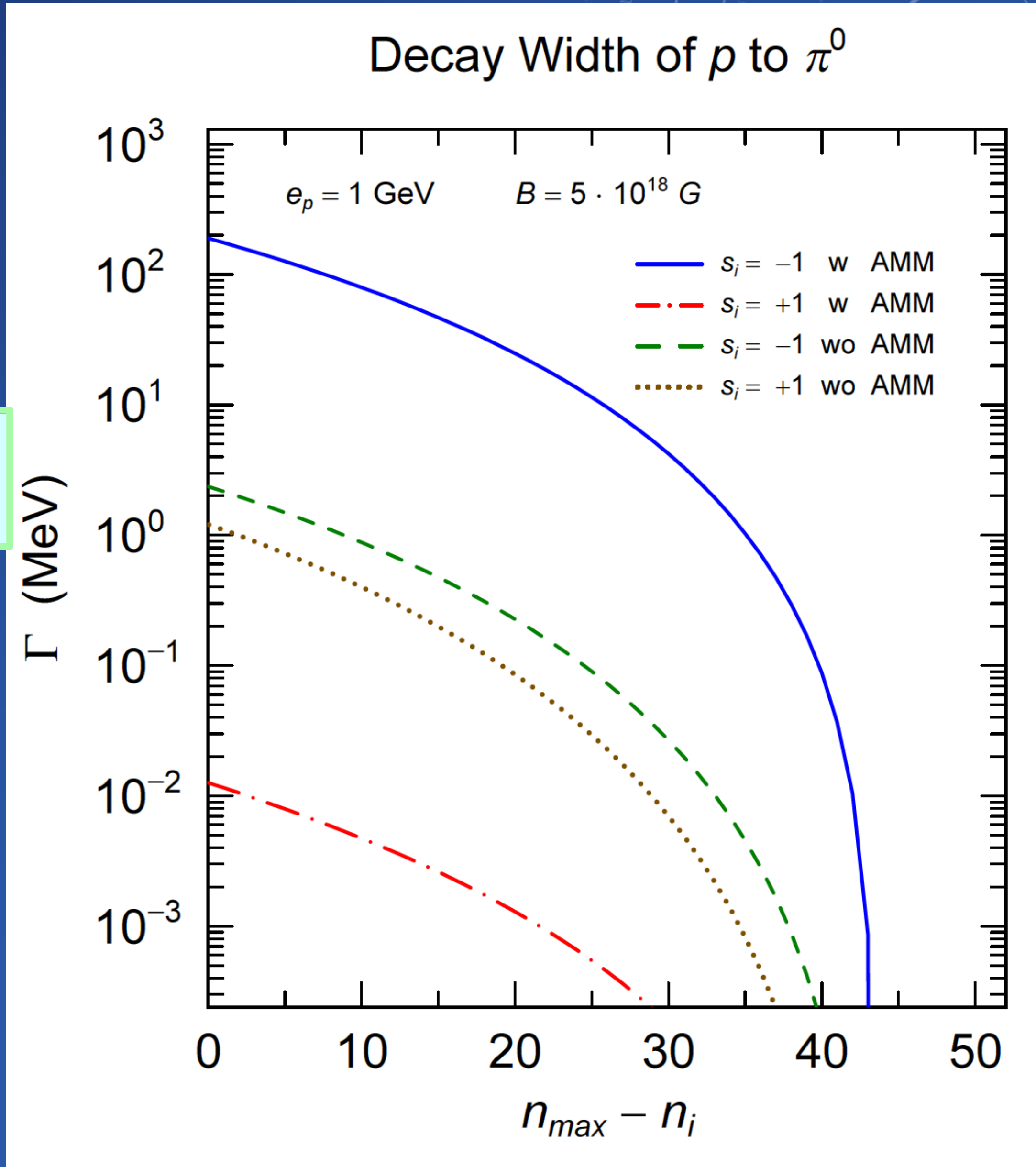
$$\sqrt{eB} = 17.2\ \mathrm{MeV}\,, \quad \frac{e\kappa_p}{2m_N} B = 28.3\ \mathrm{MeV}$$

