NAOJ 14th. May (2016)
25 min. including discussion

Rotational equilibria by variational Lagrange scheme with realistic EOS

Chiba Institute of Technology

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MNRAS Letter (2015) 446, 56

MNRAS (2016) submitted

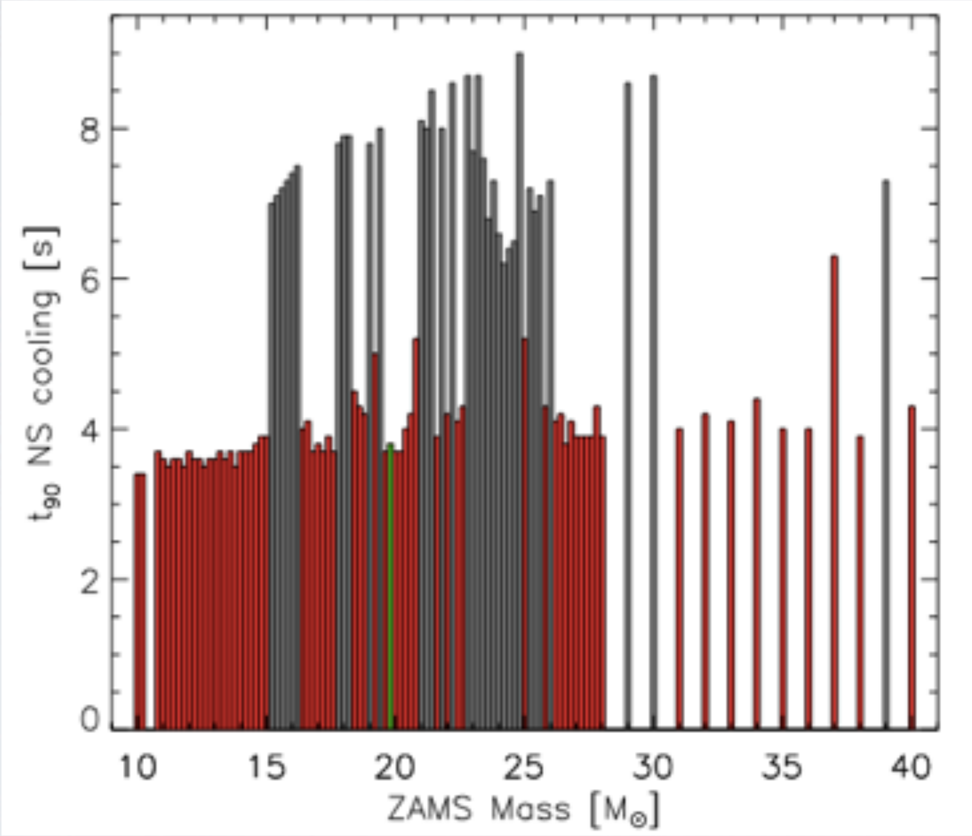
NY, K.Fujisawa, S.Yamada

“MOTIVATION”

IMPORTANCE OF STELLAR EVOLUTION

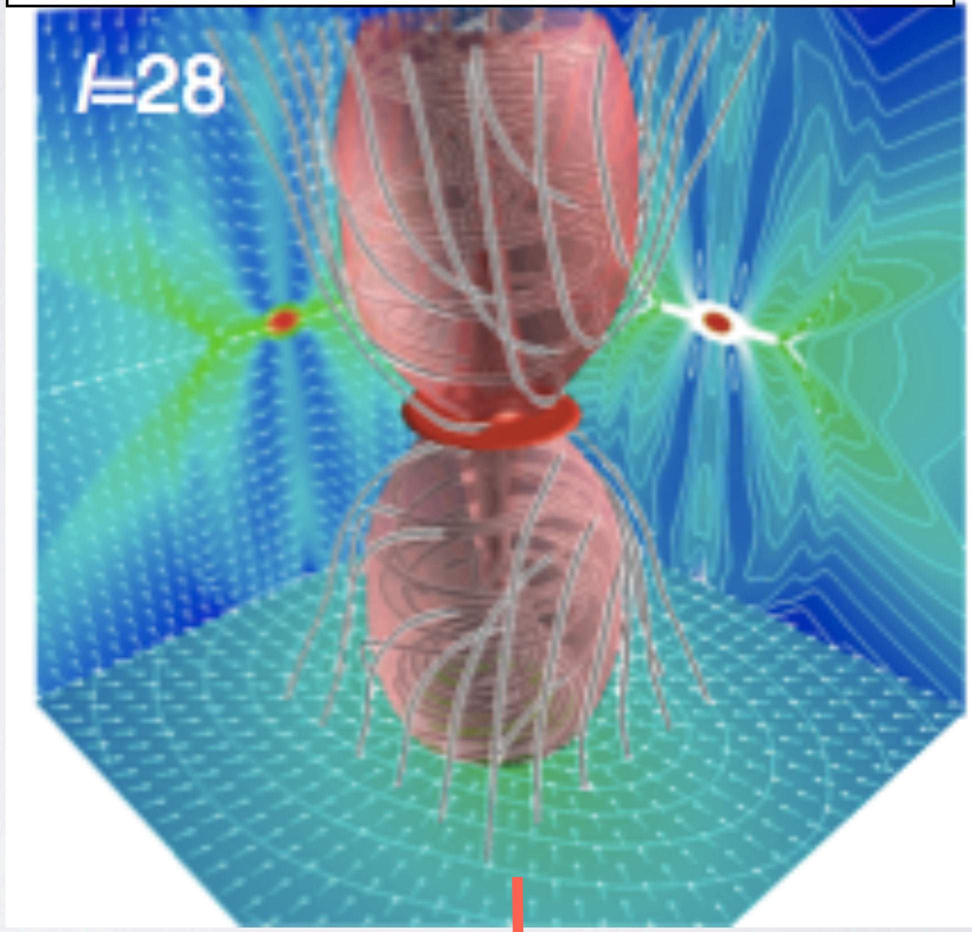
- Supernovae mechanism
  mass → compactness

But how does it appear ?



Ugliano et al. (2012)APJ, 757, 69

dynamical simulation(3D) of star formation



Machida et al. (2009)APJ, 685, 690

?

evolution(∼1D)

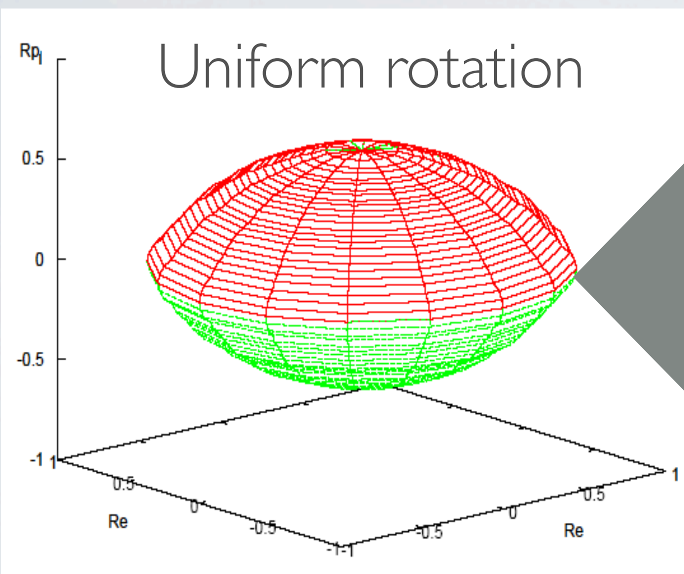
Most of general toolkits for stellar evolution calculations are based on Heney method since 1964.

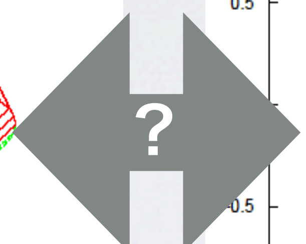
Rotational law ?

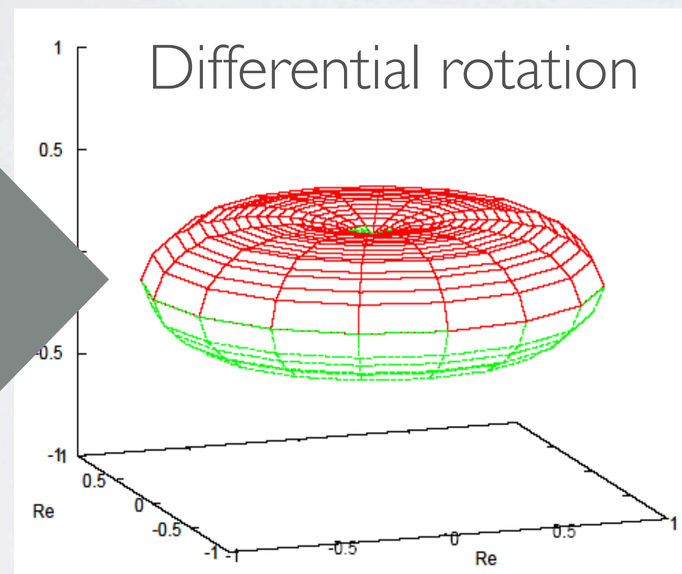
numerical simulation

astroseismology(Sun)

