Hyperon equation of state for supernovae and neutron stars with the variational method

H. Togashi (RIKEN)

E. Hiyama (RIKEN) , M. Takano (Waseda University)

Outline

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2 : Nuclear EOS for core-collapse simulations

3 : Extension to hyperonic nuclear matter

4 : Summary

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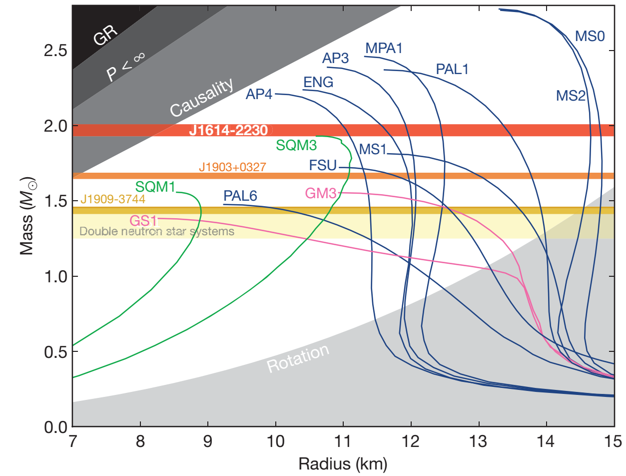
1: Introduction

The nuclear equation of state (EOS) plays important roles for astrophysical studies.

At zero temperature, a variety of nuclear EOSs have been applied to neutron stars.

EOS by APR (A. Akmal et al., PRC58(1998)1804)

Potential: AV18+UIX
Wave function: Jastrow wave function
Method: Fermi Hypernetted chain



P. B. Demorest et al., NATURE 467 (2010)

The hyperon mixing in neutron stars has been studied with various nuclear theories.

- **Relativistic mean field theory** (C. Ishizuka et al., J. Phys. G 35 (2008) 085201)
- **Relativistic Hartree-Fock theory** (T. Miyatsu, et al., PRC 88 (2013) 01802)
- **Brueckner-Hatree-Fock theory** (H. Schulze, T. Rijken, PRC 84 (2011) 035801)
- **Variational many-body theory** (D. Lonardoni et al., PRL 114 (2015) 092301)

EOS table for core-collapse simulations

Nuclear EOS at finite temperature → Core-collapse simulations

1. Lattimer-Swesty EOS : *The Skyrme-type interaction* (NPA 535 (1991) 331)

2. Shen EOS : *The Relativistic Mean Field Theory* (NPA 637 (1998) 435)

Nuclear Interaction	$n_{\rm sat}$ ($\rm fm^{-3}$)	BE/A (MeV)	K (MeV)	Q ($\frac{\rm MeV}{\rm fm^3}$)	J (MeV)	L (MeV)	type of int.	used in
SKa	0.155	16.0	263	-300	32.9	74.6	Skyrme	H&W
LS180	0.155	16.0	180	-451	28.6	73.8	Skyrme	LS180
LS220	0.155	16.0	220	-411	28.6	73.8	Skyrme	LS220, LS220Λ, LS220π
LS375	0.155	16.0	375	176	28.6	73.8	Skyrme	LS375
TMA	0.147	16.0	318	-572	30.7	90.1	RMF	HS(TMA)
NL3	0.148	16.2	272	203	37.3	118.2	RMF	SHT, HS(NL3)
FSUgold	0.148	16.3	230	-524	32.6	60.5	RMF	SHO(FSU1.7), HS(FSUgold)
FSUgold2.1	0.148	16.3	230	-524	32.6	60.5	RMF	SHO(FSU2.1)
IUFSU	0.155	16.4	231	-290	31.3	47.2	RMF	HS(IUFSU)
DD2	0.149	16.0	243	169	31.7	55.0	RMF	HS(DD2), BHBΛ, BHBΛφ
SFHo	0.158	16.2	245	-468	31.6	47.1	RMF	SFHo
SFHx	0.160	16.2	239	-457	28.7	23.2	RMF	SFHx
TM1	0.145	16.3	281	-285	36.9	110.8	RMF	STOS, FYSS, HS(TM1), STOSΛ, STOSY, STOSYπ, STOSπ, STOSπQ, STOSQ, STOSB139, STOSB145, STOSB155, STOSB162, STOSB165

Hyperon EOS

SN-EOS list by M. Hempel

There are no SN EOSs based on the microscopic many-body theory.

Our Plan to Construct the EOS for SN Simulations

Uniform Nuclear Matter

EOS constructed with ***the cluster variational method***

Non-uniform Nuclear Matter

EOS constructed with ***the Thomas-Fermi calculation***

Completion of a Nuclear EOS table for SN simulations

★We are here.★

Density ρ_{B} : $10^{5.1} \leq \rho_{\mathrm{B}} \leq 10^{16.0}\mathrm{g/cm}^3$	110 point
Temperature T : $0 \leq T \leq 400$ MeV	92 point
Proton fraction Y_{p} : $0 \leq Y_{\mathrm{p}} \leq 0.65$	66 point

Our Plan to Construct the EOS for SN Simulations

Uniform Nuclear Matter

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EOS constructed with ***the Thomas-Fermi calculation***