$$n_{\max} + \frac{s_i + 1}{2} = 50 \ \text{ for } s_i = -1$$

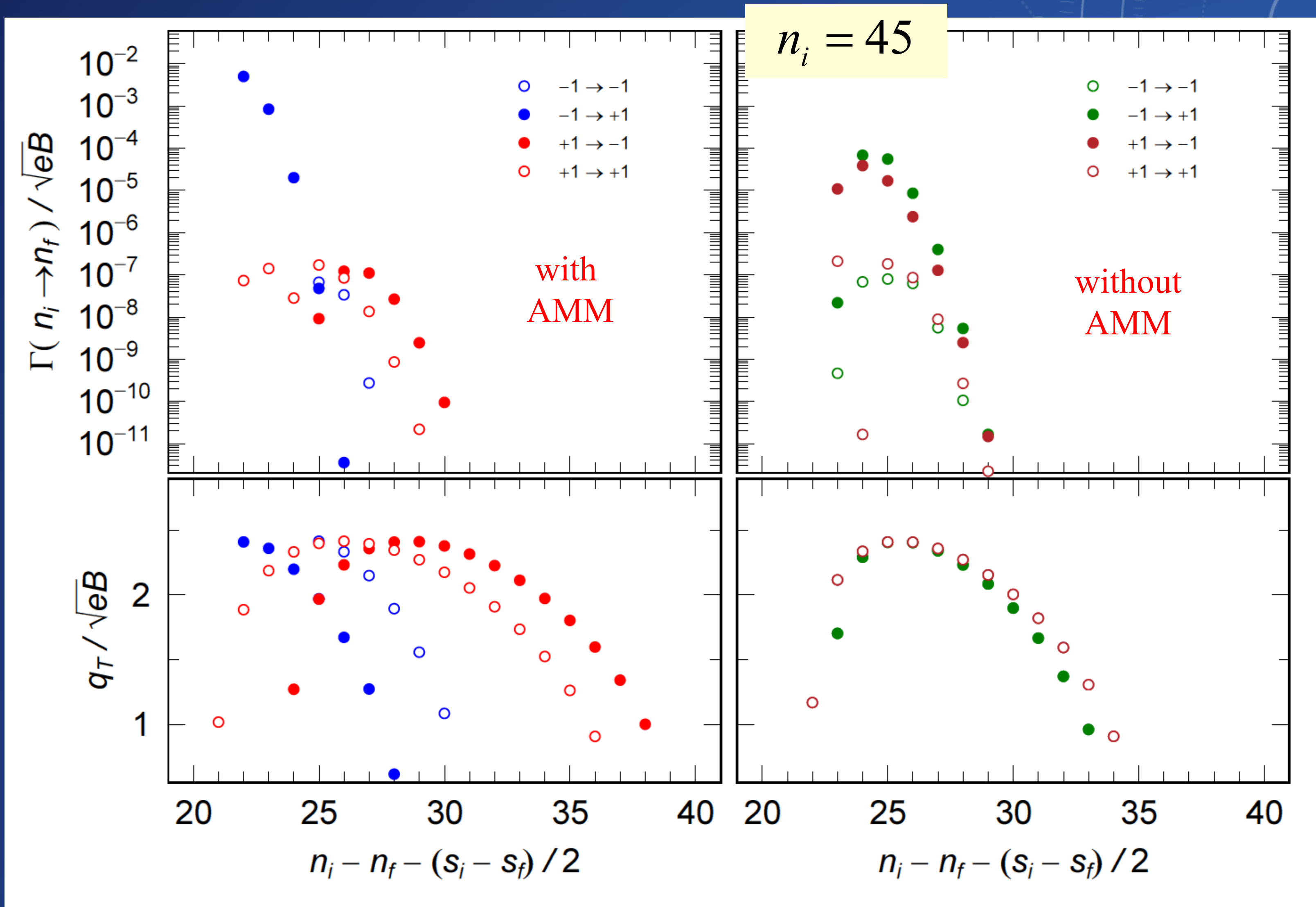
$$= 45 \ \text{ for } \ s_i = +1$$

no AM $\quad n_{\max} + \dfrac{s_i + 1}{2} = 47$

Decay Width



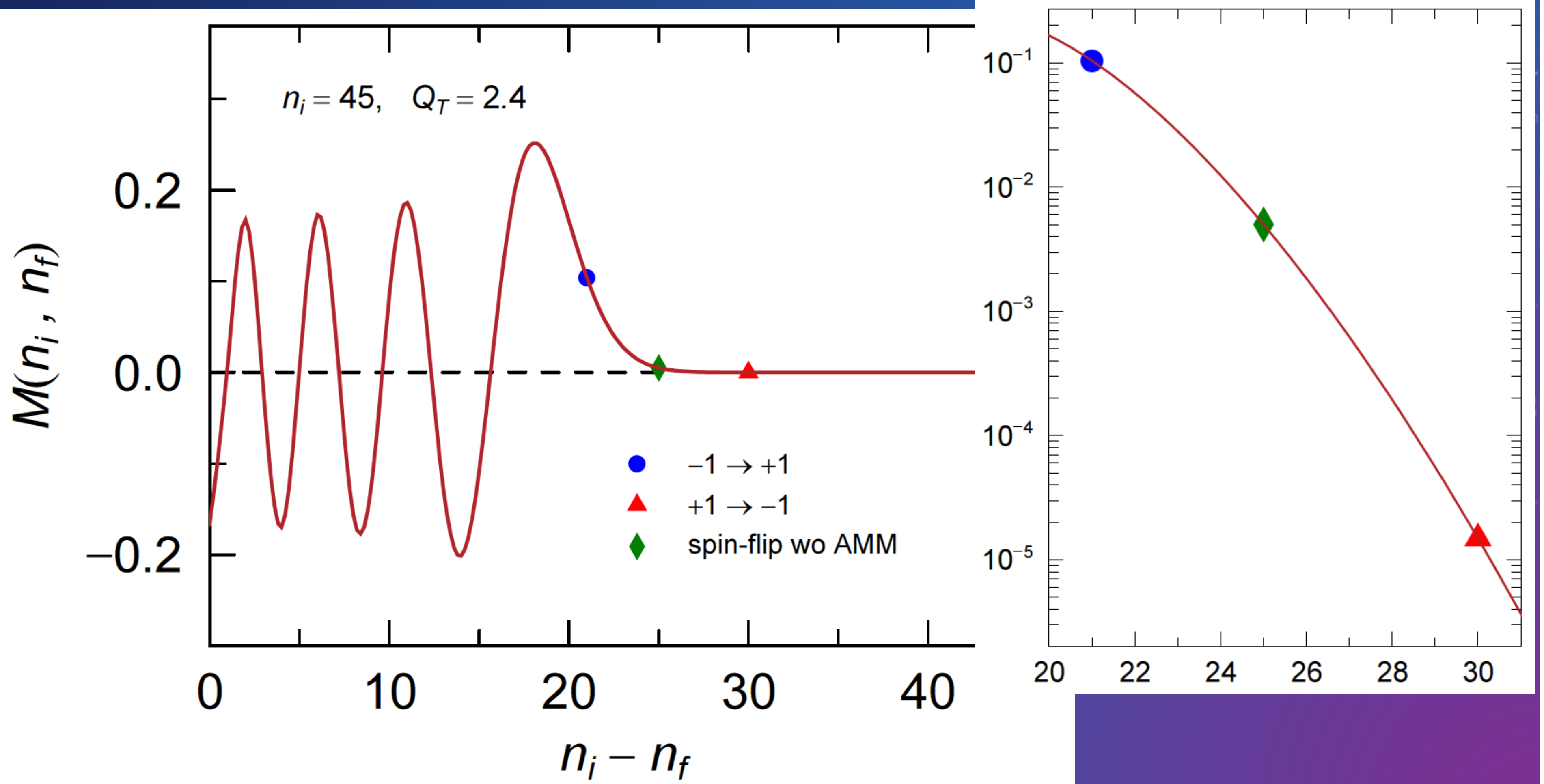
Transition Strengths between two Landau Levels



−1 —> +1 small Landau-level difference

Transition Strength 2

$$
\begin{aligned}
M(n_1,n_2) &= \int dx f_{n_1}\left(x+\frac{Q_y}{2}\right) f_{n_2}\left(x-\frac{Q_y}{2}\right). \\
&= (2^{n_1+n_2}\pi n_1! n_2!)^{-1/2} e^{-Q_T^2/4} \int dx e^{-x^2} H_{n_1}\left(x+\frac{Q_T}{2}\right) H_{n_2}\left(x-\frac{Q_T}{2}\right) \\
&= \sqrt{\frac{n_2!}{n_1!}} \left(-\frac{Q_T^2}{\sqrt{2}}\right)^{n_1-n_2} e^{-\frac{Q_T^2}{4}} L_{n_2}^{n_1-n_2}\left(\frac{Q_T^2}{2}\right) \qquad (n_1 \le n_2) \\
&= \sqrt{\frac{n_1!}{n_2!}} \left(\frac{Q_T^2}{\sqrt{2}}\right)^{n_2-n_1} e^{-\frac{Q_T^2}{4}} L_{n_1}^{n_2-n_1}\left(\frac{Q_T^2}{2}\right) \qquad (n_1 \ge n_2)
\end{aligned}
$$