NY, Hashimoto, Eriguchi, PTP(2005)







$$\Omega = \Omega_C$$

$$\Omega = \frac{\Omega_C A^2}{(r/r_{eq})^2 + A^2}$$

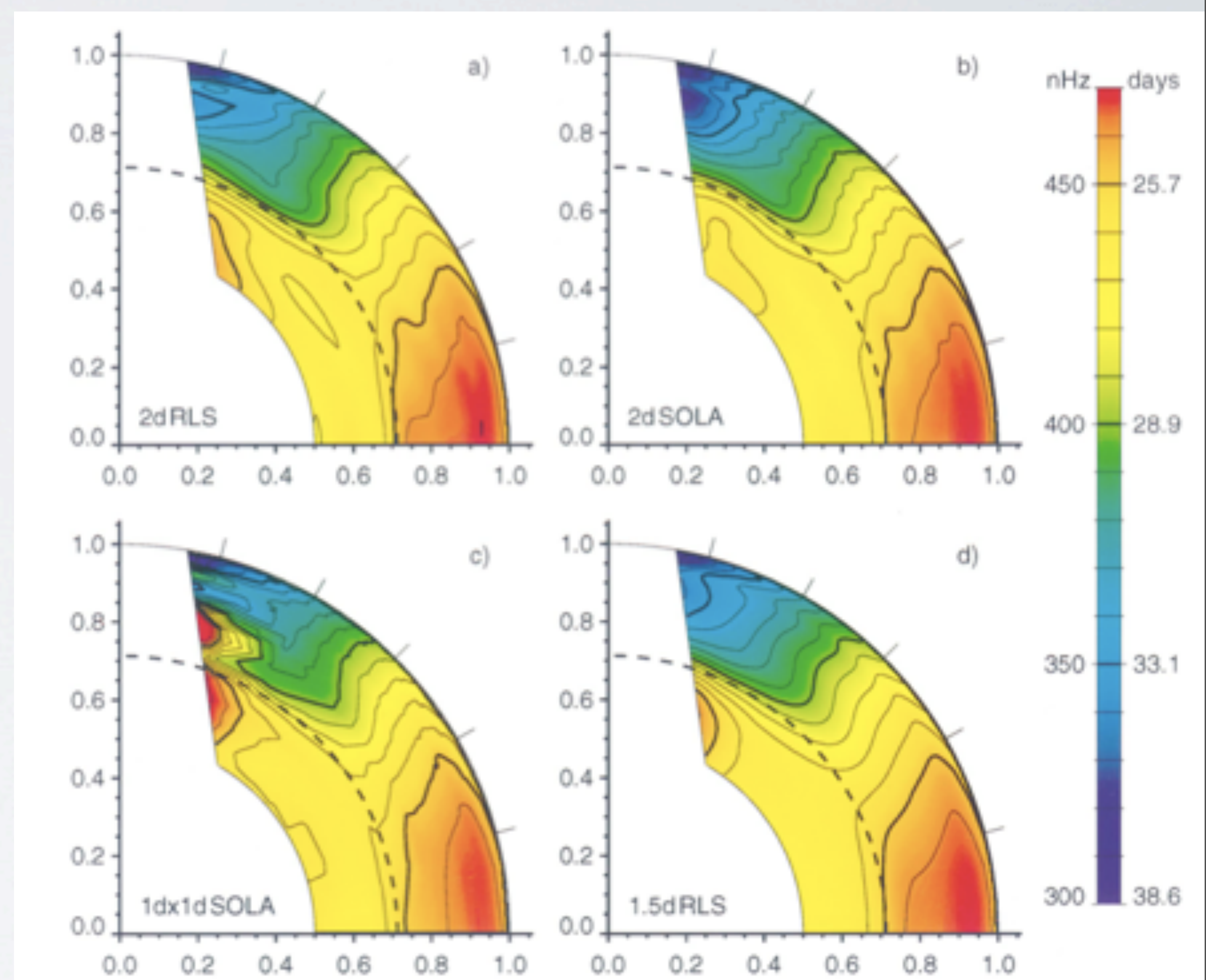


FIG. 5.—As a companion to Fig. 3, inversions for rotation rate Ω/2π with radius and latitude for four inversion methods: (a) 2dRLS; (b) 2dSOI
(c) 1d × 1dSOLA; (d) 1.5dRLS.

Schou et al.(1998)

The other difficulties of deformed stellar evolution

① Barotropic EOS, P(ρ) → Realistic (baroclinic) EOS, P(ρ,Yi,T)

Uryu&Eriguchi (1994,1995), Roxburgh (2006), Espinoza-lala&Rietord(2007), Fujisawa(2015)

② Trajectory of mass(element).

③ Convection.

④ Mass ejection.

etc. ...

The other difficulties of deformed stellar evolution

This work

① Barotropic EOS, $P(\rho)$ → Realistic (baroclinic) EOS, $P(\rho, Y_i, T)$

Uryu&Eriguchi (1994,1995), Roxburgh (2006), Espinoza-lala&Rietord(2007), Fujisawa(2015)

② Trajectory of mass(element).

③ Convection.

④ Mass ejection.

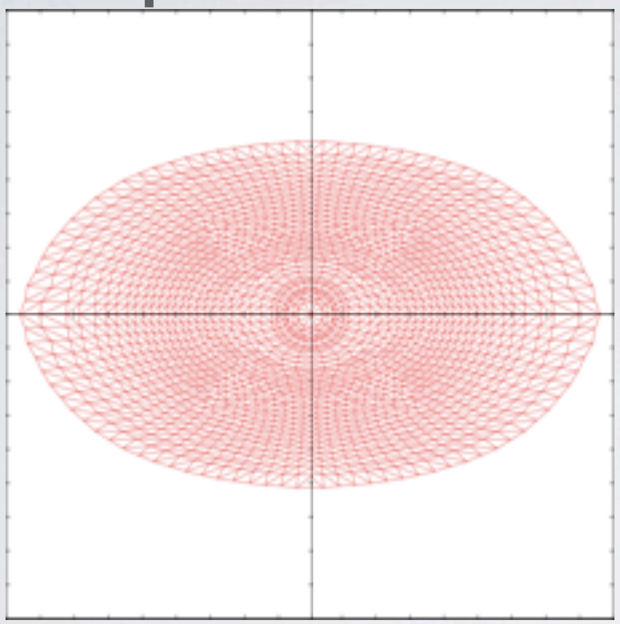
⇔ dynamical simulation(not this study)

etc. ...

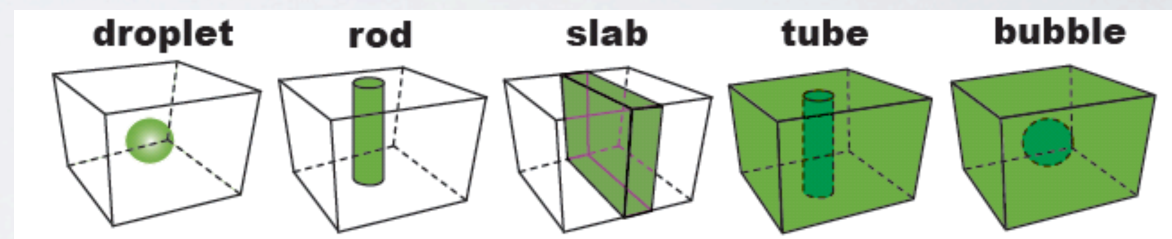
“OUR NEW METHOD”

"DEFORMED STAR/PASTA DUALITY"

Deformed stars
(non-spherical stars)



Pasta structures which are non-uniform structures in phase transitions; neutron drip, quark-hadron phase transition, etc.)



Hydro-static equilibrium

$$\frac{\delta E[\boldsymbol{\xi}]}{\delta \boldsymbol{\xi}} = \nabla P + \rho \nabla \phi - \frac{\rho j^2}{\varpi^3} \mathbf{e}_\varpi = 0.$$

Chemical equilibrium

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_C^Q, \qquad \mu_d = \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_C^Q,$$

$$\mu_n = \mu_\Lambda = \mu_B, \qquad \mu_p = \mu_B + \mu_{C,H}, \qquad \mu_{\Sigma^-} + \mu_p = 2\mu_B,$$

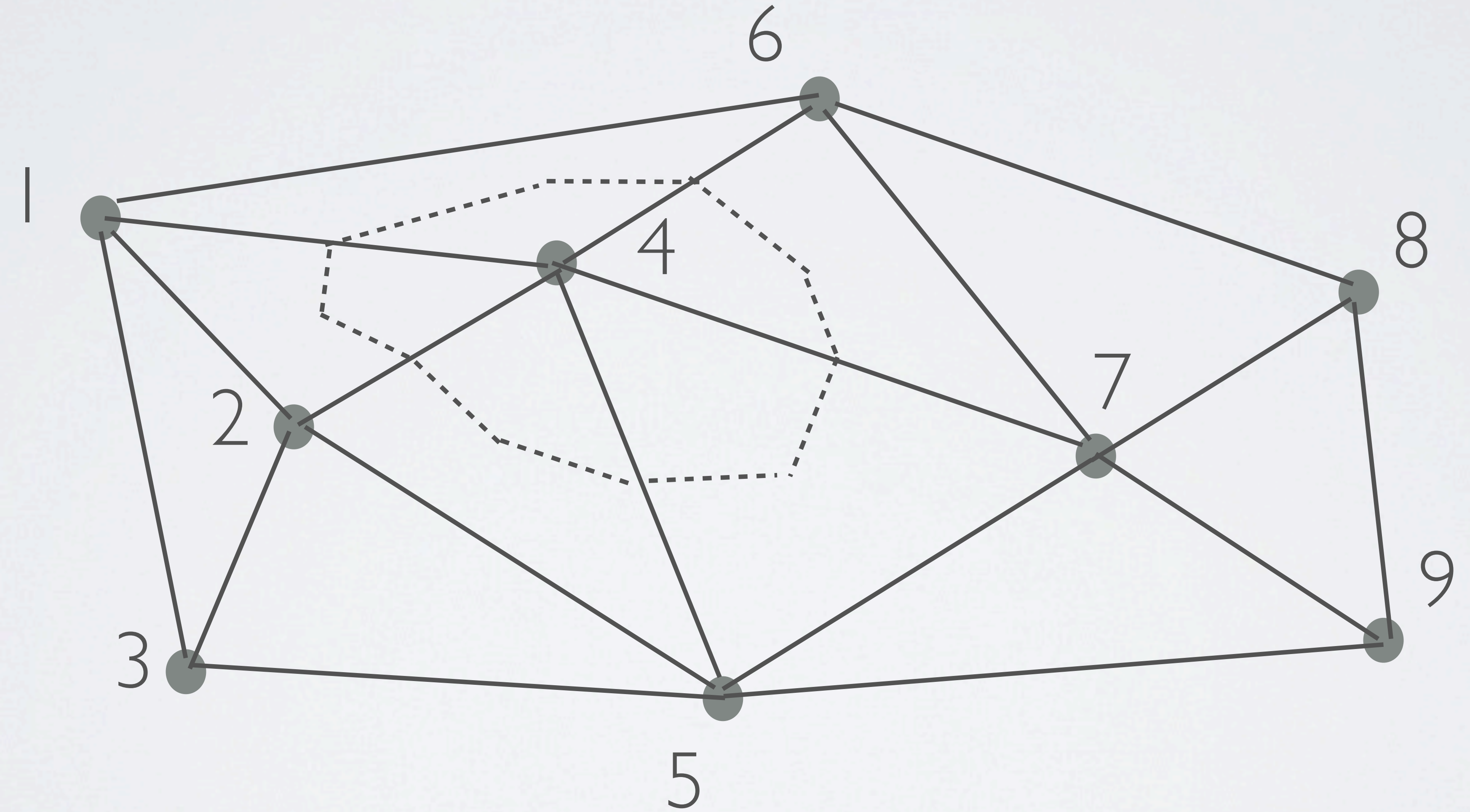
$$\mu_L^{H(Q)} = \mu_{\nu_e}^{H(Q)}, \qquad \mu_C^{H(Q)} = \mu_L^{H(Q)} - \mu_e^{H(Q)},$$

ex) quark-hadron phase transition

repulsion = pressure and rotation
attraction = gravitation

repulsion = Coulomb interaction
attraction = surface tension

StepIV If you give each node the physical quantities; position, and Lagrange values(mass, angular momentum, entropy, fractions ...), you can get Eulerian values (partial volume, density, ...) considering with the relation between neighboring nodes (and/or the adjacency matrix).