Completion of a Nuclear EOS table for SN simulations

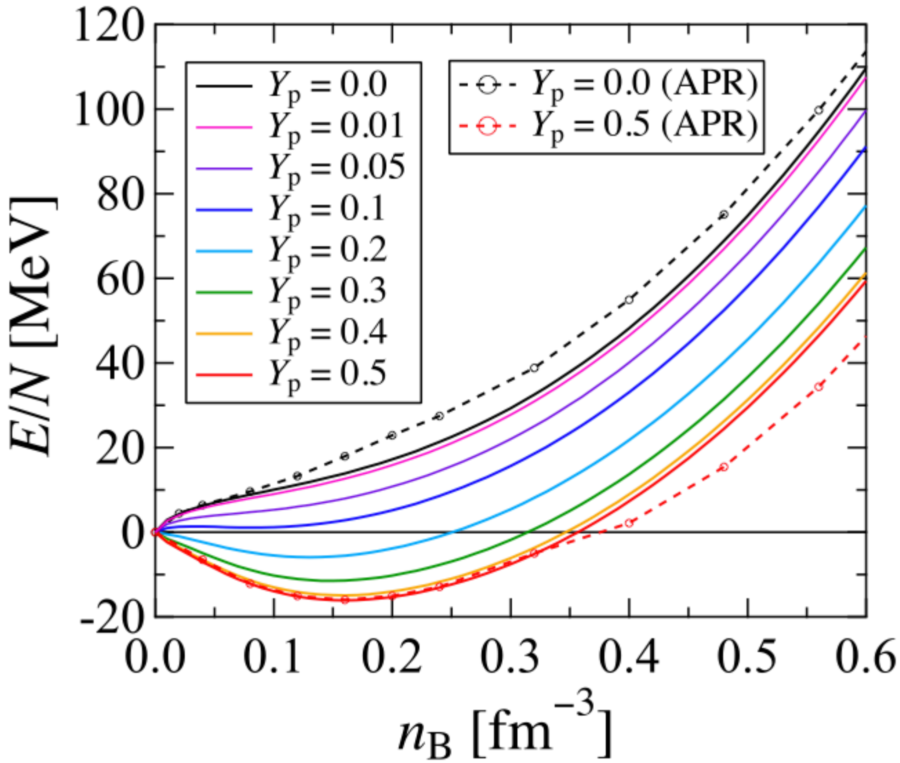
★We are here.★

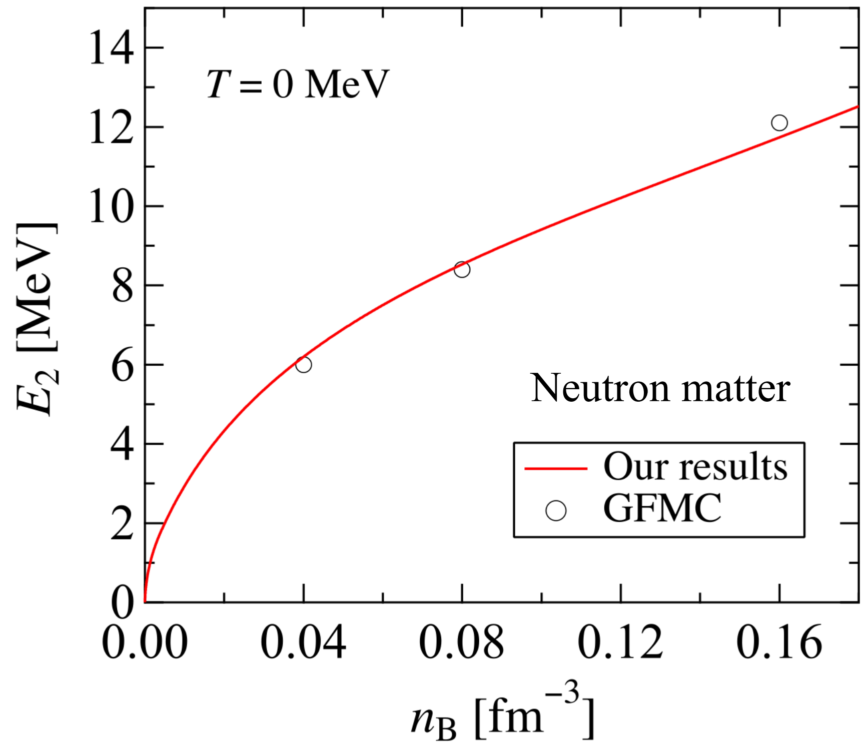
We extend the nuclear EOS table to consider Λ hyperon mixing.

2. Nuclear EOS for core-collapse simulations

Potential: AV18+UIX   Wave function: Jastrow wave function

Method : Cluster variational method





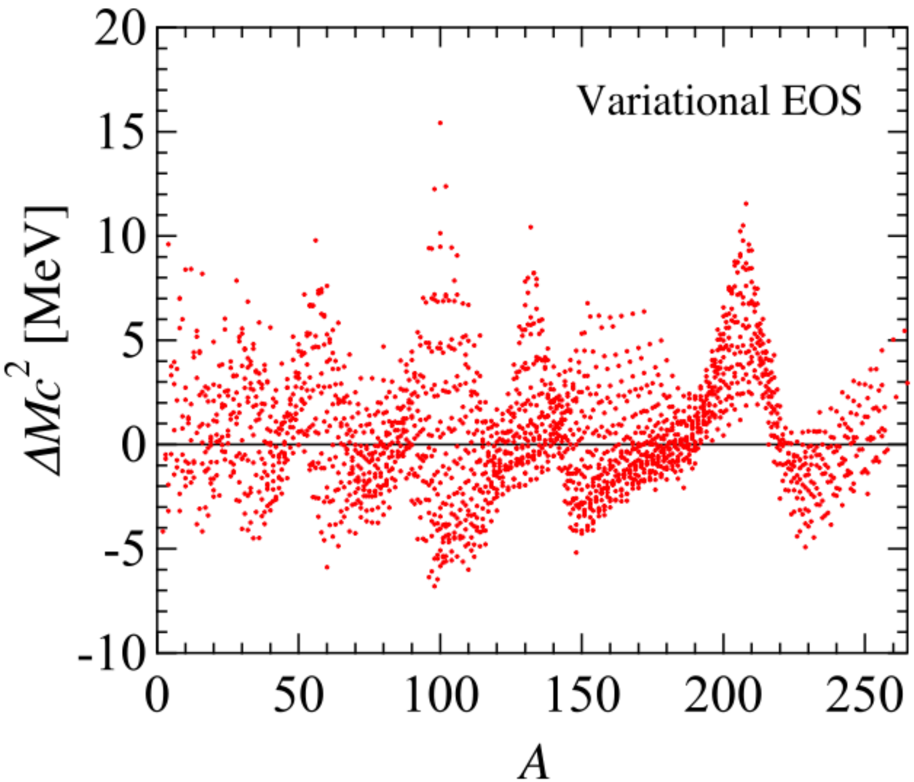
n_0[fm^{-3}]	E_0[MeV]	K [MeV]	E_{sym}[MeV]
0.16	−16.1	245	30.0