Very Large AMM Effects

$p \rightarrow p + \pi^0$ **Energy Momentum Conservation is not satisfied in the free kinematics**

Mag. Fld.＋AMM **Tensor Type Mean-Field**

$s = -1$ **(repulsive),** $s = +1$ **(attractive)**

Level Interval of Transition $n_{\mathrm{i}} - n_{\mathrm{f}}$

$s_i = -1 \rightarrow s_f = +1$ **Smaller Intervals**

$\Rightarrow$ **Enhances Transition Strength**

$s_i = +1 \rightarrow s_f = -1$ **Larger Intervals**

$\Rightarrow$ **Reduces Transition StrengthV**

Small Shifts $n_{\mathrm{i}} - n_{\mathrm{f}}$ **make Large change of Transition Strength**

§4 Realistic System

Pion Production Dominant Energy Region $\boxed{\chi = eBe_p / m_N^3 \approx 0.01 - 1}$

$B = 10^{15}$ G Landau Number : $n_i \approx 10^{12} - 10^{13}$

Actual calculations are almost impossible

Problem : HO overlap integral

$$\mathcal{M}(n_1, n_2) = \int dx f_{n_1}\left(x - \frac{Q_T}{2}\right) f_{n_2}\left(x + \frac{Q_T}{2}\right) = \sqrt{\frac{n_2!}{n_1!}} \left(\frac{Q_T}{\sqrt{2}}\right)^{n_1 - n_2} e^{-\frac{Q_T^2}{4}} L_{n_2}^{n_1 - n_2}\left(\frac{Q_T^2}{2}\right)$$

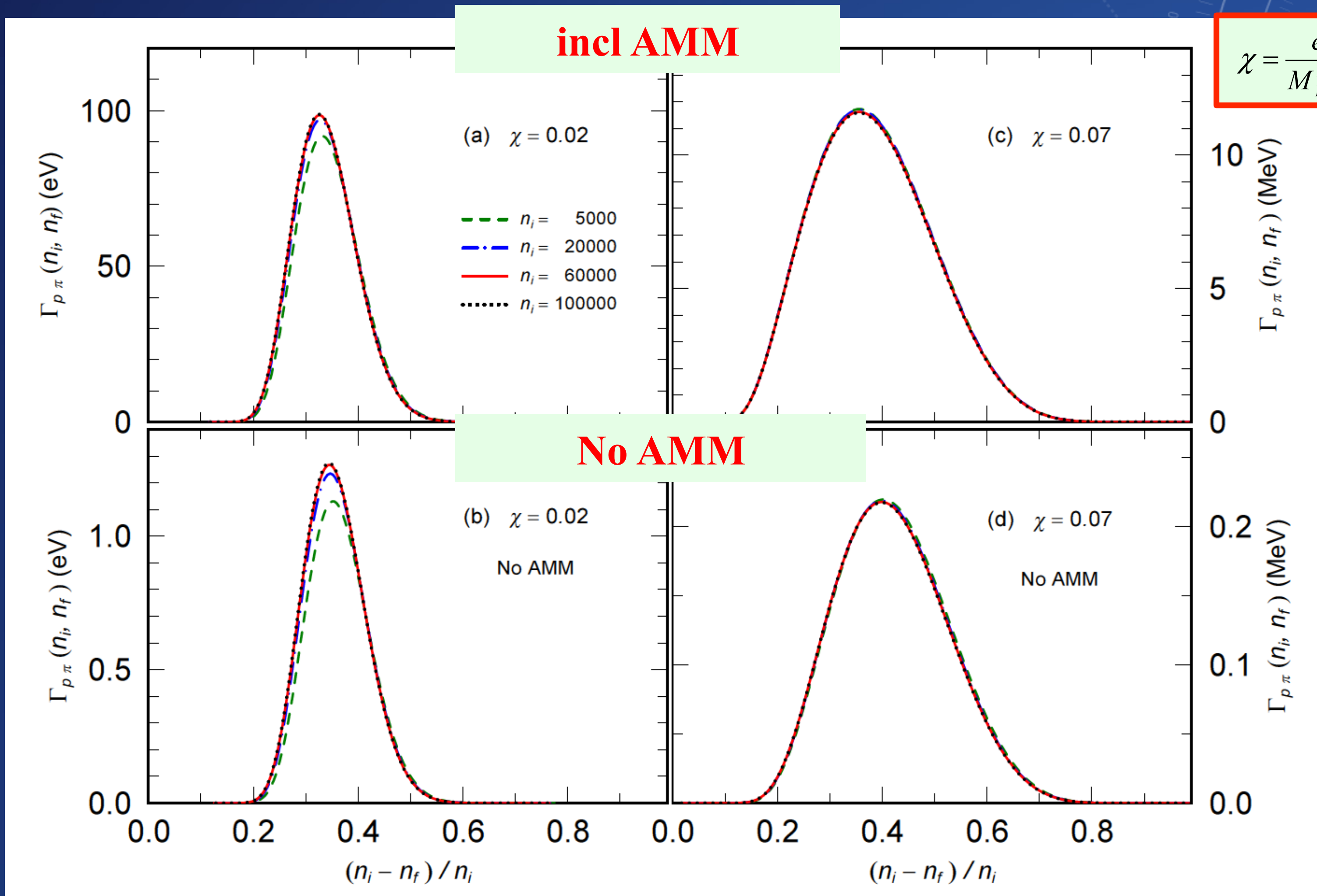
It is possible to make a Lorentz Transportation along z-direction

$$\Gamma(n_i, P_{iz}) = \frac{\sqrt{E_i^2 - P_{iz}^2}}{E_i} \Gamma(n_i, P_{iz} = 0)$$

Semi-Classical Theory $\Rightarrow$ Scaling, Dep. Only on χ

Contribution at Fixed Final Landau Number

$$\chi = \frac{e_i^3}{M_N^3 R_L} = \frac{eBe_i}{M_N^3}$$



Scaling Law **Function of χ, $(n_i - n_f)/n_i$**

Prediction Results $n_i \approx 10^4$ $\Rightarrow$ Results $n_i \approx 10^{12-13}$ (B ~ 10^{15}G)

Huge Effects of AMM remain even in B ~ 10^{15}G

Small χ

Larger n_i → Scaling

Total Decay Width

Scaling Relation

(All Semi-Classial Theoryies Show)