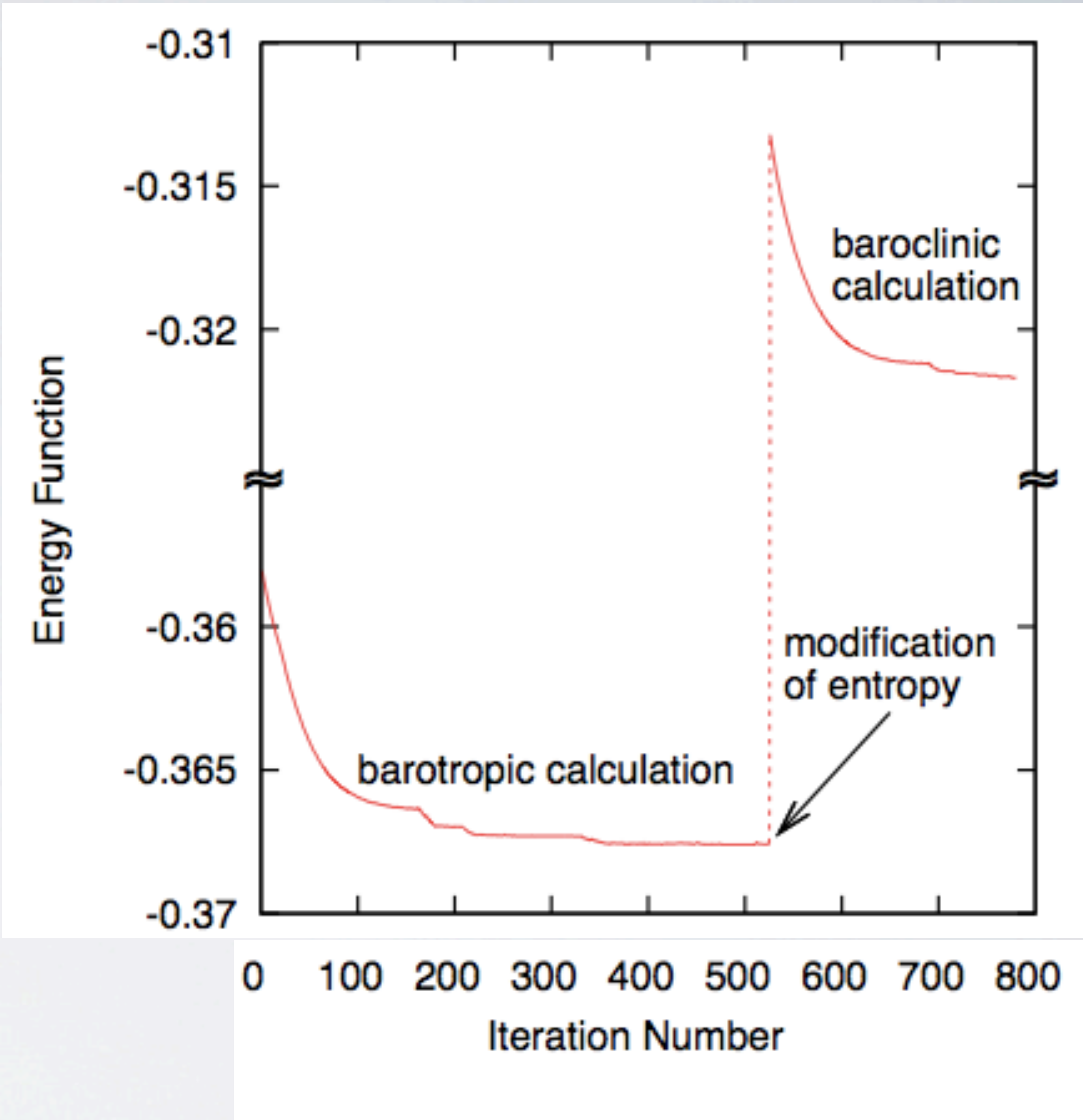


each node: $x, y, m, j, s, Ye, Yn, Yp, Y_{He}$ ..... → dv, $\boldsymbol{\rho}$, P, T, u, ...

StepV

Find the most optimal arrangement by changing the positions of nodes finding the minimum total energy.

$$E_{\mathrm{FEM}}(\boldsymbol{r}_i) = \sum_i \varepsilon_i m_i + \frac{1}{2}\sum_i \phi_i m_i + \sum_i \frac{1}{2}\left(\frac{j_i}{\varpi_i}\right)^2 m_i$$



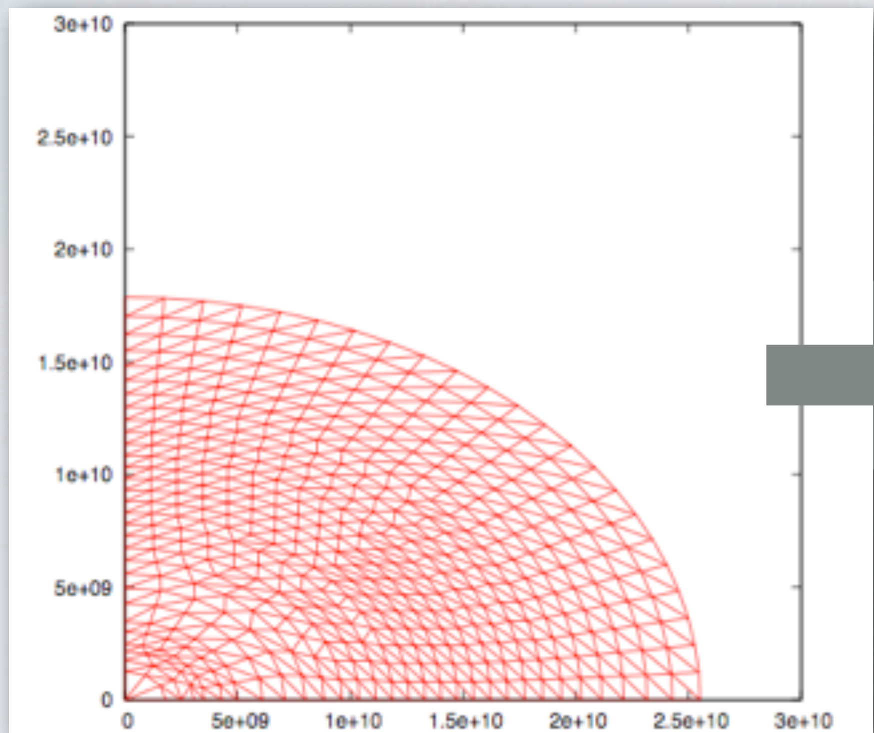
Caution

Now, you may think that this is about two dimension. Of course, if you give (x_i,y_i) as the positions on nodes, it becomes a two-dimensional topic. But, if you give (x_i,y_i, z_i, ...), it becomes the multi-dimensional.

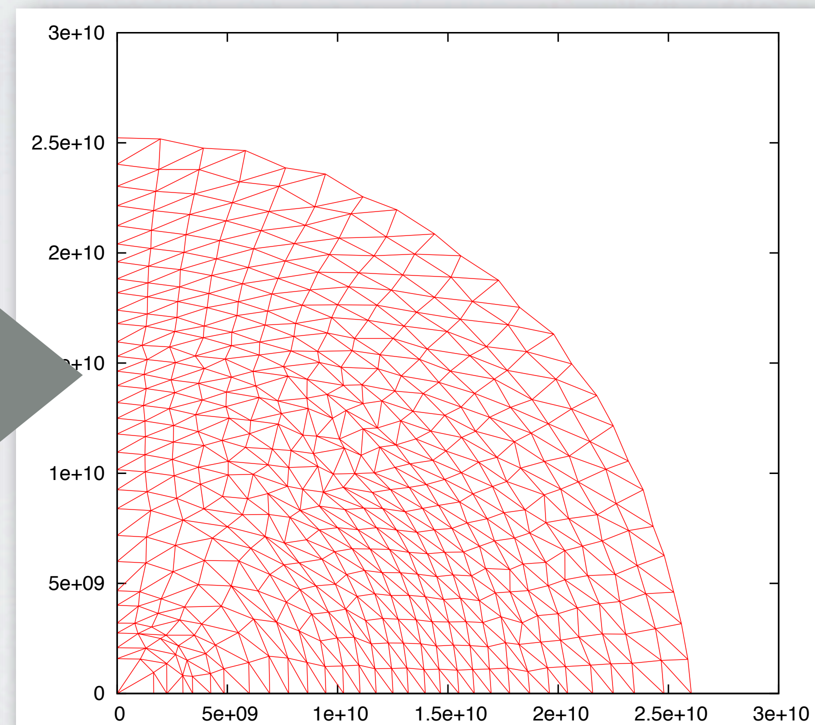
“APPLICATIONAL RESULT”

