

星の内部磁場

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第2回DTAシンポジウム「コンパクト天体の活動性と磁氣的性質」
国立天文台(2014年10月27-29日)

研究会での内容の抜粋

多くは口頭で話し、進行中の内容はここでは省略したので、必要があれば、連絡ください。

Contents

Topics + Preliminary Results

- Comment on Equilibrium and Stability
- Long term Evolution
 - Hall drift
- Oscillations in Giant Flares
 - Developing codes to examine magnetic coupling

Equilibrium and Stability of Magnetized Star

磁場の形状は弱いから何でもよいか？ No

$$\vec{j} \times \vec{B} - \nabla p - \rho \nabla \Phi = 0 \quad \vec{B} = \frac{1}{R} \vec{\nabla} G \times \vec{e}_\phi + \frac{S}{R} \vec{e}_\phi,$$

軸対称性を仮定

$$4\pi \vec{j} = \frac{1}{R} \vec{\nabla} S \times \vec{e}_\phi - \frac{D(G)}{R} \vec{e}_\phi$$

GS 方程式

$$S = S(G), \quad (\vec{\nabla} G \times \vec{\nabla} S)_\phi = 0$$



$$D(G) = -4\pi(SS' + \rho R^2 f'(G))$$

Unknown functions S, f

Precession of Magnetar 4U0142+61

Makishima + 2014

Magnetic deformation $\varepsilon \approx 10^{-4}$

Dipole B from p-pdot $B \propto (p\dot{p})^{1/2}$

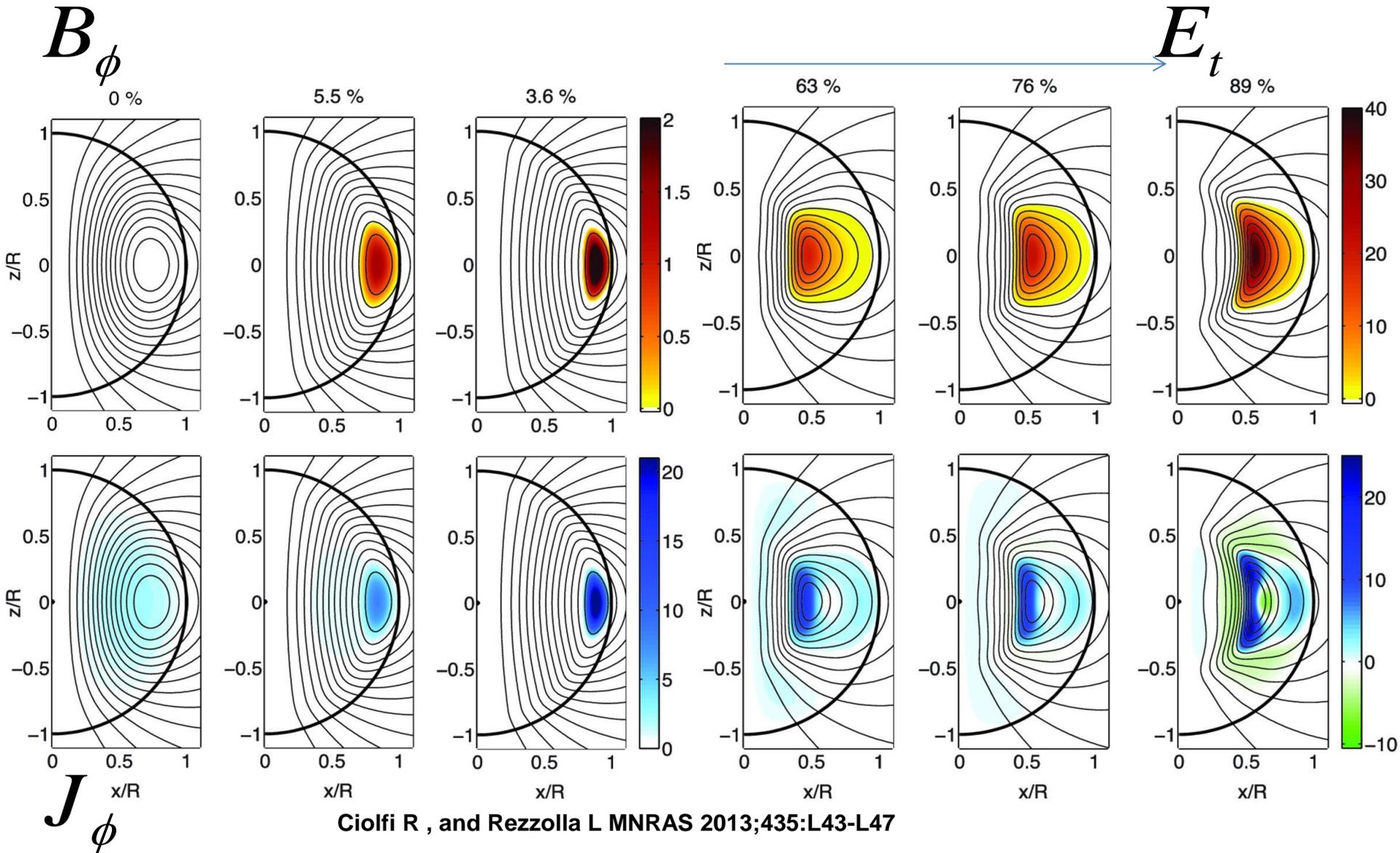
Toroidal B from defromation

$$\varepsilon \approx E_{mag} / E_{grav} \approx 5 \times 10^{-8} (B / 10^{14} G)^2$$

→ $B_d \approx 10^{14} G, B_\phi \approx 10^{16} G (P = 8.7 s)$

$$E_{pol} \approx 10^{46}, E_{tor} \approx 10^{50} (< E_{grav} \approx 10^{53}) \text{ ergs}$$