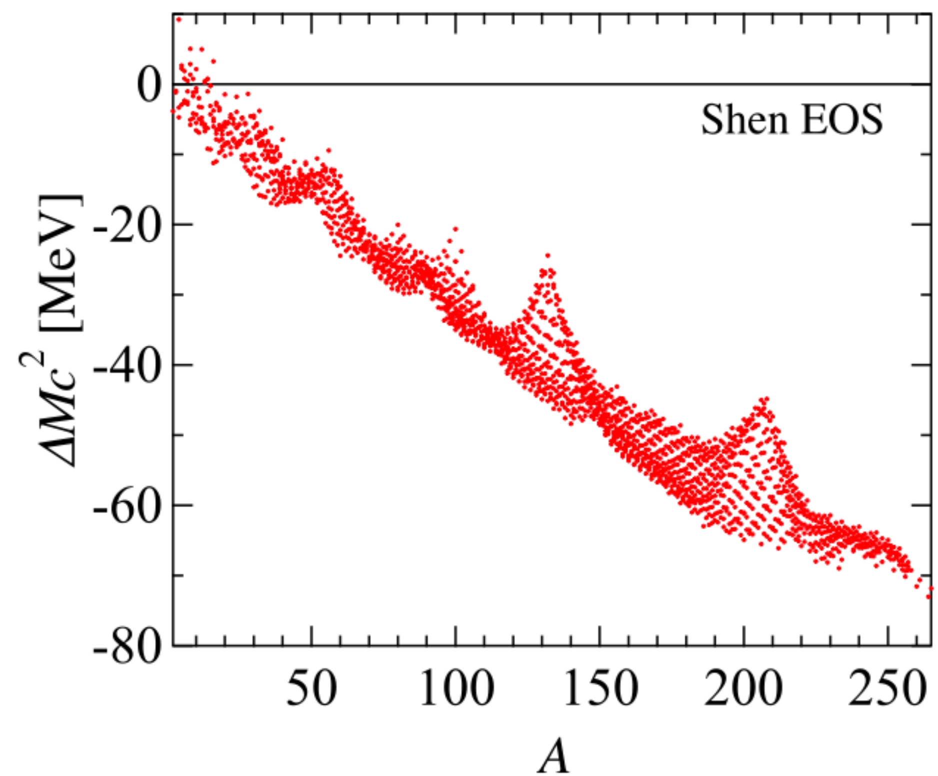
Our EOS : NPA902 (2013) 53

APR : PRC58(1998)1804

GFMC : PRC56(1997)1720

Thomas-Fermi calculation for isolated atomic nuclei





$\Delta M = M_{TF} - M_{exp}$

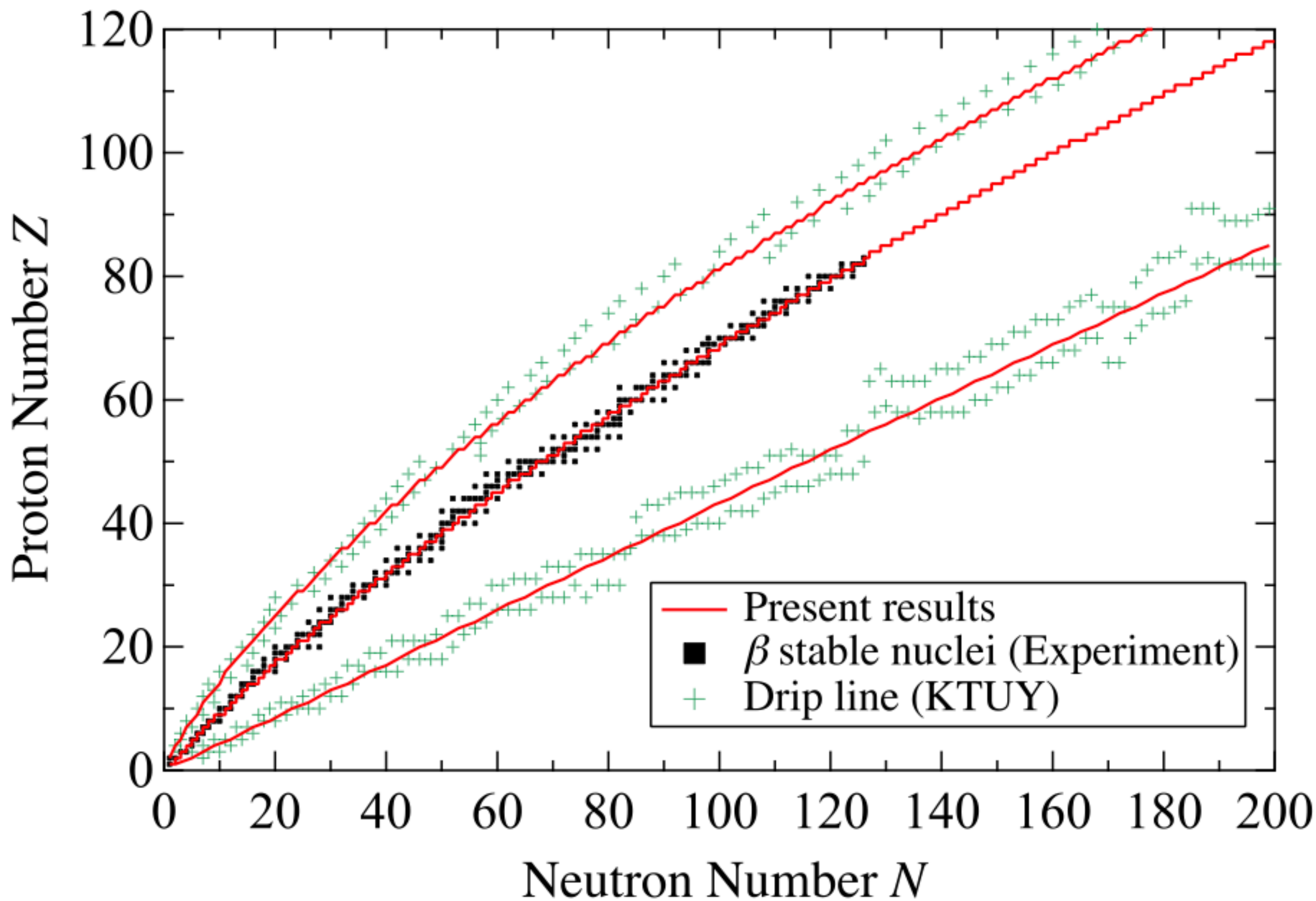
M_{TF} : Mass by the Thomas-Fermi calculation

M_{exp} : Experimental data
(G. Audi et al., Nucl. Phys. A **729** (2003) 337)

RMS deviation (for 2226 nuclei)