A simple example

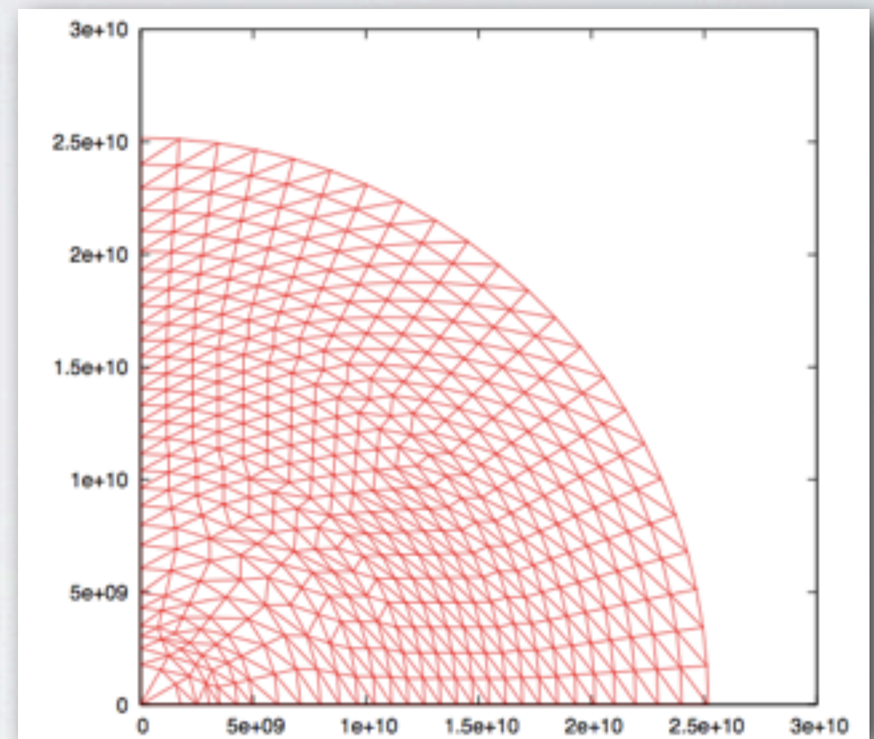
initial guess



result

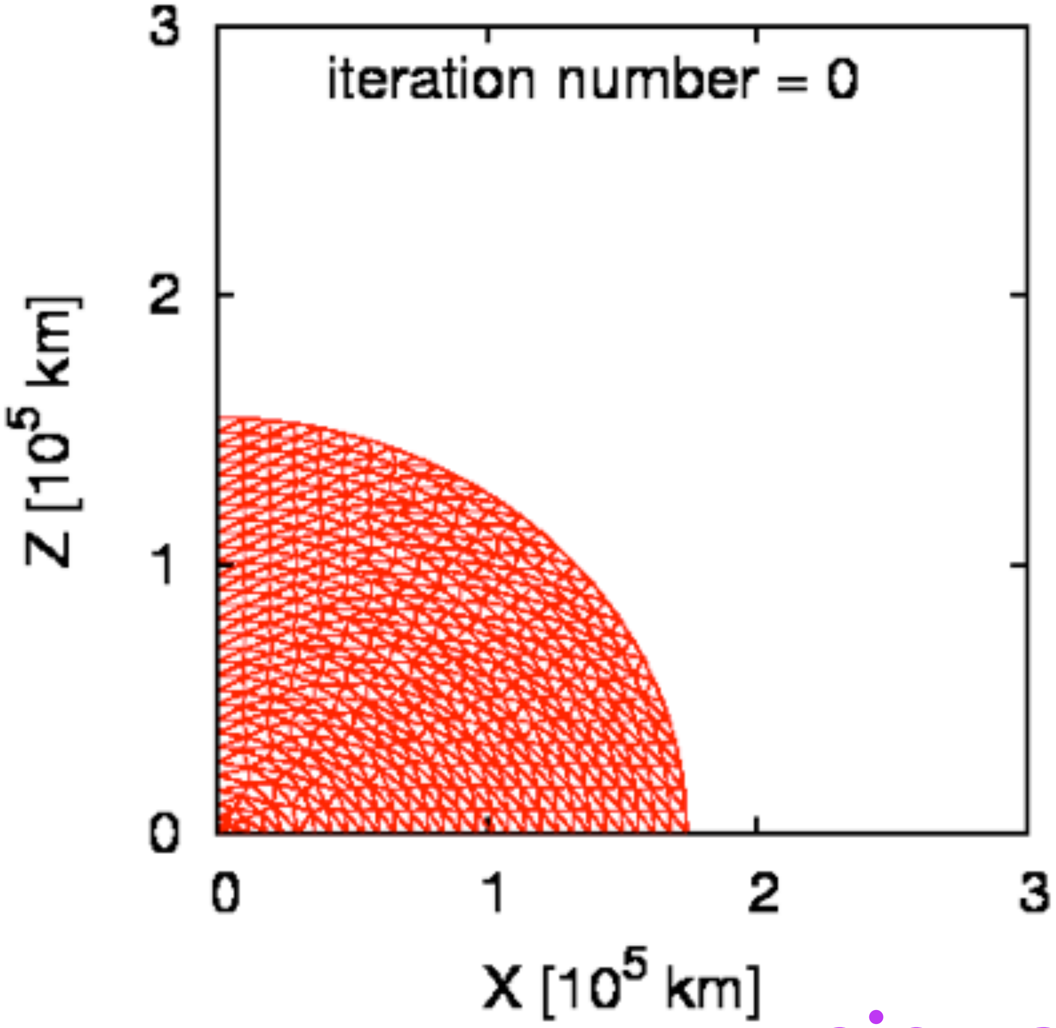


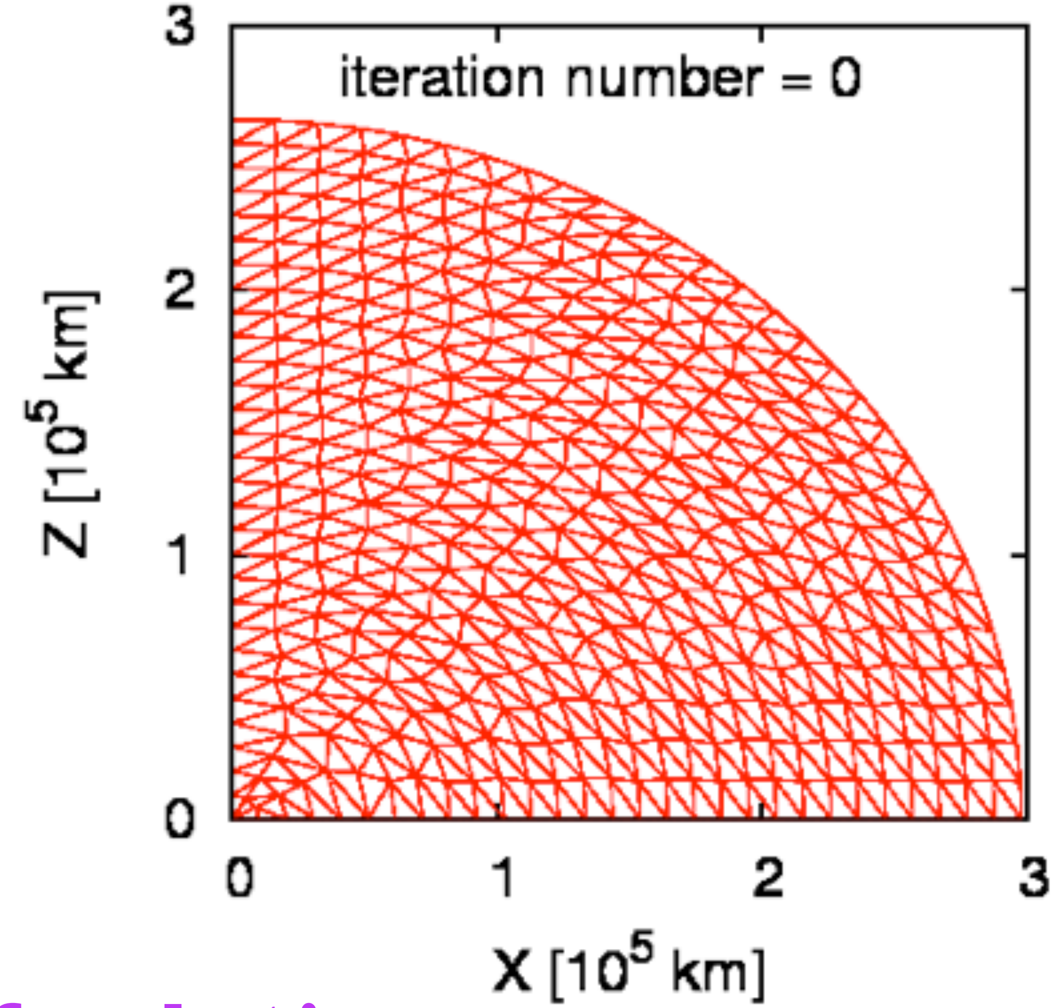