Poloidal / toroidal dominated



Stability of Magnetized Star

Long history (Taylor(1973), Wright,+++)

Pure toroidal -> unstable / Pure poloidal -> unstable

Braithwaite & Spruit 2004 MHD simulation

Stable configuration in a range of B_T/B_P

Lander & Jones 2012 No stable model (perturbation)

Akgun + 2013

Importance of Non-barotropic effect

found by energy principle technique(Bernstein + 1958)

結果

NR MHD である種のモデルの解析の結果

安定な範囲 (おおざっぱな評価式)

$$E_{pol}/4 < E_{tor} < [(\Gamma/\gamma - 1)E_{grav}E_{pol}]^{1/2}/2$$

余分な浮力

エントロピー勾配

Barotropic case -> 安定なもの無

主系列星 $\Gamma/\gamma - 1 \approx 1/4$

中性子星 $\Gamma/\gamma - 1 \approx 10^{-2}$

高精度が要求される

安定な範囲

観測

A puzzle in 4U0421

$$10^{46} < E_{tor} < 10^{48} \text{ ergs}$$

$$E_{tor} \approx 10^{50} (B_{tor}/10^{16} \text{ G})^2 \text{ ergs}$$

この部のまとめ

- トロイダルが支配的な平衡形状も計算されるようになってきた。

関数形をどう選ぶか？

- 安定性からトロイダル成分を限定

→しかし、非常に小さいnon-barotropic 効果が安定領域をひろげる。

高次の多重極磁場？

回転は？ 小さいけど??

$$E_g \approx 10^{53}, \underline{E_R} \approx 10^{45}, E_B \approx 10^{48}, E_T \approx 10^{44}, \dots$$

In typical Magnetars

中性子星の磁場は変化(減衰)するか？

現状の観測的な観点からは未確定

異なる種類(種族)は存在

パルサー(1968-) : 10^{12} G

-> 統計的議論もあるが、？

ミリ秒パルサー(1982-) : $< 10^8$ G

連星系(質量降着)

マグネター(1995-) : $> 10^{13}$ G : 熱源(X線)

> 磁場の減衰？ 種族？

+ CCO, RRAT, INS,..... (2000-) : 10^{12} G ?