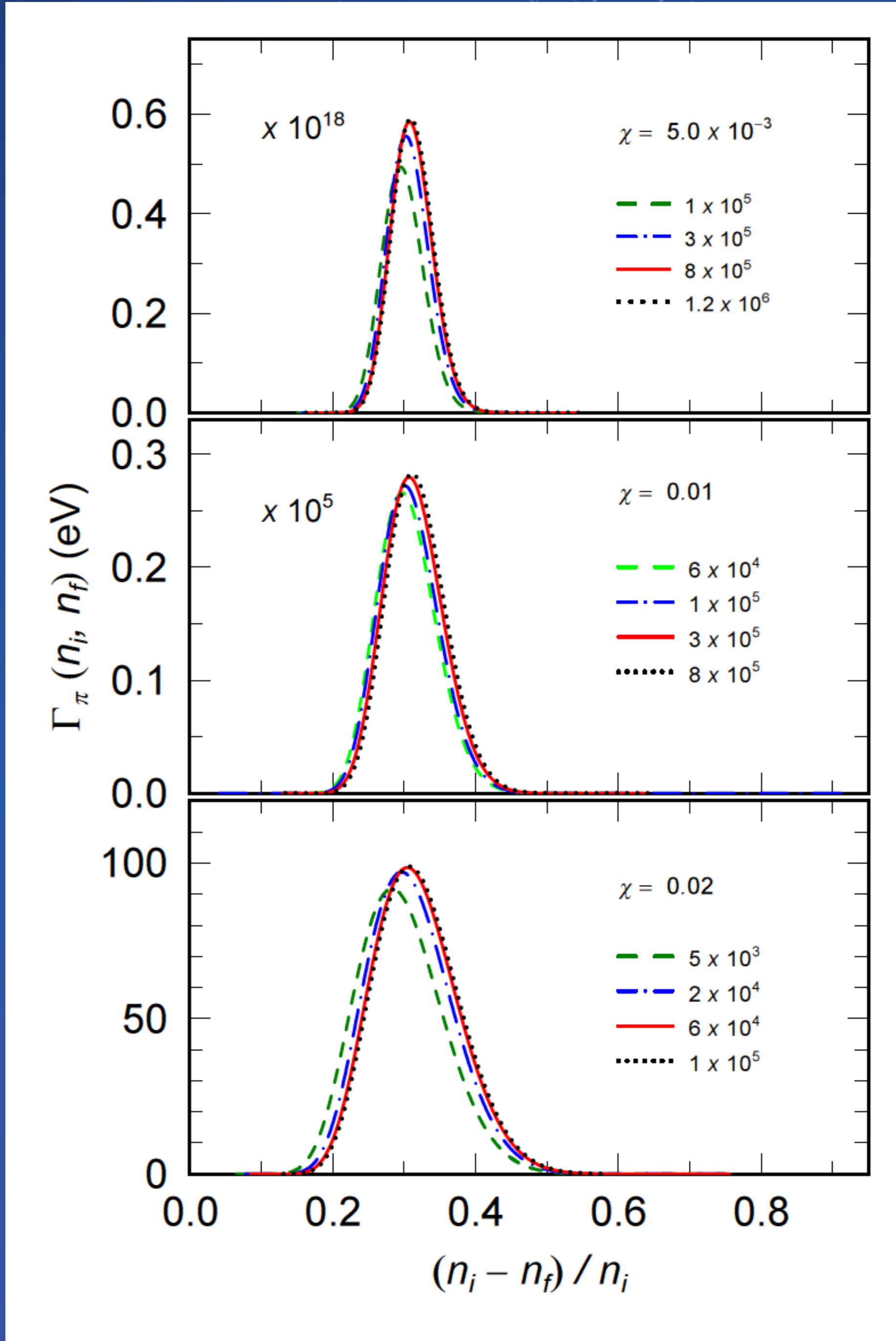
3 Variables B, n_i, n_f

⇒ 2 Variables

$$\chi = eBEe_i / m_N^3,\ (n_i - n_f)/n_i$$

Peak position

$(n_i - n_f) / n_i \rightarrow\ 0.3$



Adiabatic Limit

Relative Momentum between
Final **Proton** and Pion
is Zero,

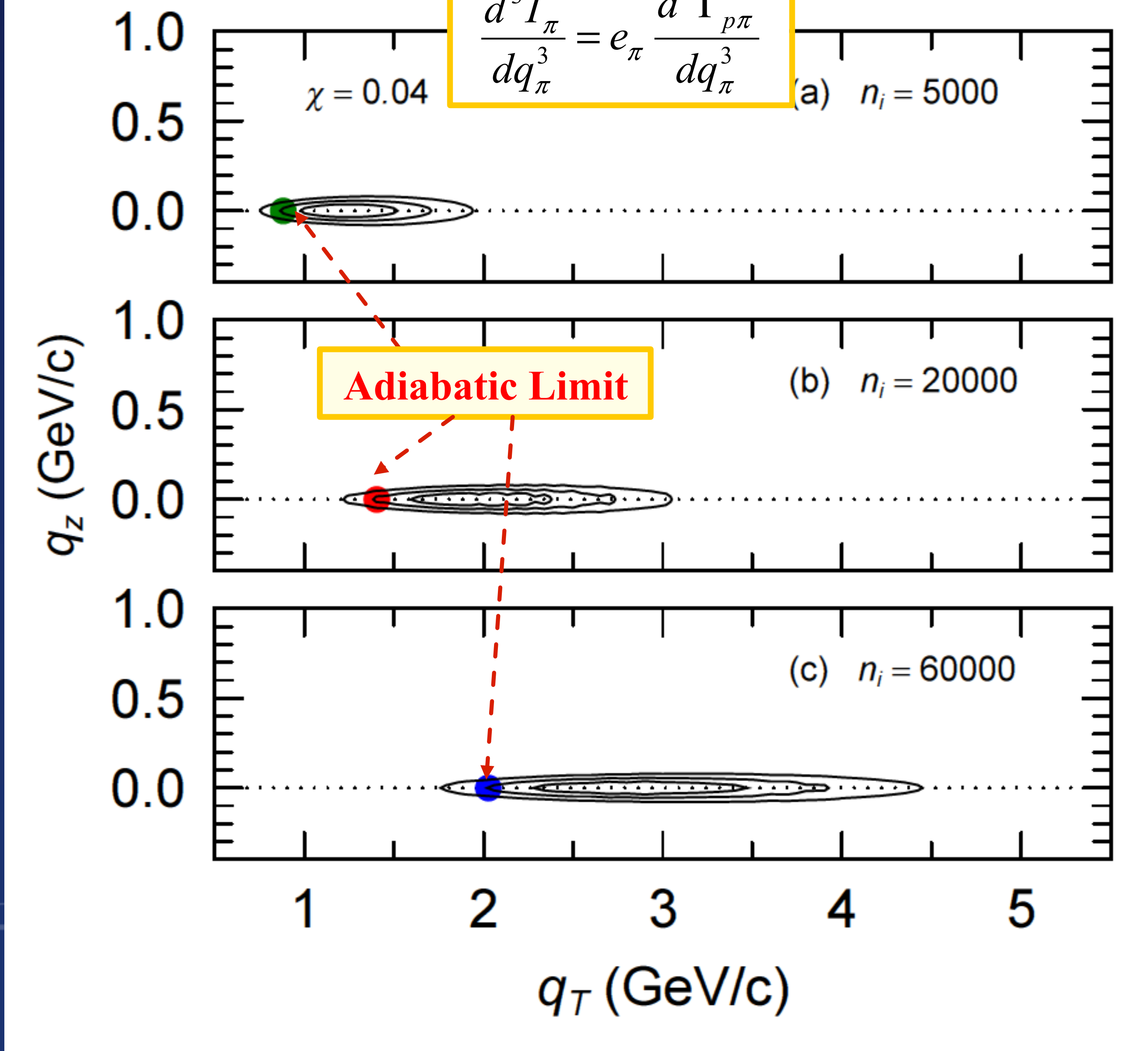
Same Velocity

$$e_\pi = \frac{m_\pi}{m_N + m_\pi} e_i, \quad e_f = \frac{m_\pi}{m_N + m_\pi} e_i \quad \left(e_{i,f} \approx \sqrt{2n_{i,f}} \right)$$

$$\rightarrow \boxed{\frac{n_i - n_f}{n_i} \approx 0.28} \Leftrightarrow \text{Semi-Classical}: \frac{n_i - n_f}{n_i} << 1$$

Angular Distrbution at $p_{iz} = 0$

$$\frac{d^3 I_\pi}{dq_\pi^3} = e_\pi \frac{d^3 \Gamma_{p\pi}}{dq_\pi^3}$$



Δq_z
indep. on I