exact solution

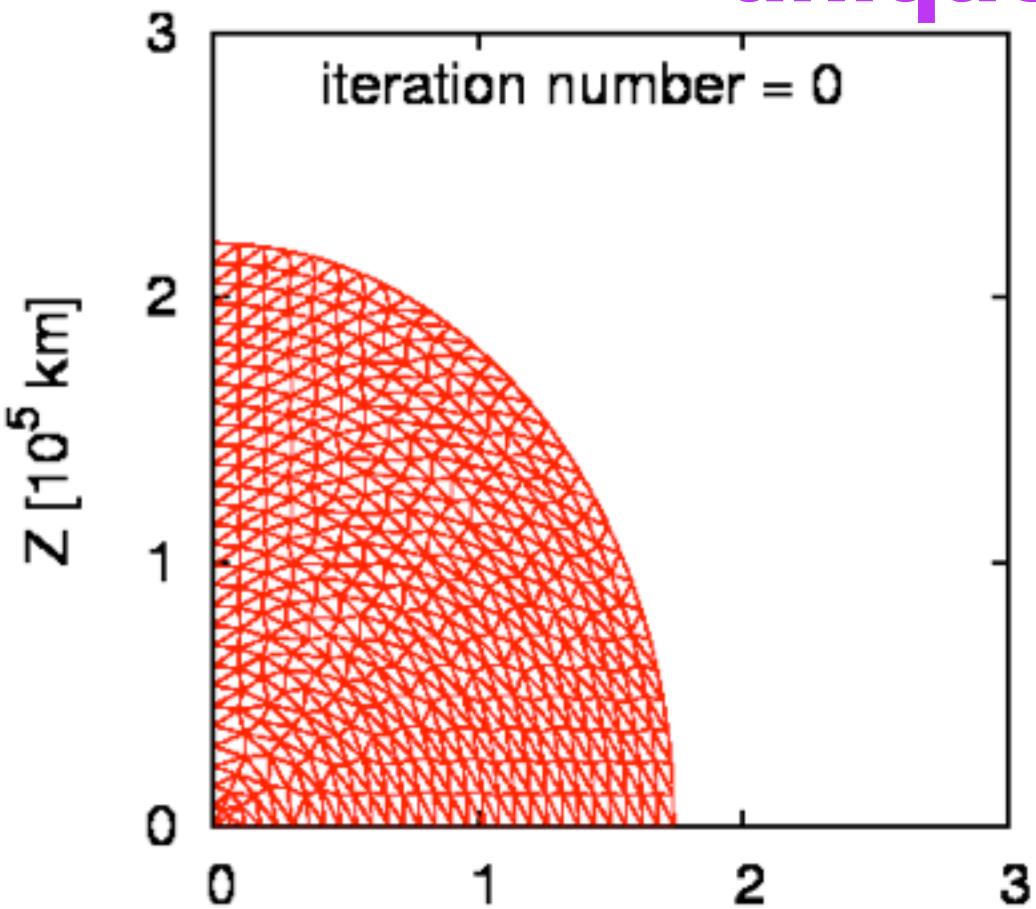


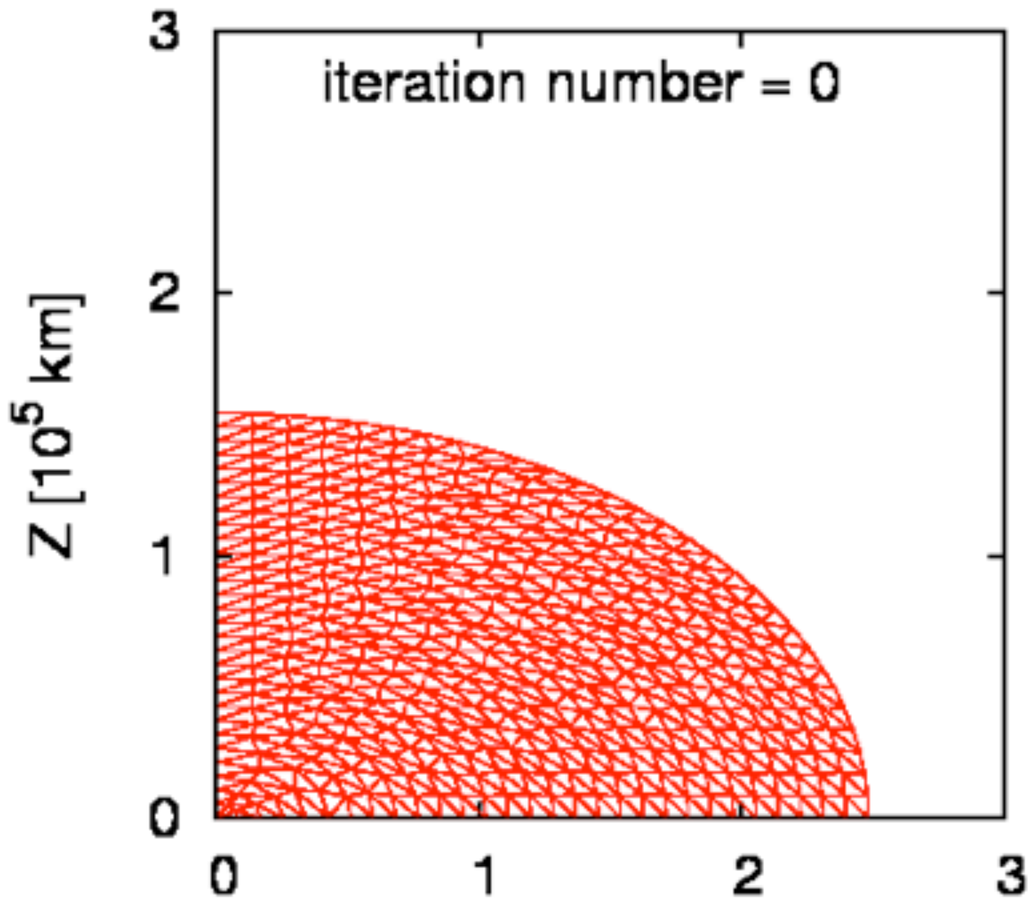
Appropriate initial guesses go to exact solutions.