Braking index in PRS

Evolution of spin down

$$\dot{\Omega} = -K\Omega^n, (P\Omega = 2\pi)$$

$$\ddot{\Omega} / \dot{\Omega}^2 = n - \dot{K} / (K^2 \Omega^{n-1})$$

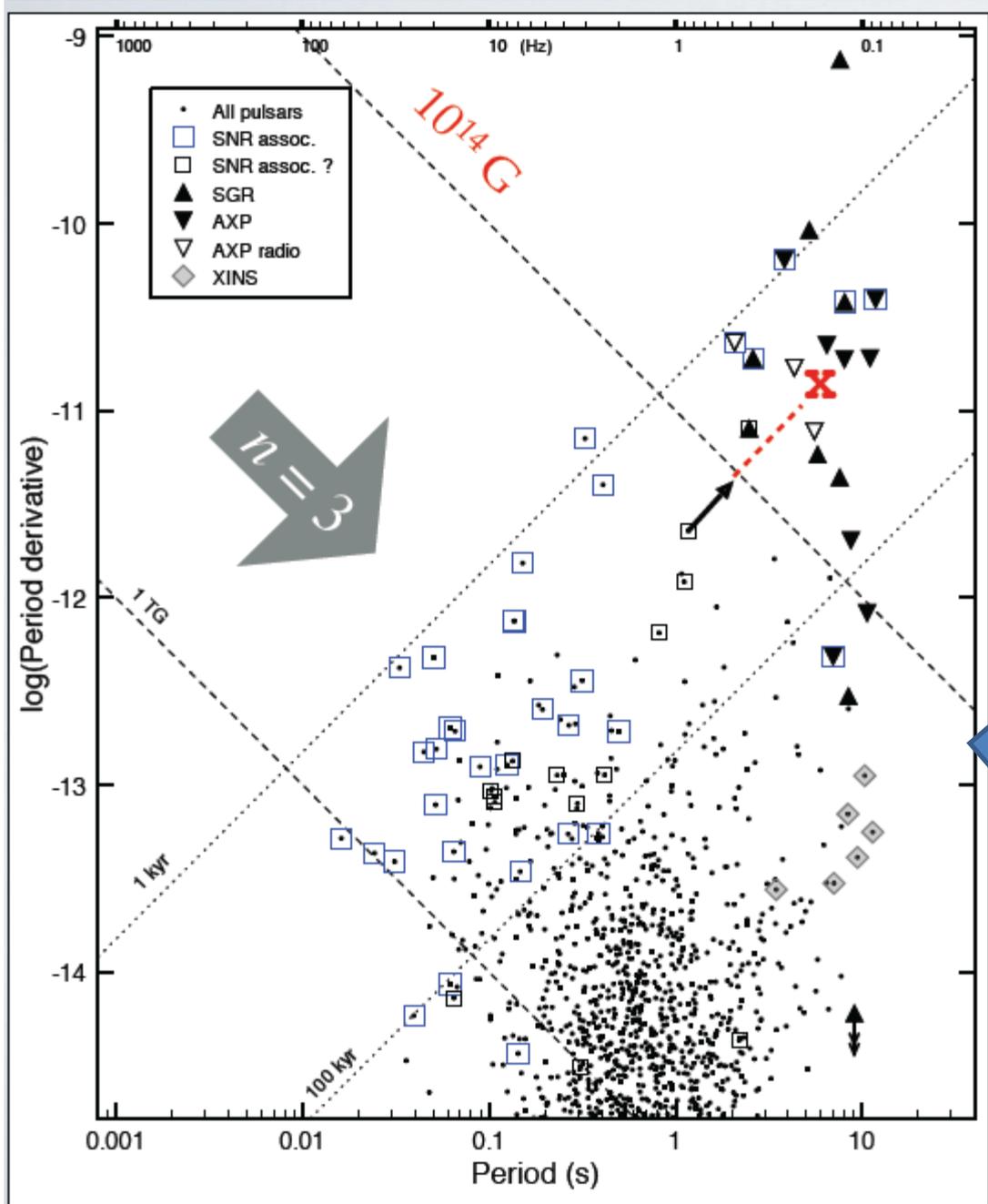
K depends on B_dip, Inertia moment,
Magnetosphere,...

n=3 for dipole radiation,

but, not in general.