Incidet Energy

Norrow Angular Ditsr.

$$\Gamma(n_i, n_f; P_{iz} = 0) \propto \delta(Q_z)$$

Lorentz Trans. along z-direction

Angular Distribution of Pion Luminocity

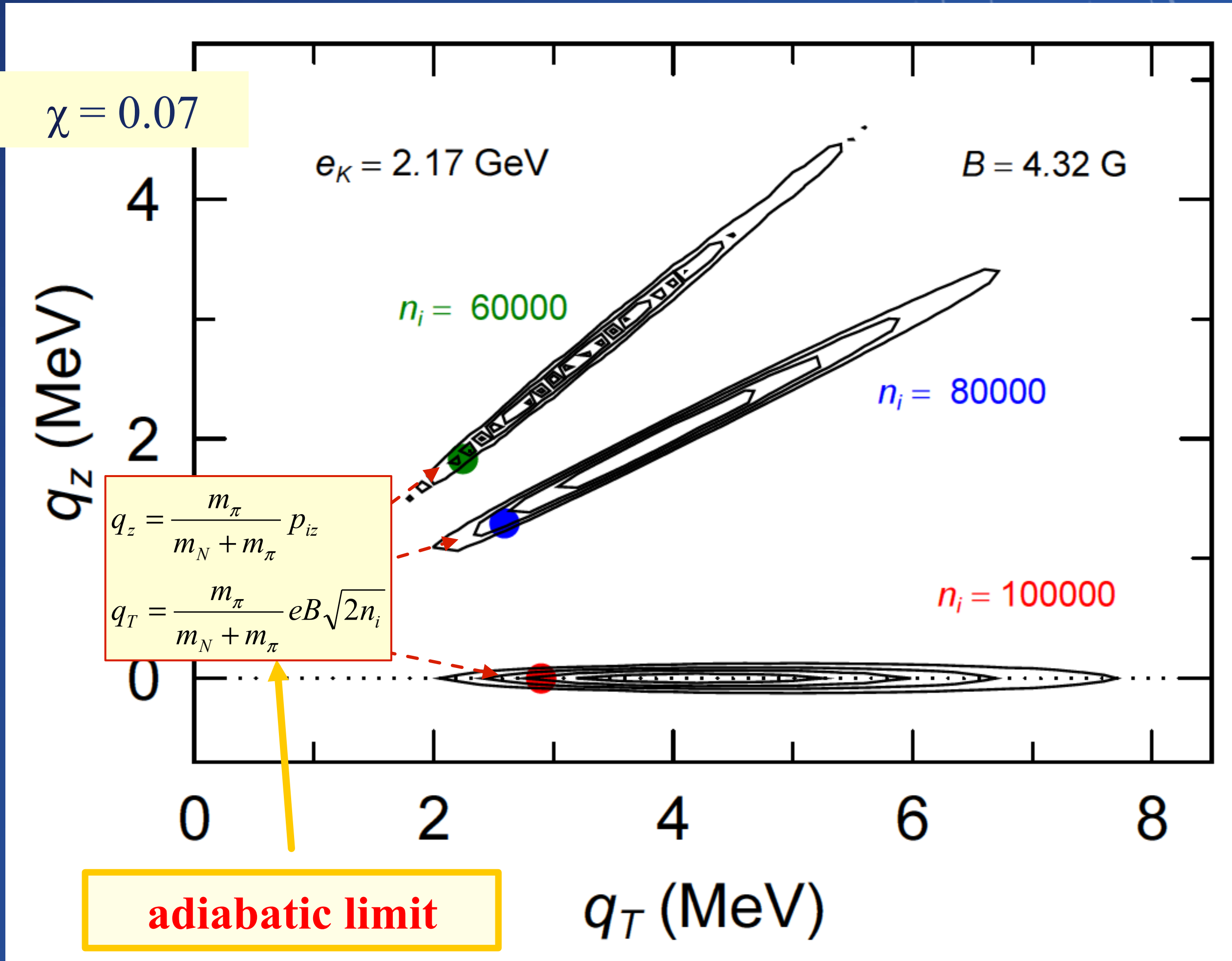
$$\frac{d^3 I_\pi}{dq_\pi^3} = e_\pi \frac{d^3 \Gamma_{p\pi}}{dq_\pi^3}$$

when $n_i >> 1$,

$\boldsymbol{q}_T \,//\, \boldsymbol{p}_f \,//\, \boldsymbol{p}_i$

Same Polar Angle

Width is very small



$$q_z = \frac{m_\pi}{m_N + m_\pi} p_{iz}$$

$$q_T = \frac{m_\pi}{m_N + m_\pi} eB\sqrt{2n_i}$$

Proton Decay Width $n_{\mathrm{i}} \gg 1$

$p_{\mathrm{iz}} = 0$

$$\frac{d\Gamma_{p\pi}(p_{iz}=0,s_i)}{dq^3} = \frac{1}{e_\pi}\sum_{n_f}\Gamma_{p\pi}(n_i,n_f)\delta\left(e_i - e_f - q_0\right)\delta(q_z)$$