uniqueness of solution



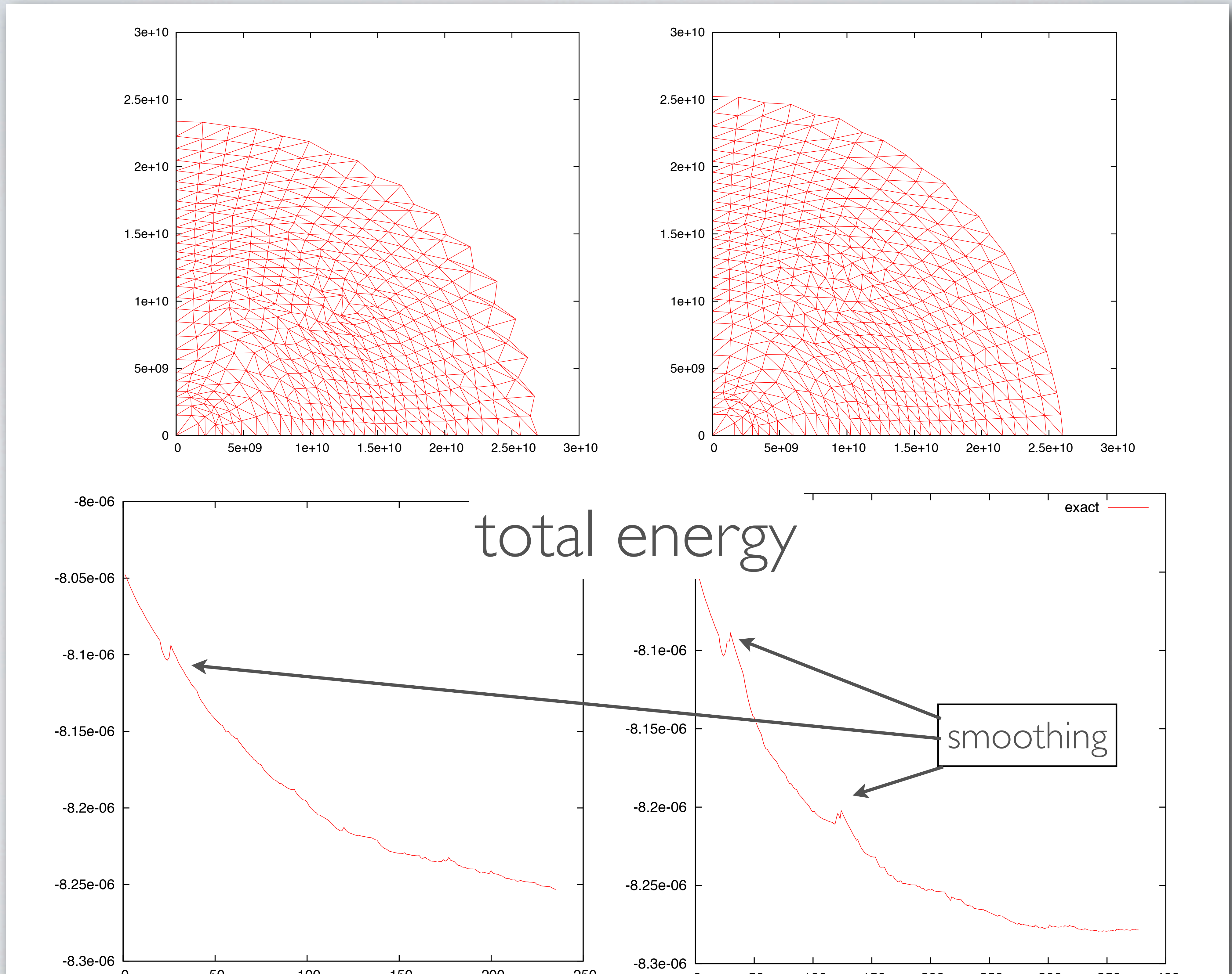




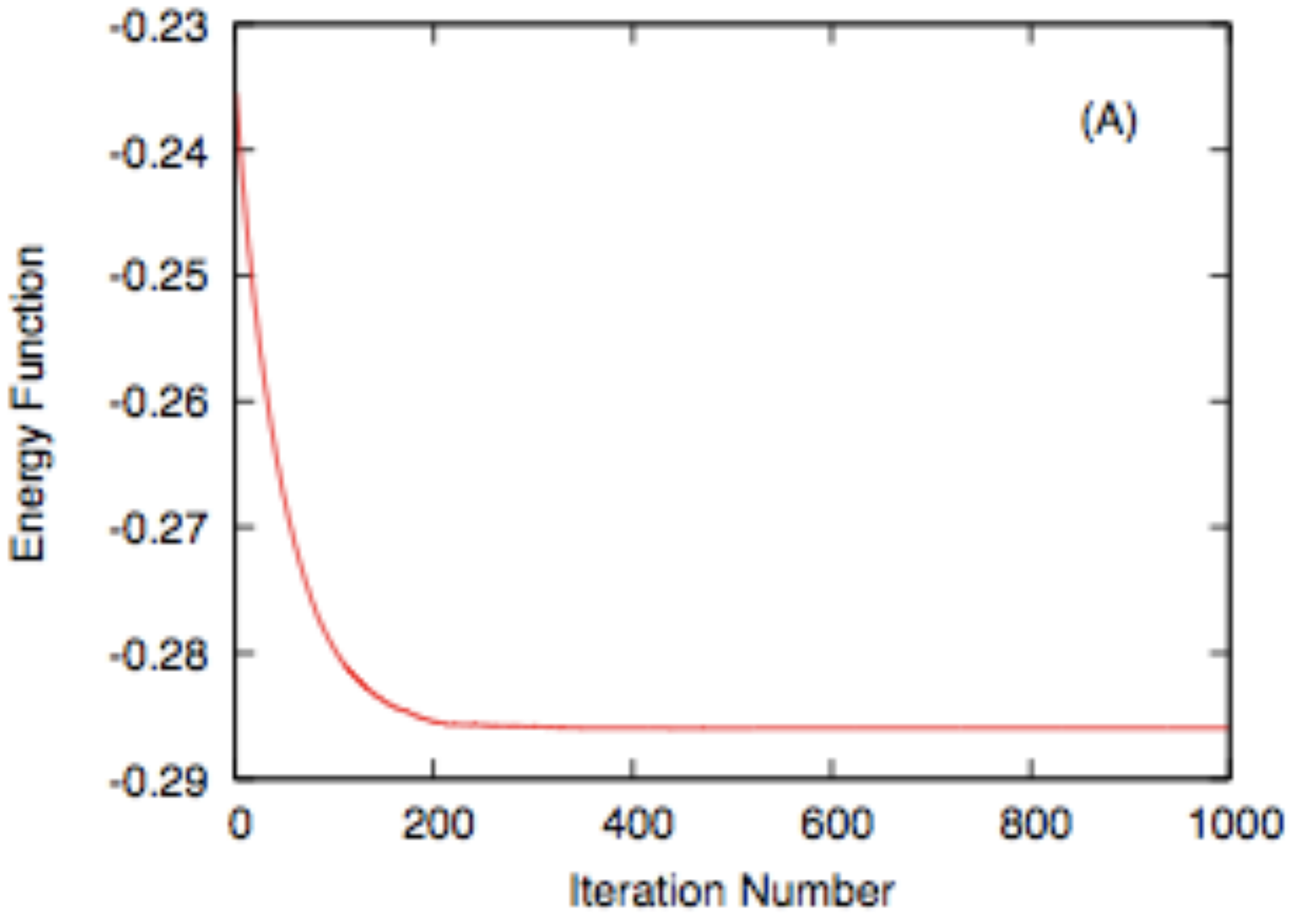


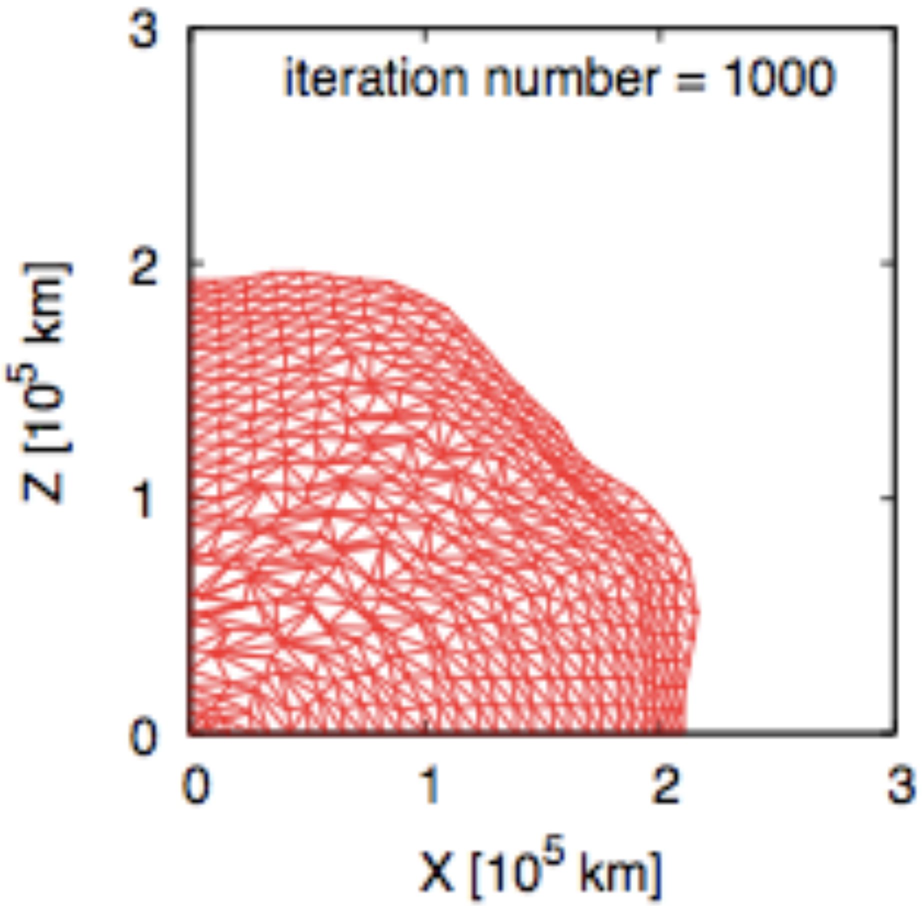
A part of local minimums comes from deformation of triangles

failure success



TRAP OF LOCAL ENERGY MINIMUM (WITHOUT SMOOTHING)





MATHEMATICAL/NUMERICAL PROBLEMS

Lagrangian perturbation fails because of the guarge freedom.

LAGRANGIAN PERTURBATION THEORY OF NONRELATIVISTIC FLUIDS*

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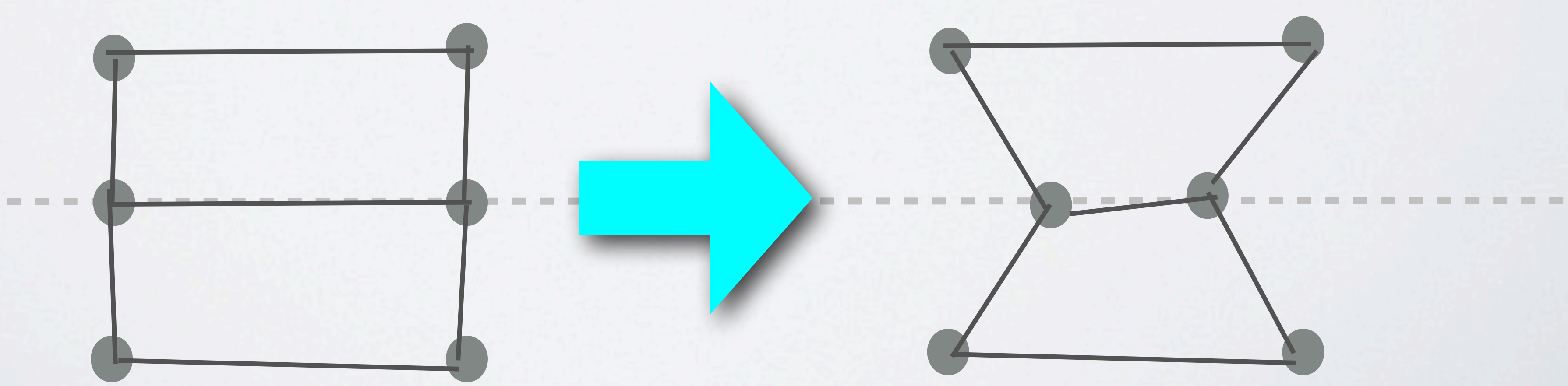
Received 1977 July 11; accepted 1977 November 7

ABSTRACT

In this paper the conventional description of adiabatic perturbations of stationary fluids in terms of a Lagrangian displacement is reexamined, to take account of certain difficulties that have been overlooked in other treatments. A class of displacements—called trivials—that leave the physical variables unchanged is identified; these define "gauge" transformations of the initial data in the Lagrangian picture. The conserved canonical energy E_c (Hamiltonian) and angular momentum J_c (in the case of axisymmetric unperturbed fluids) associated with the dynamical equations are shown not to be invariant under these gauge transformations. Since E_c has formed the basis of previous criteria for secular stability of stars, it is necessary to eliminate the gauge freedom in order to regain a meaningful criterion. To this end a conserved inner product (the symplectic structure) is introduced and used to define a dynamically invariant class of "canonical" displacements orthogonal to the trivials. In general, canonical displacements obey the extra