2.99 MeV (Variational EOS) **42.9 MeV (Shen EOS)**

β-stability line and drip lines

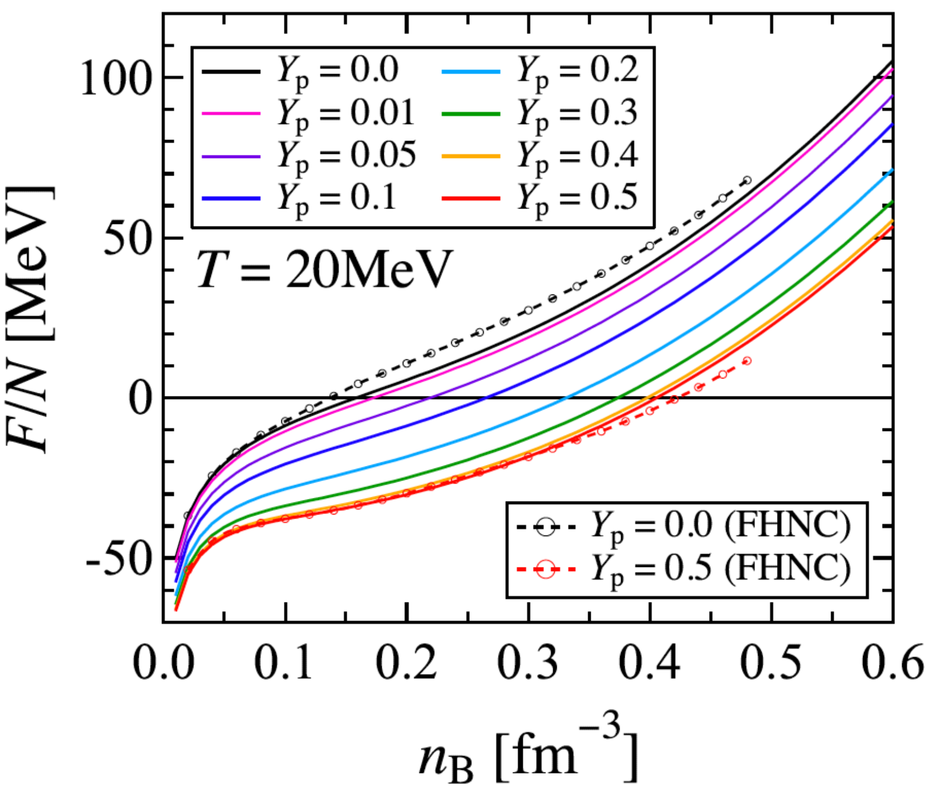


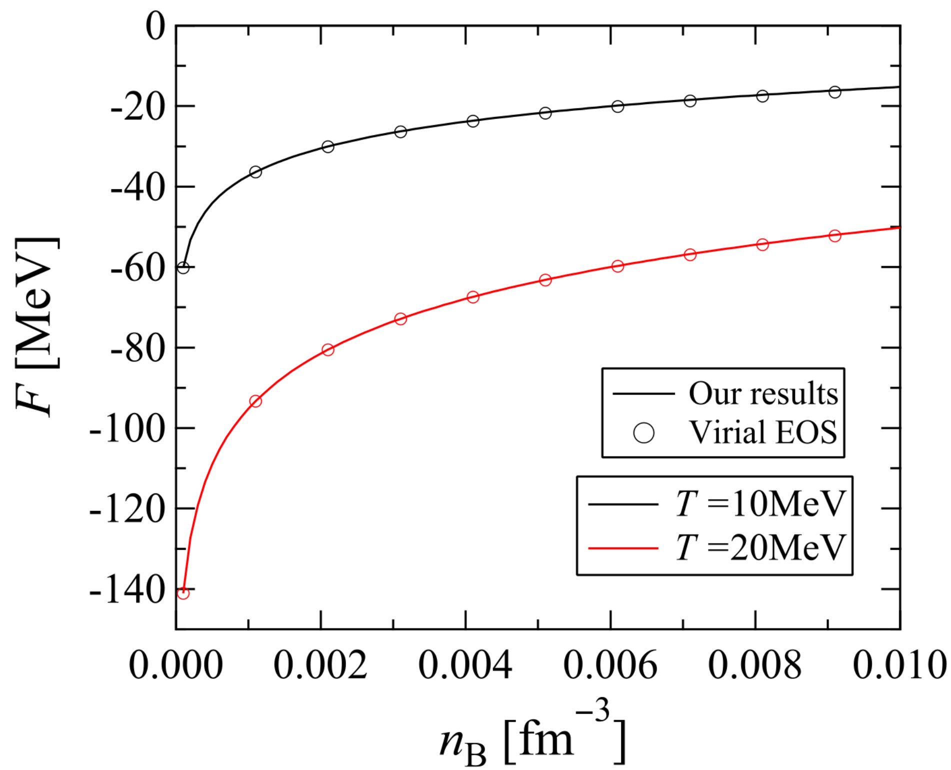
Our results are in good agreement with the experimental data and the sophisticated atomic mass formula.

EOS of Uniform Nuclear Matter at Finite Temperature

We follow the prescription proposed by *Schmidt and Pandharipande.*

(Phys. Lett. 87B(1979) 11) (A. Mukherjee et al., PRC 75(2007) 035802)