Lorentz Transformation

$p_{\mathrm{iz}} \neq 0$

$$\frac{d\Gamma_{p\pi}(p_{iz},s_i)}{dq^3} = \frac{1}{e_\pi}\frac{e_{iT}}{e_i}\sum_{n_f}\Gamma_{p\pi}(n_i,n_f)\delta\left(e_i - e_f - q_0\right)\delta\left(q_z - \frac{e_\pi}{e_i}p_z\right)$$

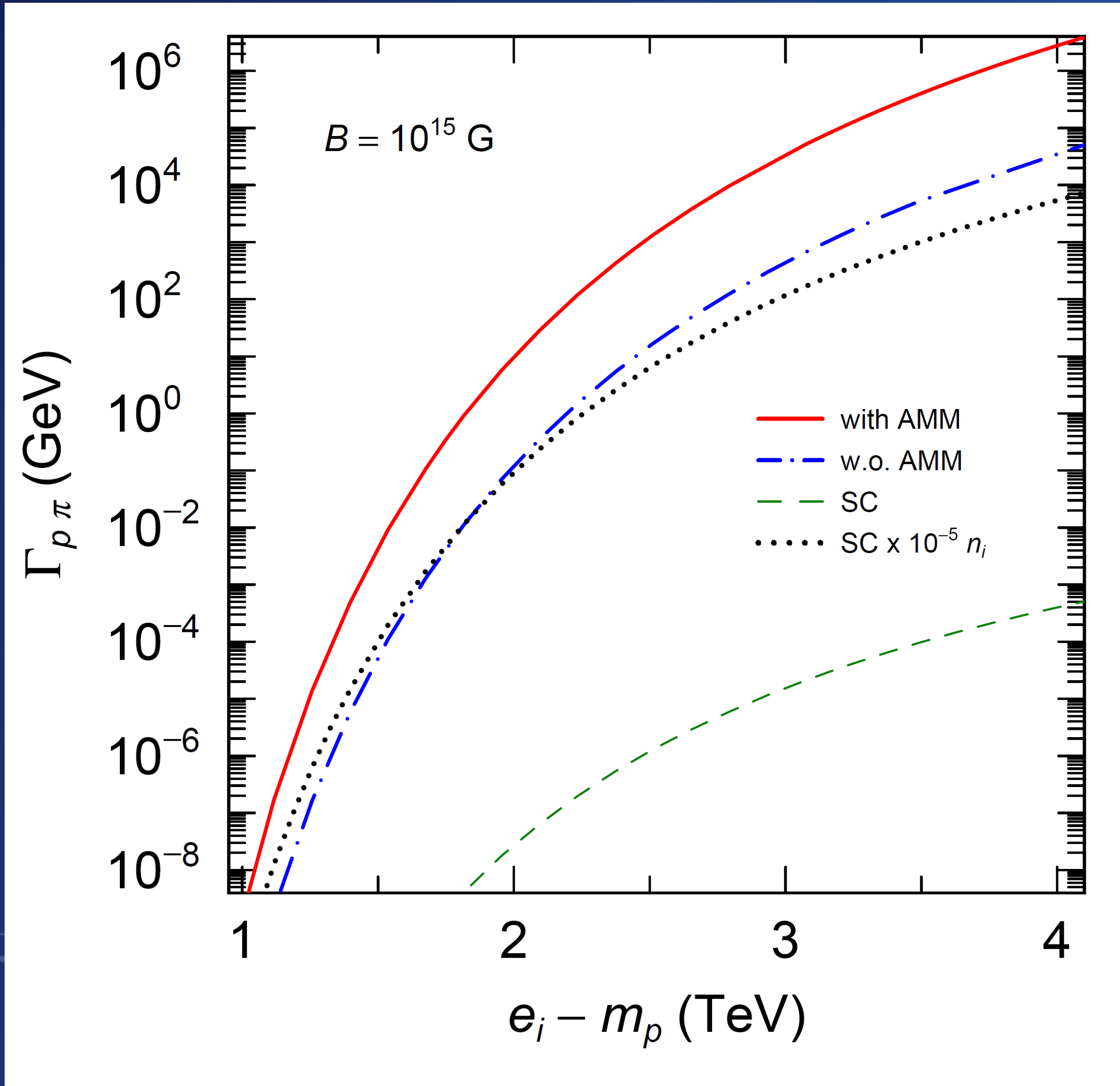
Scaling **Results with** $n_{\mathrm{i}}, n_{\mathrm{f}} \sim 10^4 \Rightarrow$ Results with 10^{12}

Semi-Classical Approximation **assume** $n_{\mathrm{i}} - n_{\mathrm{f}} \ll n_{\mathrm{i}}$

π has mass **This Assumpyion is wrong**

$$\sqrt{n_i} - \sqrt{n_f} > \frac{m_\pi}{m_N + m_\pi}\sqrt{n_i}$$

Total Decay Width



Semi-Classical
A.Tokushita and T. Kajino,
ApJ. 525, L117 (99).