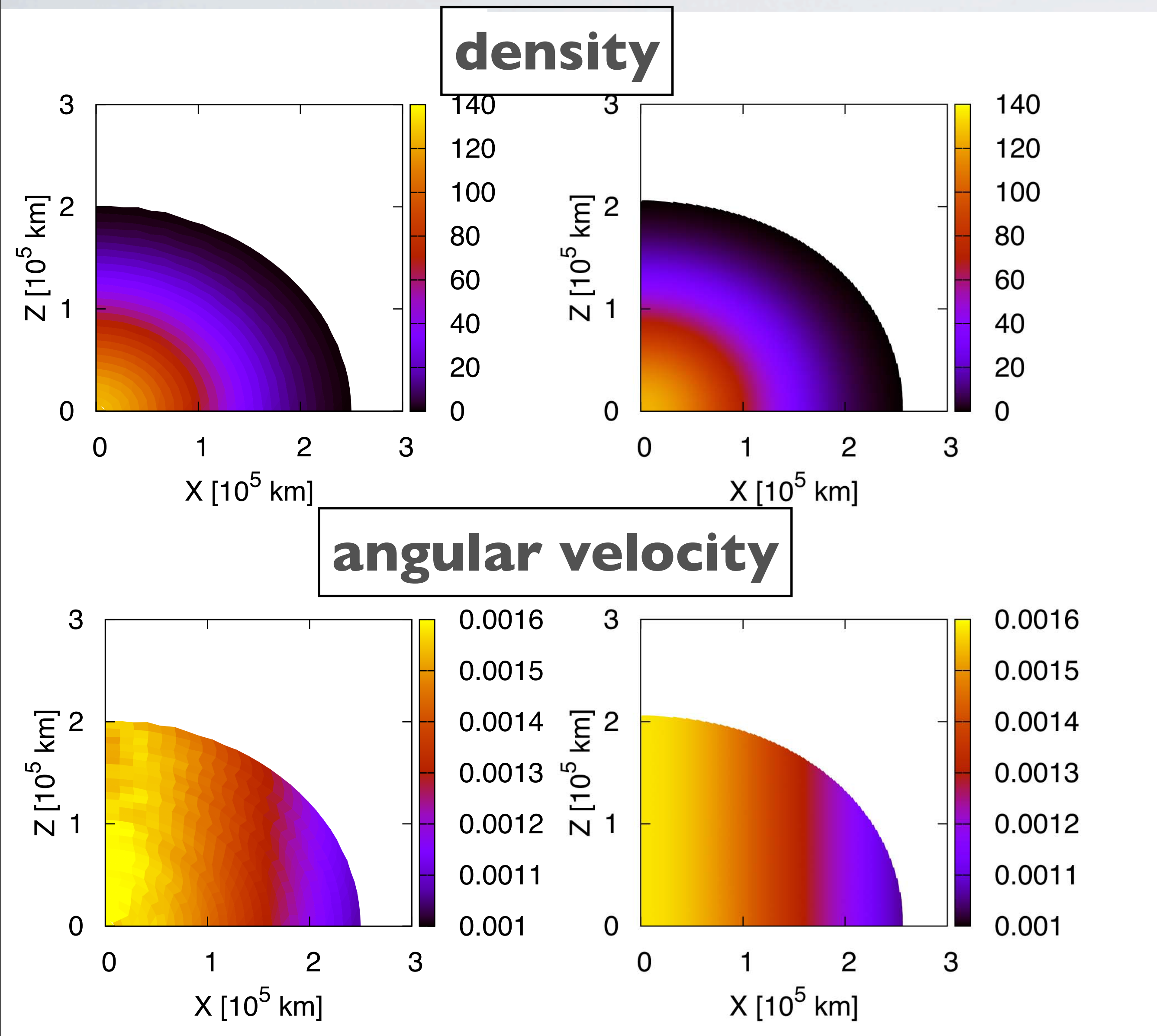
Hour-glass problem

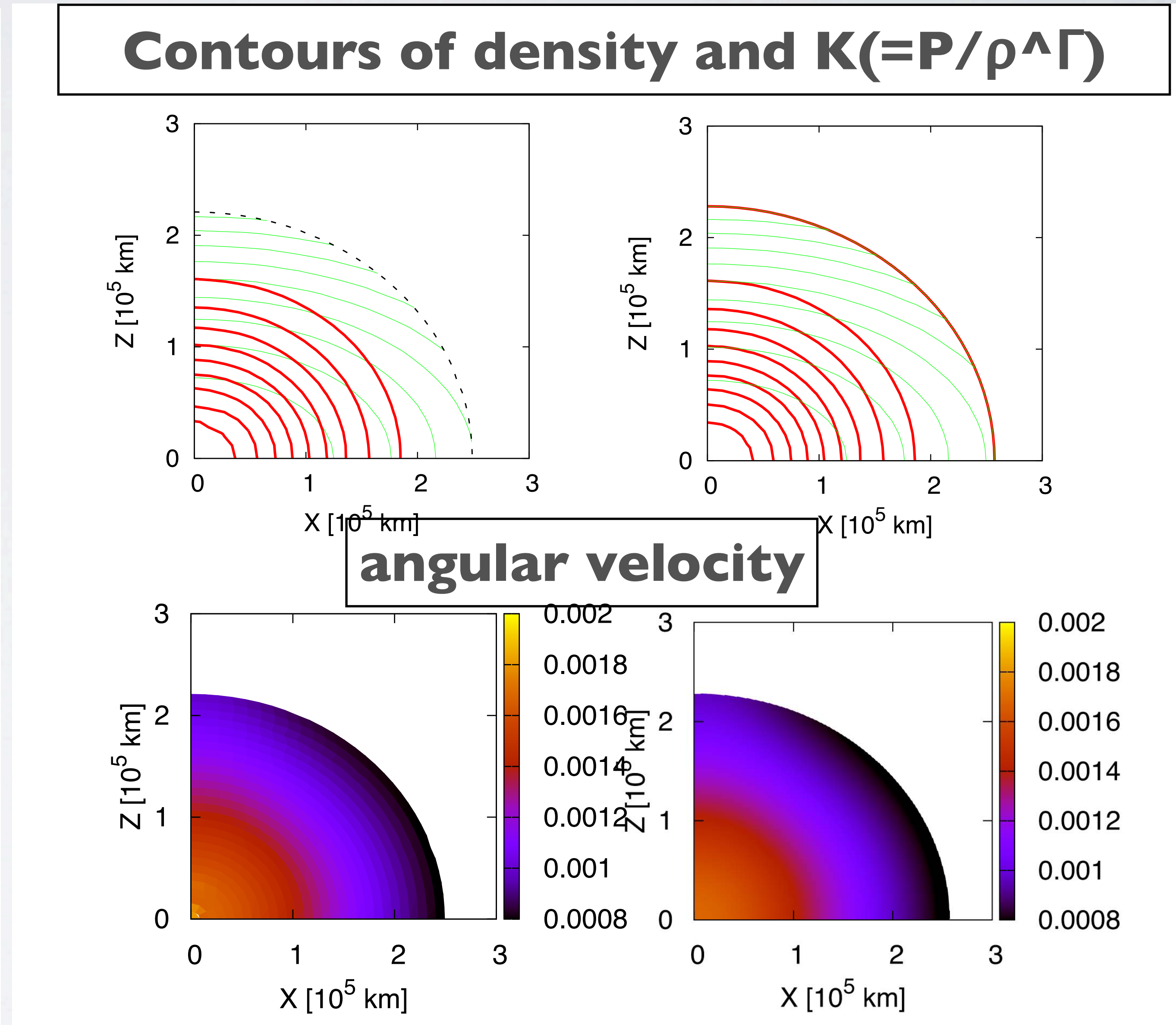
iso potential surface→



COMPARISON WITH OTHER METHODS

Ours | Hachisu method | Ours | Fujisawa method





P=P(ρ) | P=P(ρ,Yi,T)

Stellar structures without H¢iland criteria

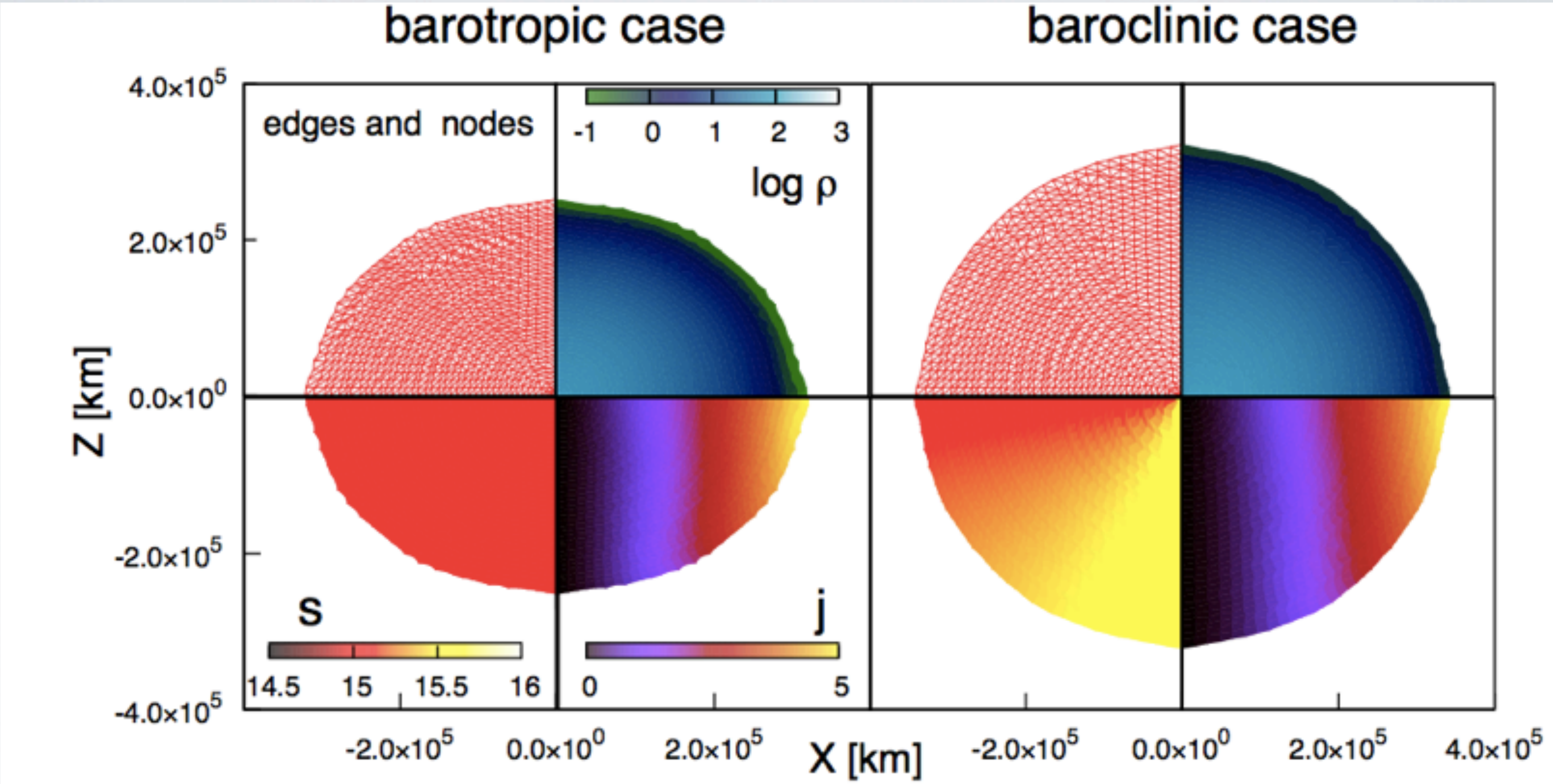


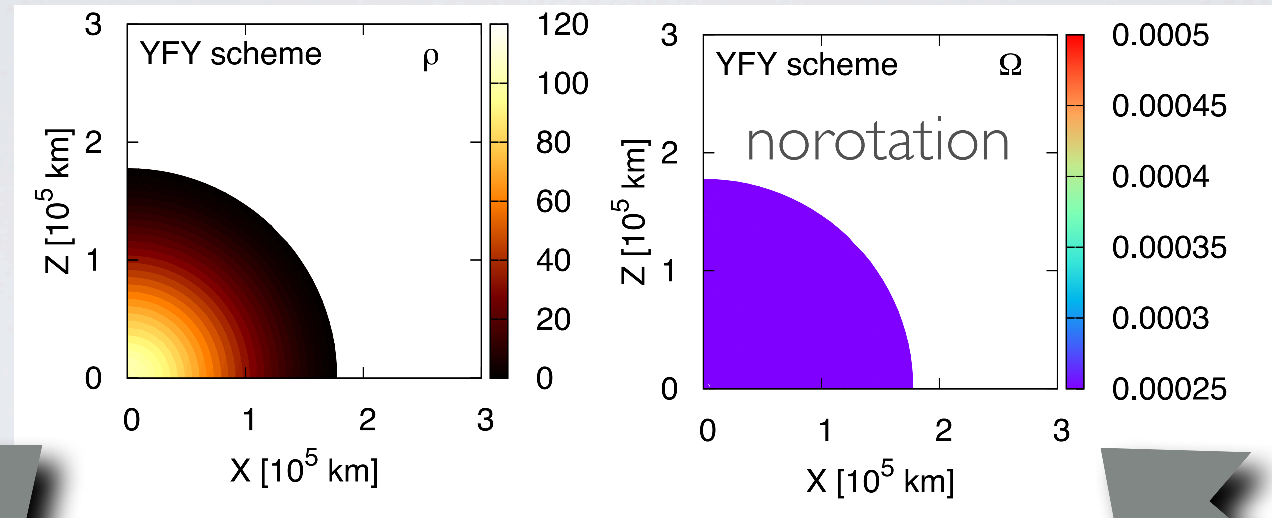
Figure 1. (color on line). Structures of a star in rotational equilibria for barotropic (left four panels) and baroclinic (right four panels) EOS's. The upper left quadrants show the nodes and edges in the triangulated mesh. The other panels display clockwise the color contours of logarithmic density in g/cm^3, specific entropy in k_B and specific angular momentum in $10^{18}cm^2/s$. The color scales are identical for both cases.