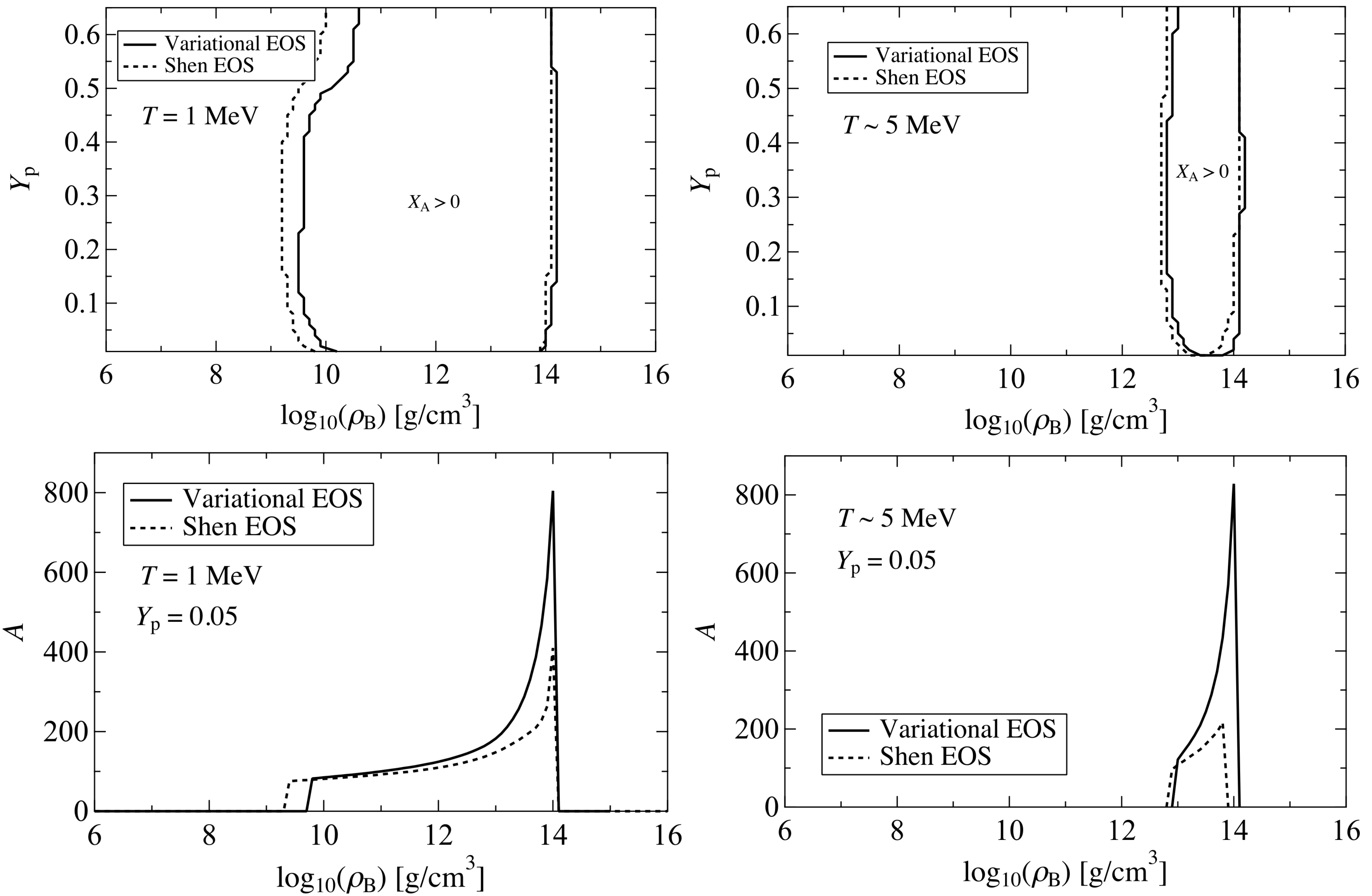




Our EOS : NPA902 (2013) 53

FHNC : A. Mukherjee, PRC 79(2009) 045811

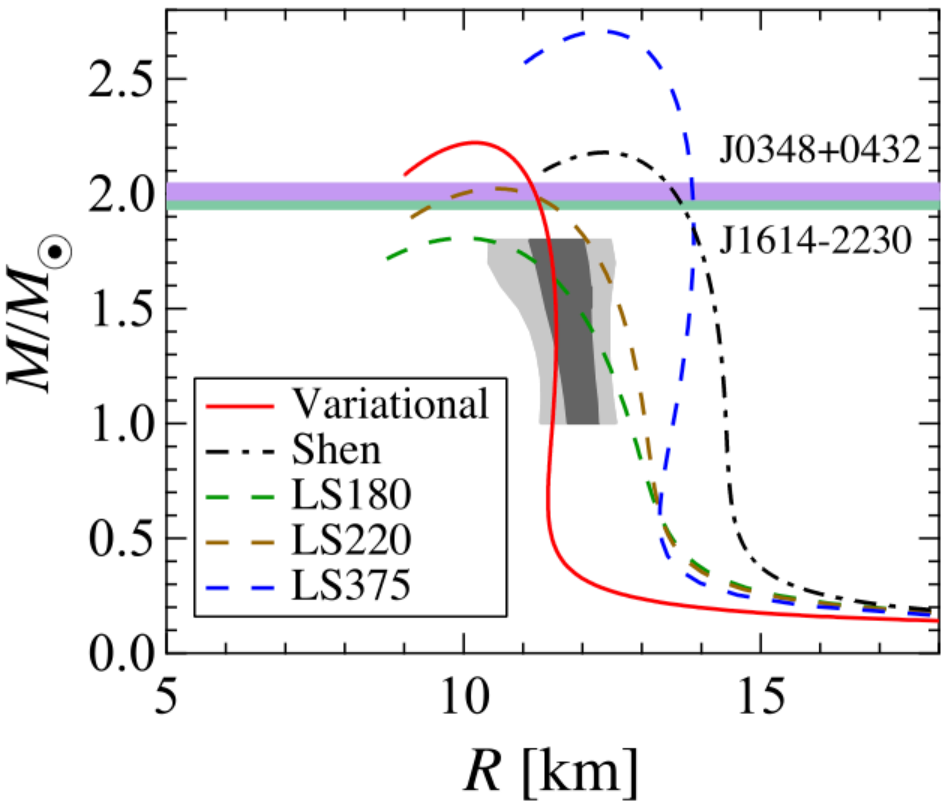
Numerical results of Thomas-Fermi Calculation



Shen EOS (APJ Suppl.197 (2011) 20)

Application to neutron stars

Application of the EOS at zero temperature to neutron stars



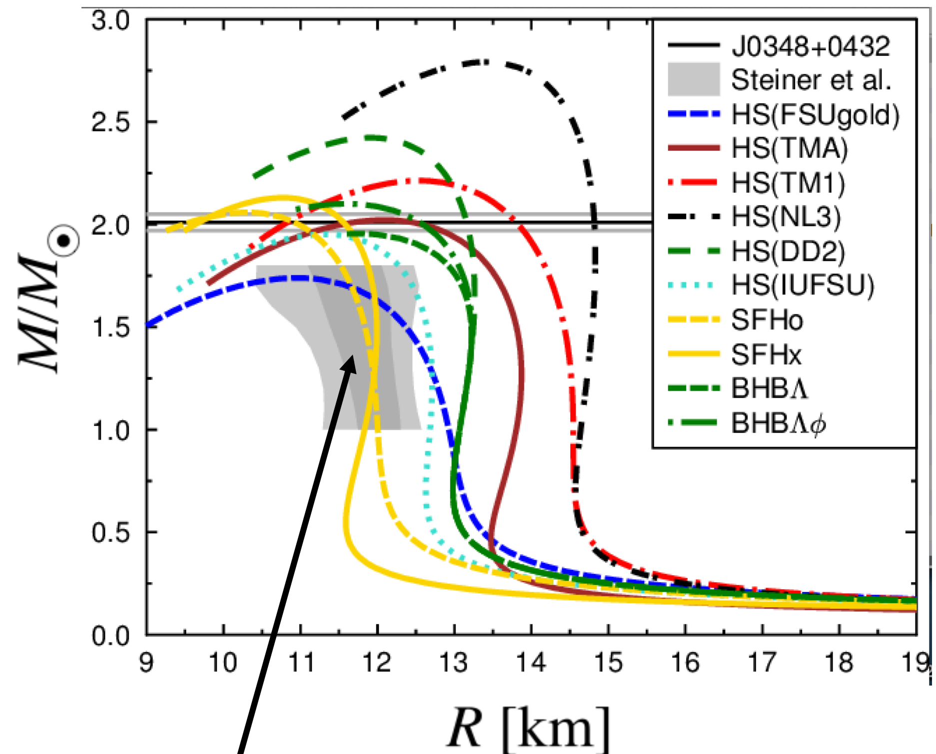


Figure by M. Hempel

Radius of a 1.4 $M_\odot$ neutron star: $R_{1.4}$ = 11.56 km (Variational)