Observation of n < 3 suggests increasing B, or ...

$$n = 3, \dot{K} / (K^2 \Omega^2) = 2 \quad \rightarrow \quad \text{Exponential growth}$$



PSRJ1734-3333

$$n = 0.9 \pm 0.2$$

Magnetar in

$$\tau = 2.9 \times 10^4 \text{ yrs}$$

Espinoza + 2011

磁場の永年変化

$$\partial_t \vec{B} = -\vec{\nabla} \times \left(\underbrace{(\vec{\nabla} \times \vec{B}) \times \vec{B}} \right) + \underbrace{\nabla^2 \vec{B} / R_m}$$



誘導方程式＋一般化されたオームの法則

$$\partial_t \vec{B} = -\vec{\nabla} \times \vec{E}, \quad \vec{J} = \vec{\nabla} \times \vec{B},$$

$$\vec{E} = -\vec{v} \times \vec{B} + \underbrace{\vec{J} \times \vec{B} / ne} + \underbrace{\vec{J} / \sigma}$$

$$\approx (\propto 1 \quad \propto c^2 \omega_{pi}^{-2} / l^2) \times B^2 / \rho^{1/2}$$

☆プラズマ物理：小スケールではホール項が重要

☆中性子星の殻(金属)では電子のみが動く

Order estimate

二つの変化する時間スケール

- Ohmic decay $\tau_D = 4\pi R_S^2 \sigma / c^2 \approx 10^9 (\sigma / 10^{25}) yrs$
- Hall drift time $\tau_H = 4\pi en R_S^2 / c B_0 \approx 10^7 (B_0 / 10^{13})^{-1} yrs$

場所とサイズによるが R_S or $\Delta R \approx 0.1 R_S$

+ α の効果 (質量降着は無関係 single star)

- 冷却の効果 高温で抵抗大

$$\sigma = \sigma(T) \quad n_e = n_e(T) \quad T = T(t)$$

Evolution of braking index by Hall effect

- Virgono + 2012....
- Gourgouliatos & Cumming 2014

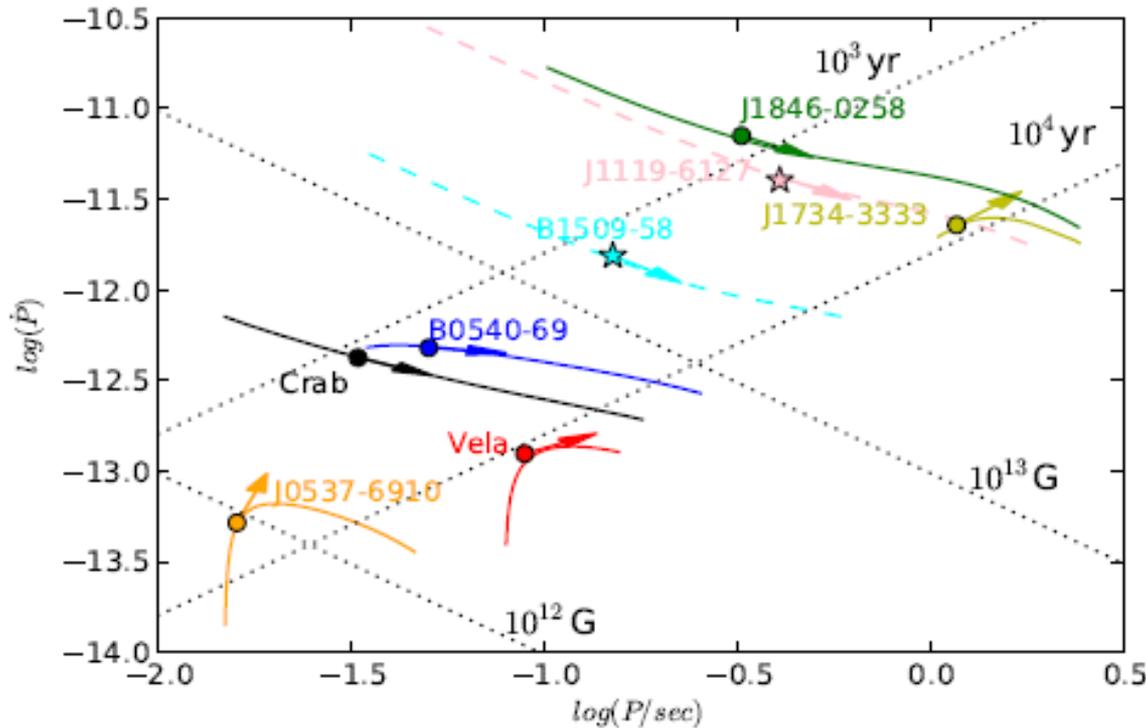
? Initial configuration

B_ϕ Toroidal dominated (10^{14} - 10^{15} G)

=> increase of dipole B

注: Hall項は弱い磁場(通常のPSR)では効果 小

Young PRSs



矢印：観測
線：進化モデル

?