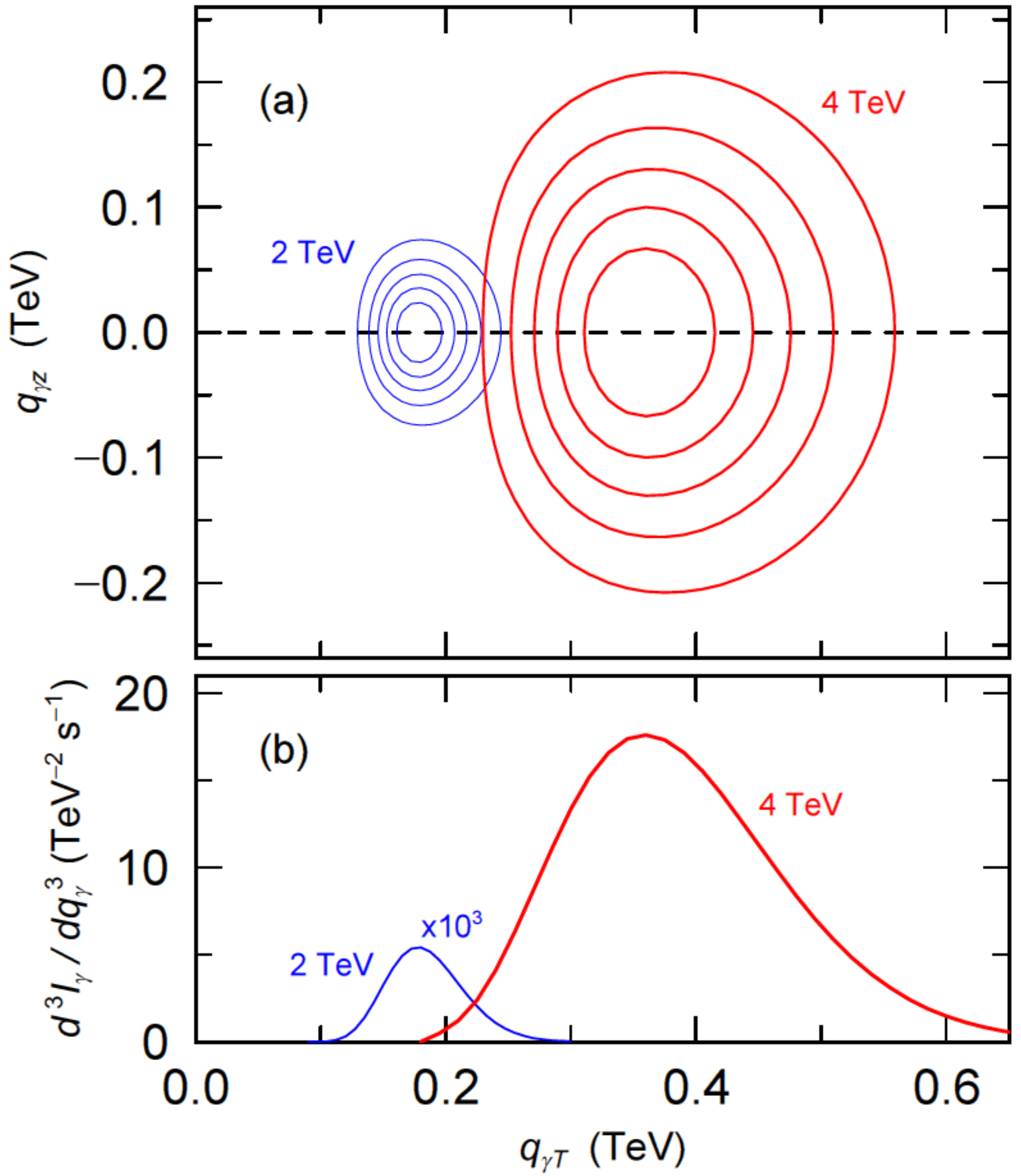
Luminocity-Distribution of Emitted Photons

$$p \longrightarrow p + \pi^0$$

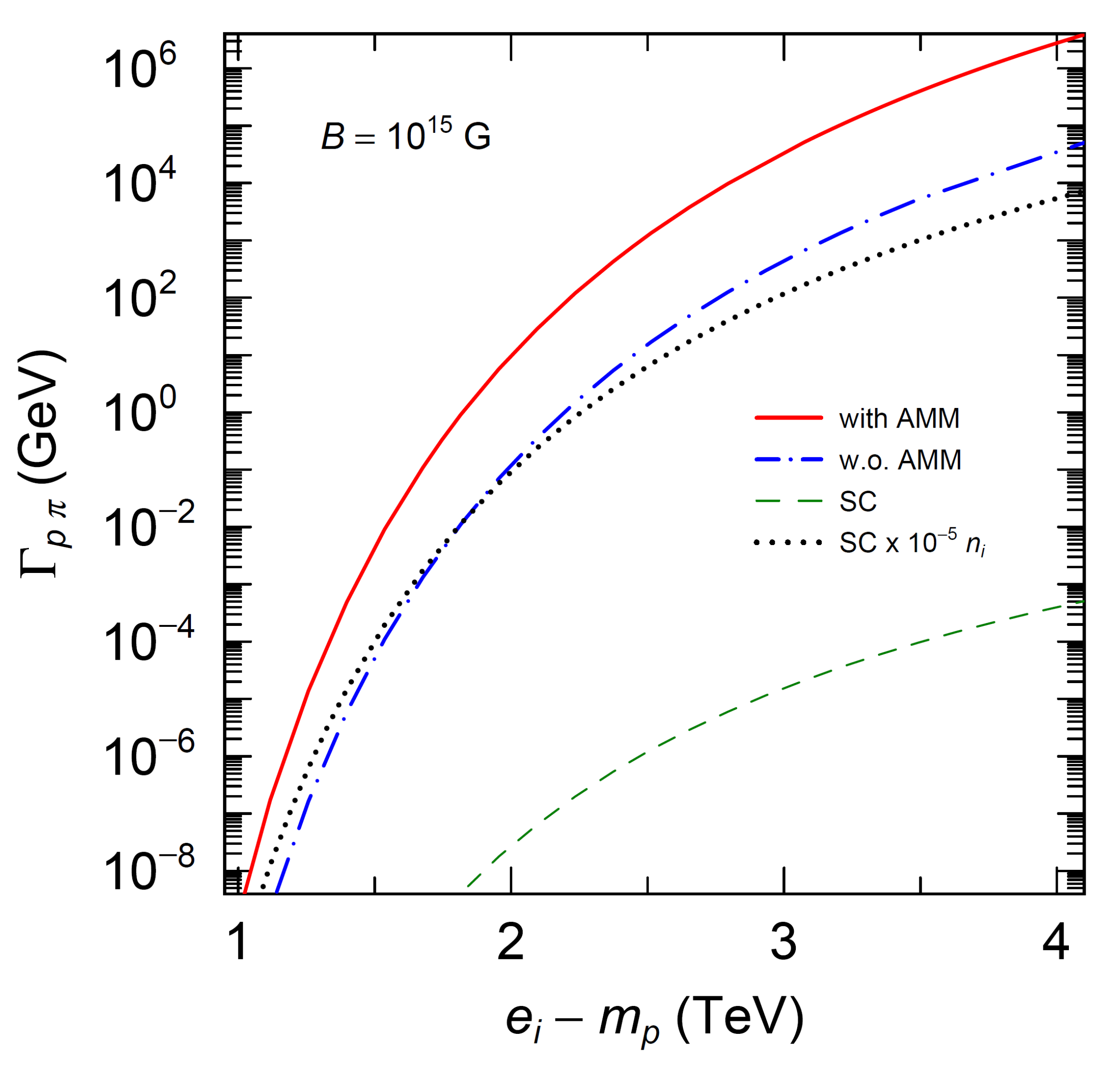
$$\pi^0 \longrightarrow 2\,\gamma$$

Average over Initial Proton Angle

Distribution is Spherical



Total Decay Width



$$\Gamma(n_i, \chi; P_{iz} = 0) \propto n_i$$

Semi-Classical
A.Tokushita and T. Kajino,
ApJ. 525, L117 (99).