$R_{1.4}$ = 11.89 km (SFHo)

J0348+0432: Science 340 (2013) 1233232 J1614-2230: Nature 467 (2010) 1081

Shaded region is the observationally suggested region by Steiner et al. (Astrophys. J. 722 (2010) 33)

3. Extension to hyperonic nuclear matter

Two-body Hamiltonian

$$H_2 = -\sum_i \left[m_i c^2 + \frac{\hbar^2}{2m_i} \nabla_i^2 \right] + \sum_{i<j} V_{ij}$$

Two-body potential

$$V_{ij} = V_{ij}^{NN} + V_{ij}^{N\Lambda} + V_{ij}^{\Lambda\Lambda}$$

Three-body Hamiltonian

$$H_3 = \sum_{i<j<k} V_{ijk}$$

Three-nucleon potential : UIX

- ***NN* potential**: AV18 two-nucleon potential (PRC 51 (1995) 38)
- **Λ*N* and ΛΛ potentials**: Central potentials
  (E. Hiyama et al., PRC 74 (2006) 054312) (E. Hiyama et al., PRC 66 (2002) 024007)
  - **The *ab initio* variational calculations for Λ hypernuclei reproduce their experimental eigenvalues.**

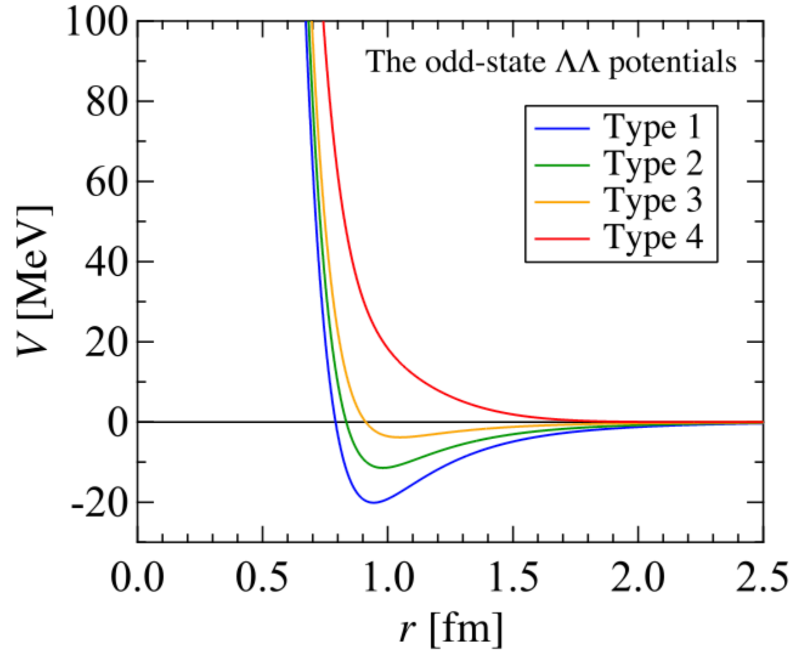
*The experimental data of hypernuclei give no information on **the odd-state part of the ΛΛ interactions**.*

The odd-state part of the ΛΛ interaction

We prepare **four different models** for the odd-state part of the ΛΛ interaction.

Type 1 : *The most attractive*
Type 2 : *Less attractive*
Type 3 : *Slightly repulsive*
Type 4 : *The most repulsive*

The repulsion strength of Type 4 is comparable to that of the odd-state repulsion of ΛN interaction.



The repulsive effect increases monotonically from Type 1 to Type 4.

- We investigate the effects of **the odd-state part of bare ΛΛ interactions** on **the compact astrophysical objects**.

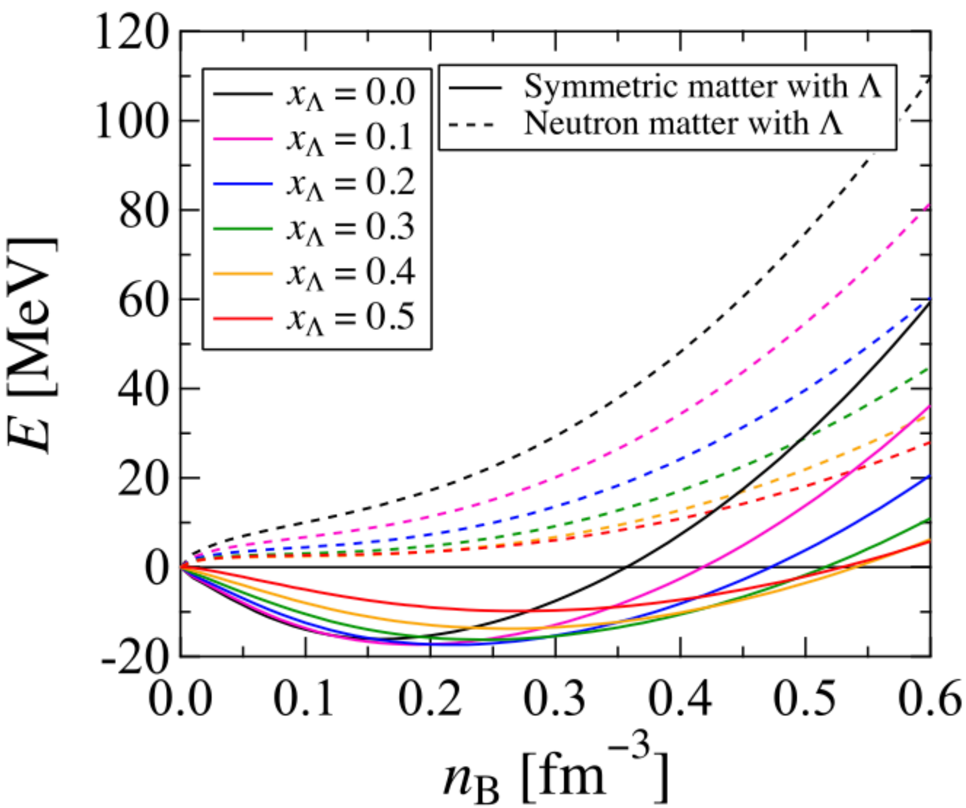
Energy of hyperonic nuclear matter

Energy per baryon $\boxed{E(n_\mathrm{n}, n_\mathrm{p}, n_\Lambda) = E_2(n_\mathrm{n}, n_\mathrm{p}, n_\Lambda) + E_3^\mathrm{N}}$

E_2 : The expectation value of H_2 with the Jastrow wave function
in the two-body cluster approximation

$$\Psi = \mathrm{Sym}\left[\prod_{i<j} f_{ij}\right]\Phi_\mathrm{F}$$

Φ_F: Fermi-gas wave function



Energy of hyperonic nuclear matter (Type 1)

E_3^N : Three-nucleon energy

Based on the expectation value of H_3 with the Fermi-gas wave function

$$E_3^\mathrm{N} = \langle \alpha H_3^\mathrm{R} + \beta H_3^{2\pi} \rangle_\mathrm{F} + E_\mathrm{corr}$$

(NPA902 (2013) 53)

Hyperon EOS at finite temperature

We follow the prescription proposed by *Schmidt and Pandharipande.*

(Phys. Lett. 87B(1979) 11)　　(A. Mukherjee et al., PRC 75(2007) 035802)

Free energy F is expressed by the average occupation probabilities.