Figure 7. P - \dot{P} diagram for the eight pulsars for which the braking index is measured, taking into account the magnetic field evolution. The points correspond to the observed P and \dot{P} . The direction of the arrow is related to the braking index (Espinoza et al. 2011) $\tan \omega = 2 - n$, where ω is the angle with horizontal axis. Examples of evolutionary tracks are plotted, which evolve to the right values of magnetic field and braking index at the pulsar's current age, assuming an initial magnetic field in the ranges shown in Figure 6. P_0 is solved for, given the evolution of the magnetic field and the pulsars position in the P - \dot{P} diagram. The tracks of B1509-58 and J1119-6127 are shown as dashed lines, as their estimated ages are much longer than the ones that can be accommodated by a steady spin-down over the pulsar's lifetime.

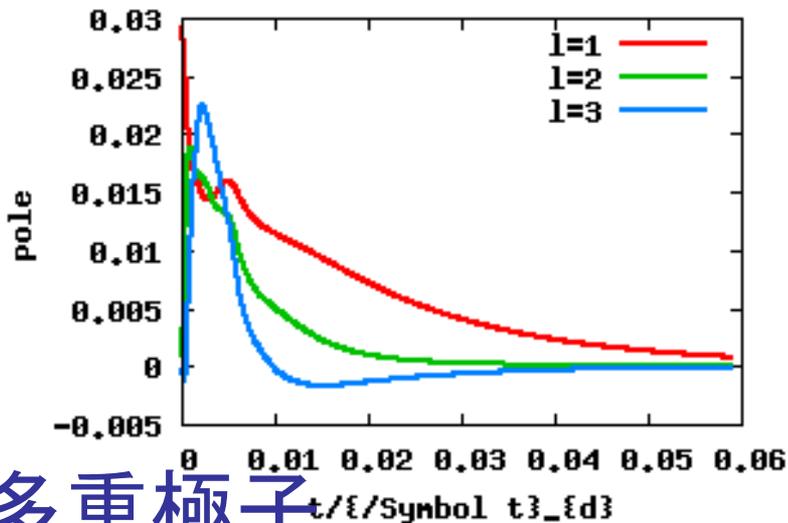
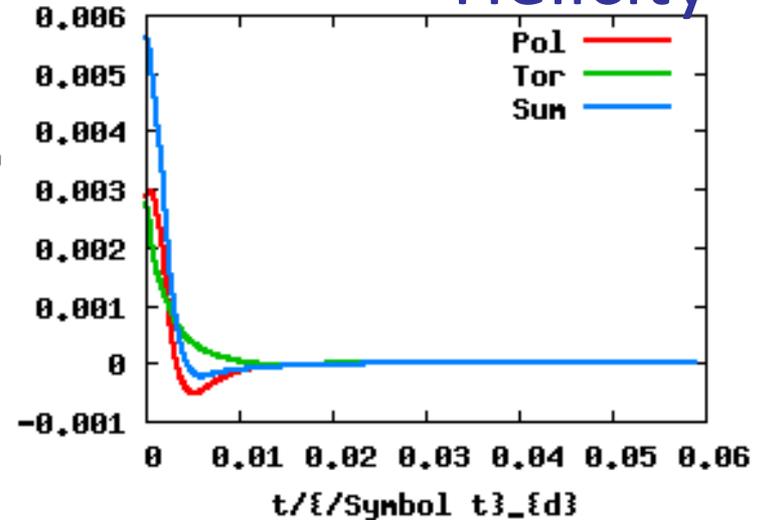
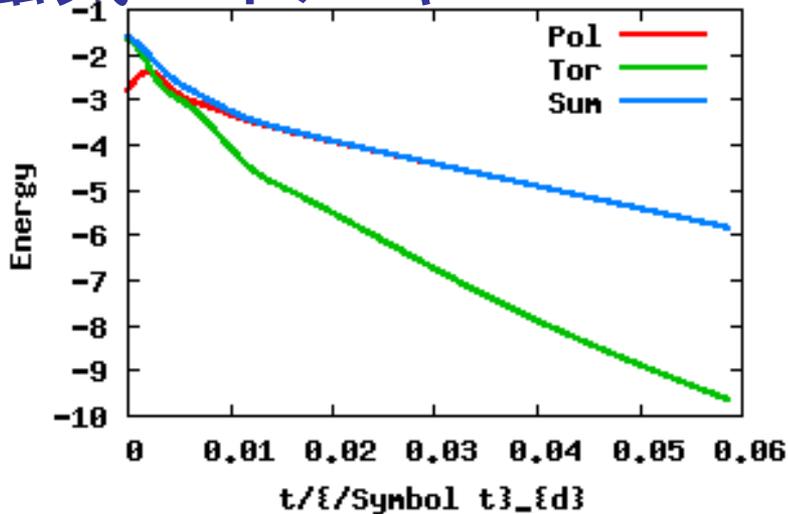
Gourgouliatos+
2014