We succeeded to get hydro-static equilibriums for stars with realistic (baroclinic) EOS in Lagrange coordinate. The distribution of specific angular moment j is not cylindrical as shown in the right panel.

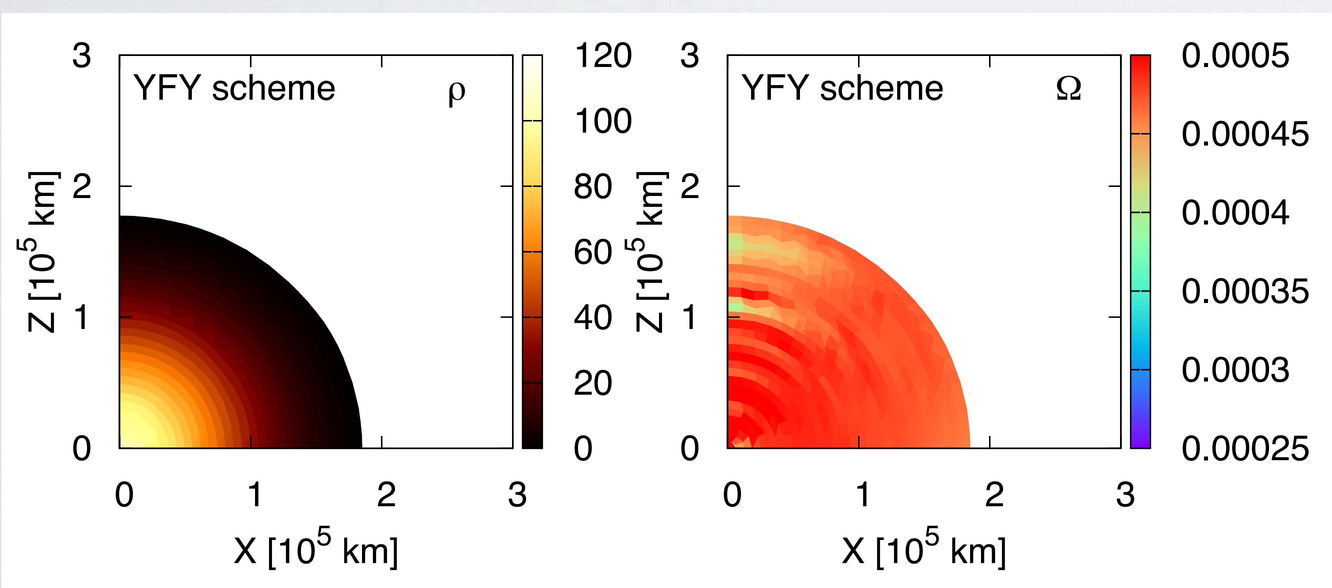
→Bjerkness theorem.

RIGID TYPE ROTATION(LEFT) & SHELLULAR ROTATION(RIGHT)

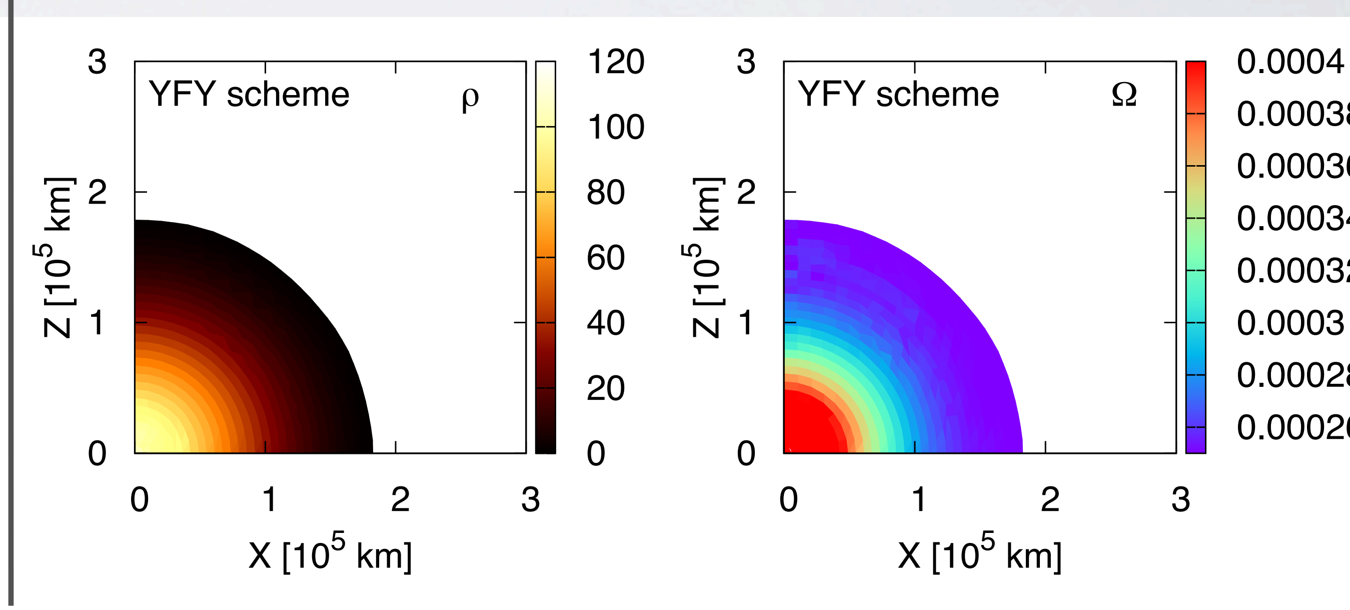
density **angular velocity**



density **angular velocity**



density **angular velocity**



【KEY POINT】

The distribution of angular momentum depends on the evolution.
→ We can check it !!

ON GOING PROJECTS

weak solution(this talk)

$$E_{\mathrm{FEM}}(\boldsymbol{r}_i) = \sum_i \varepsilon_i m_i + \frac{1}{2}\sum_i \phi_i m_i + \sum_i \frac{1}{2}\left(\frac{j_i}{\varpi_i}\right)^2 m_i$$

$$\delta E = 0$$

weak solution

$$\int w \cdot \left(\nabla P + \rho \nabla \phi - \rho \frac{j^2}{r^3} e_r\right) = 0$$

2次内積

exact solution

$$\nabla P + \rho \nabla \phi - \frac{\rho j^2}{\varpi^3} \mathbf{e}_\varpi = 0,$$

SUMMARY

We construct a new formulation to calculate structures of deformed stars with rotation.

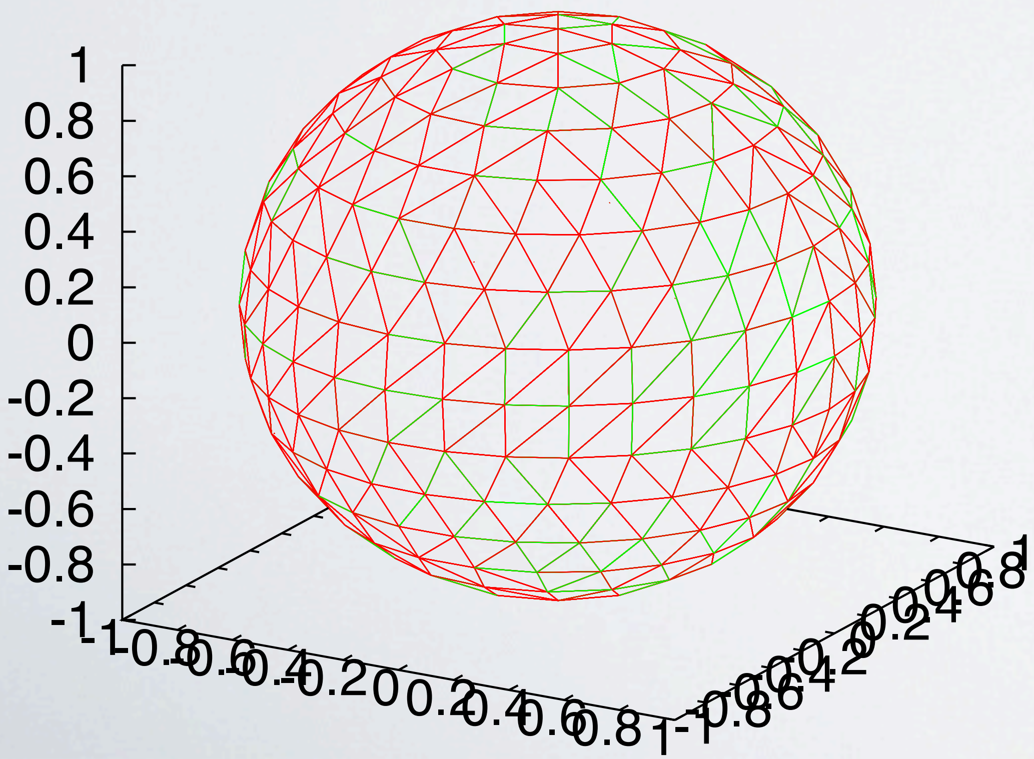
This technique has the similarity with calculations of pasta structures.

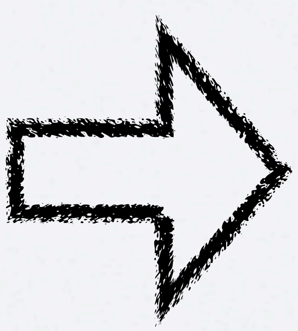
Our method is applicable for all types of deformed stars.

Future works

- Next step is the time evolution of the deformed stars.
- Triangulations in 3D is easy, and we have already prepared.

→But, it needs a new and special formulation for 3D.





inside