The average occupation probability

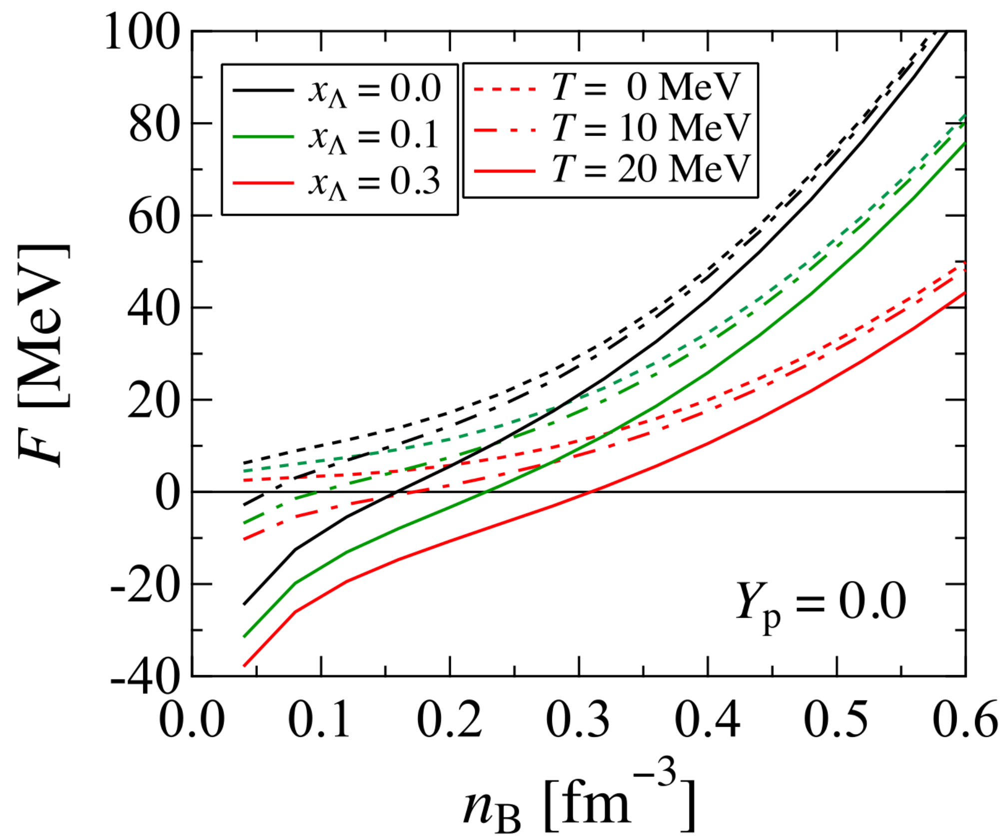
$$f_i(k) = \left\{ 1 + \exp\left[\frac{\varepsilon_i(k) - \mu_{0i}}{k_B T} \right] \right\}^{-1}$$

$\varepsilon_i(k)$: Single particle energy

$$\varepsilon_i(k) = \frac{\hbar^2 k^2}{2m_i^*} \quad (i = \mathrm{p, n}, \Lambda)$$

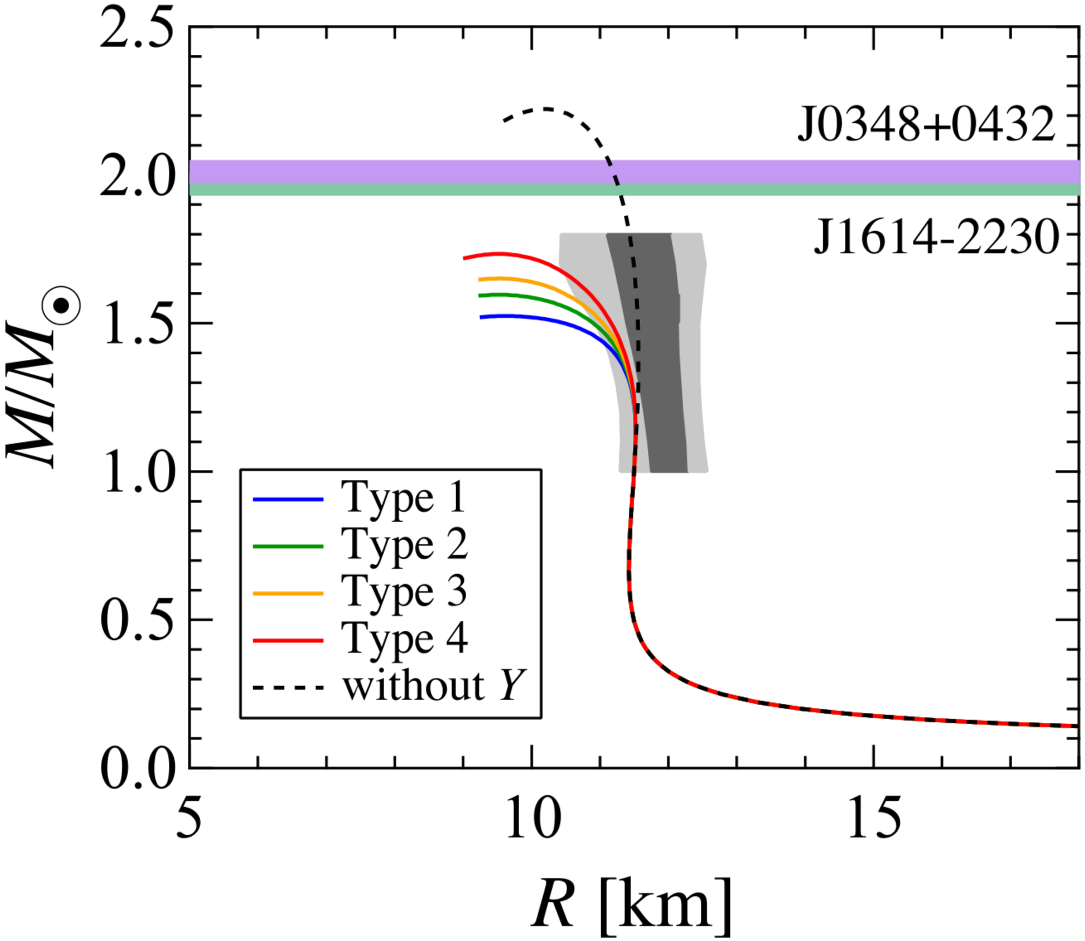
m_i^*: Effective mass of baryons

Free energies are minimized with respect to m_i^*



Free energy of hyperonic nuclear matter

Application to Neutron Star



Mass-radius relations of neutron stars

Maximum mass of neutron stars

Type 1	$1.52\ M_\odot$
Type 2	$1.60\ M_\odot$
Type 3	$1.65\ M_\odot$
Type 4	$1.73\ M_\odot$
without Y	$2.22\ M_\odot$