$$\Gamma(n_i, \chi; P_{iz} = 0)$$
$$\text{indep.of } \; n_i$$

§5 Summary

- π^0 emission from Proton Transition between two Landau Levels

  $n_i, n_f \sim 10^5 \Rightarrow B \sim 10^{17}$ G

  AMM effect $-1 \to +1$ Dccay widths become 50 – 100 times larger

- Scaling Law, predicted by the Semi-Classical theory

  3 Variables $B, n_i, n_f \Rightarrow$ 2 Variables $\chi = eBEe_i/m_N^3$, $(n_i - n_f)/n_i$

  $B \sim 10^{17}$ G $\Rightarrow B \sim 10^{15}$ G (Magnetar)

  Results with $n_i, n_f \sim 10^4 \Rightarrow$ Results with 10^{12}

- Angular Dist $\theta_i \approx \theta_f \approx \theta_\pi$

$$\frac{d\Gamma_{p\pi}(n_i, p_{iz})}{dq^3} \alpha\, \delta\left(q_z - \frac{e_\pi}{e_i} p_z\right)$$

- Pion Energies are distributed in Broad Region

$$\sqrt{n_i} - \sqrt{n_f} > \frac{m_\pi}{m_N + m_\pi}\sqrt{n_i} \quad \Leftrightarrow$$

Semi-Classical Approx.

$n_i - n_f << n_i$

The Results come from HO overlap Integral

$$\mathcal{M}(n_1,n_2) = \int dx\, f_{n_1}\left(x+\frac{Q_T}{2}\right) f_{n_2}\left(x-\frac{Q_T}{2}\right) = (2\pi)\mathcal{W}(n_i,n_f)\delta(Q_z)$$

It is a function of Q_T and very rapidly change when $n_{i,f} >> 1$

$$\mathcal{W}(n_i,n_f) \propto \frac{1}{\sqrt{n_i}}(\text{Function of } \chi)$$

Generally

$$\Gamma(n_i, P_{iz}=0) = \mathcal{W}(n_i,n_f) \times F(P_{iz}=P_{fz}=Q_z=0)$$

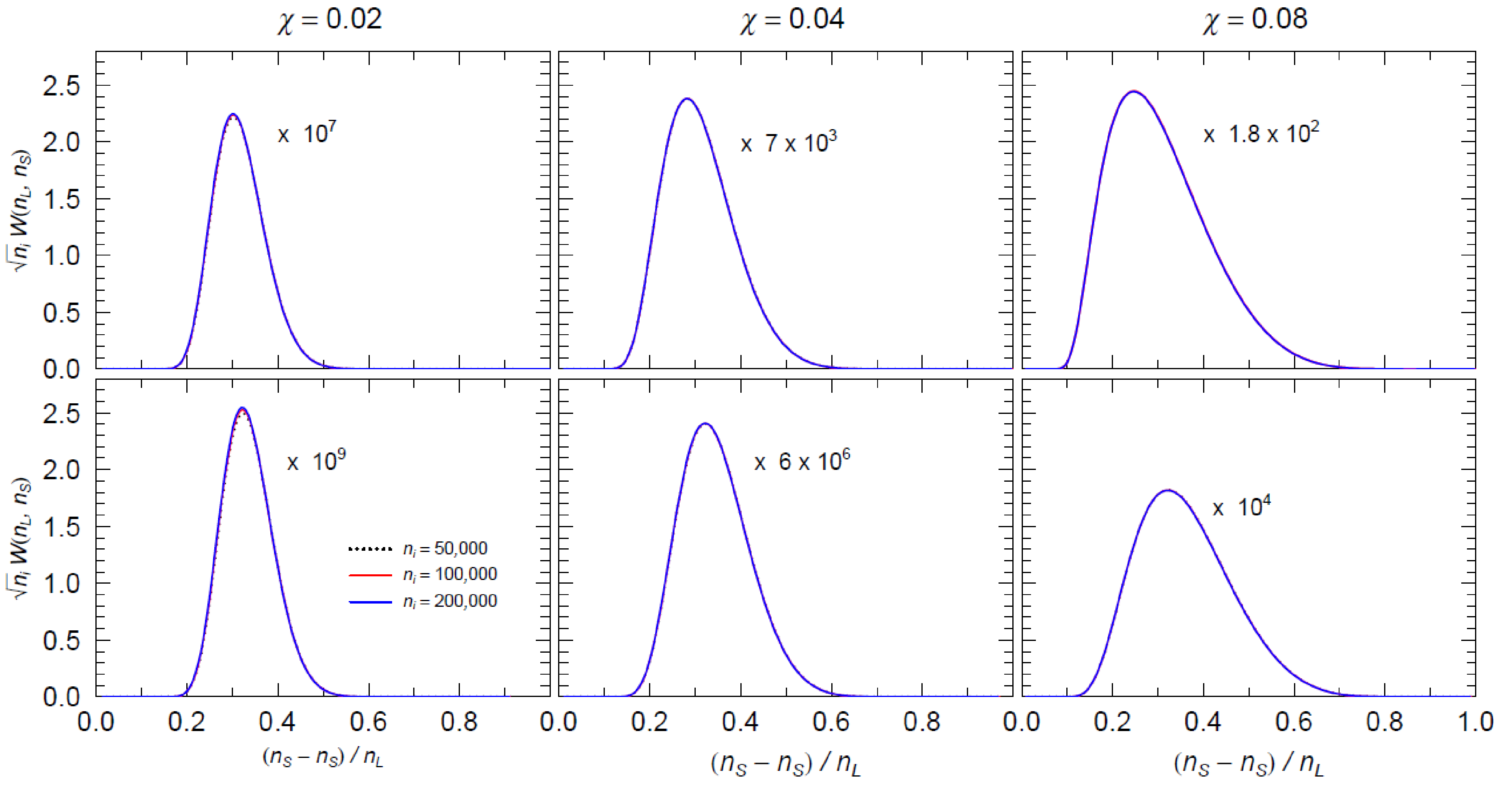
$\Rightarrow$ Other Particle Productions

$\Rightarrow$ **Magnetic Structure inside Magnetars**

HO Overlap Integral

$$\mathcal{M}(n_1, n_2) = \int dx f_{n_1}\left(x + \frac{Q_T}{2}\right) f_{n_2}\left(x - \frac{Q_T}{2}\right) = (2\pi)\mathcal{W}(n_i, n_f)\delta(Q_z)$$

$$\boxed{\mathcal{W}(n_1, n_2) = \int \frac{Q_z}{2\pi} \int dx f_{n_1}\left(x + \frac{Q_T}{2}\right) f_{n_2}\left(x - \frac{Q_T}{2}\right).}$$



In PS-coupling
$\Gamma(n_i, n_f)$ does not satisfy
Scaling Relation