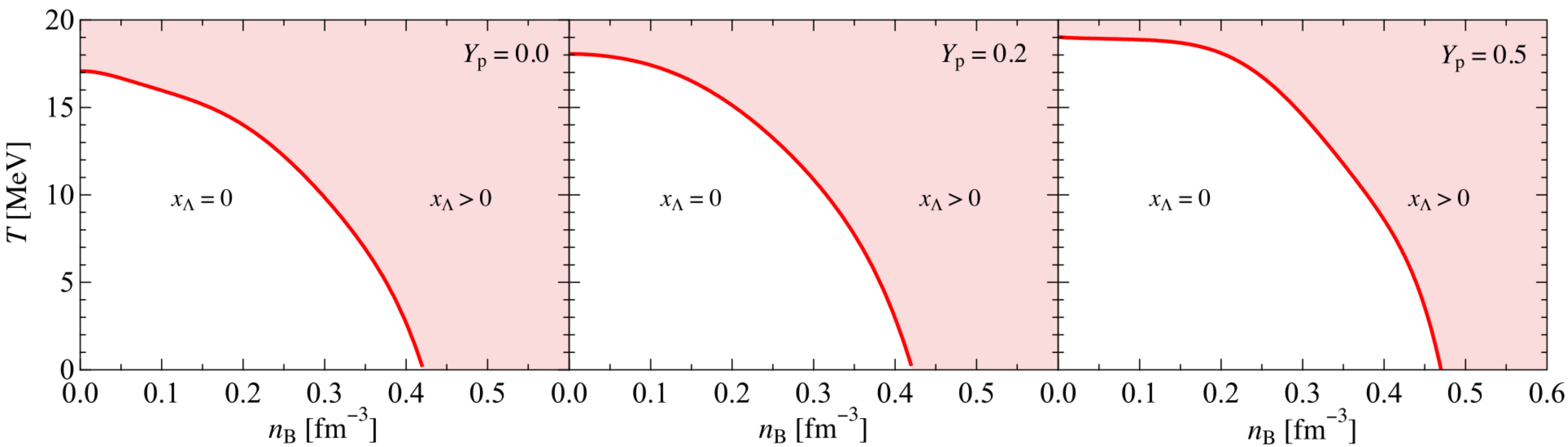
The maximum mass increases.
($1.52\ M_\odot \rightarrow 1.73\ M_\odot$)

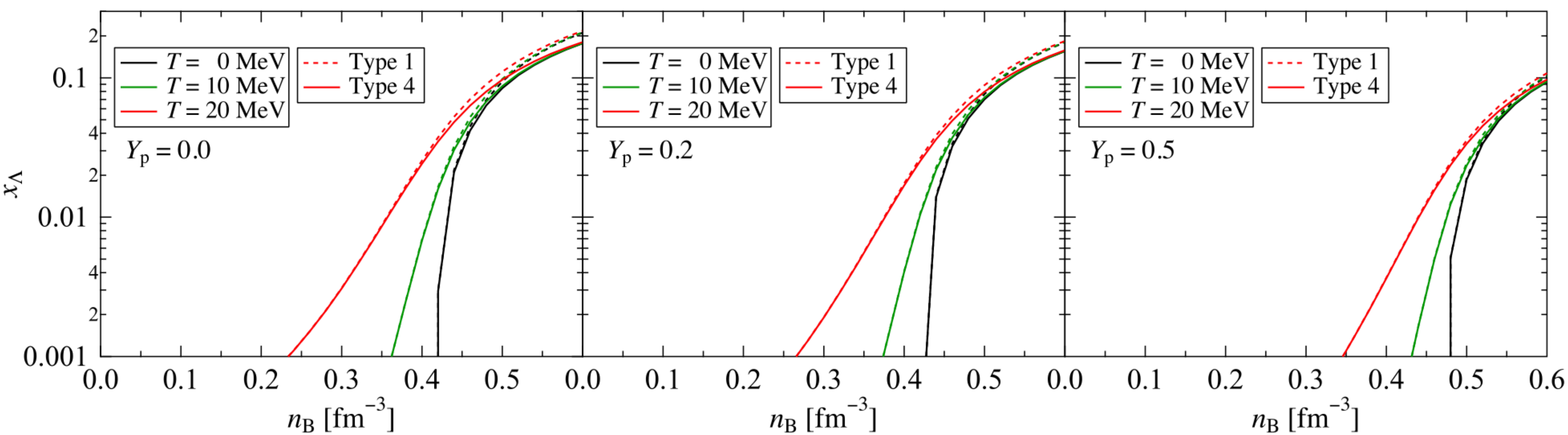
J0348+0432: Science 340 (2013) 1233232 J1614-2230: Nature 467 (2010) 1081

Shaded region is the observationally suggested region by Steiner et al.
(Astrophys. J. 722 (2010) 33)

Application to supernova matter

We calculate the onset density of Λ hyperons in hot dense matter
with the equilibrium condition $\mu_n = \mu_\Lambda$.





4. Summary

We construct the EOS of nuclear matter including Λ hyperons at zero and finite temperatures by the variational method.

- The obtained thermodynamic quantities are reasonable.
- The repulsion in the odd-state ΛΛ interaction raises the maximum mass of neutron star. ($1.52\ M_{\odot} \rightarrow 1.73\ M_{\odot}$)
- The onset density of Λ is insensitive to the odd-state ΛΛ interaction.
- TBF shifts the onset density of Λ to the higher density region at low temperatures.

Future Plans

- Construction of the EOS table for core-collapse simulations
- Taking into account mixing of other hyperons (Σ^-, Σ^0, Σ^+, Ξ^0, Ξ^-)
- Employing more sophisticated baryon interactions (e.g. Nijmegen)