Poloidal – Toroidal 3

磁気エネルギー

Seprntd1

Helicity



多重極子

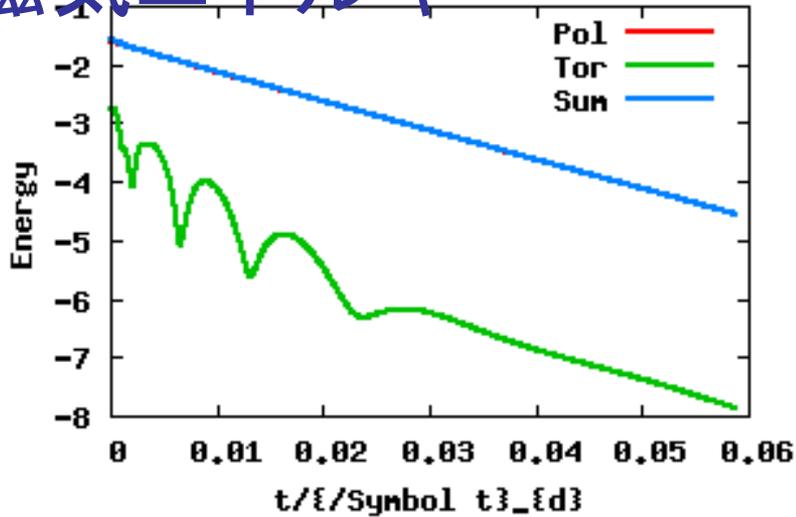
Toroidal 支配的
強い磁場 (100)
Pol(l=2,3) が大

Kojima+Kisaka 2012

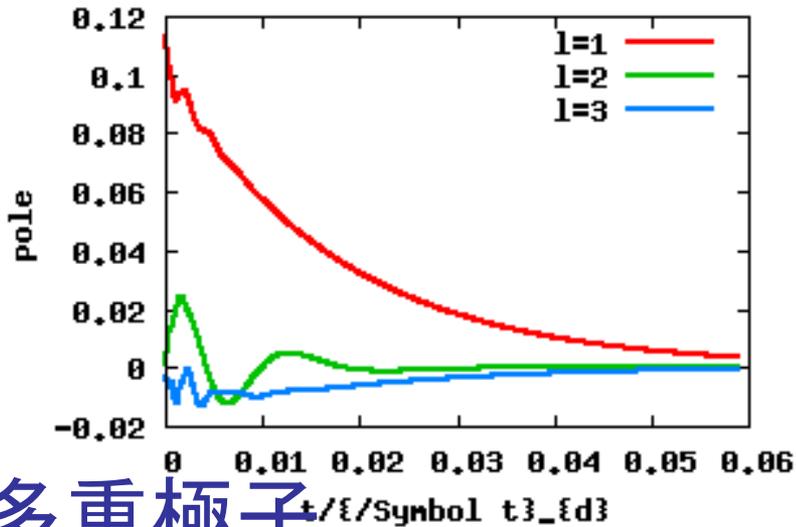
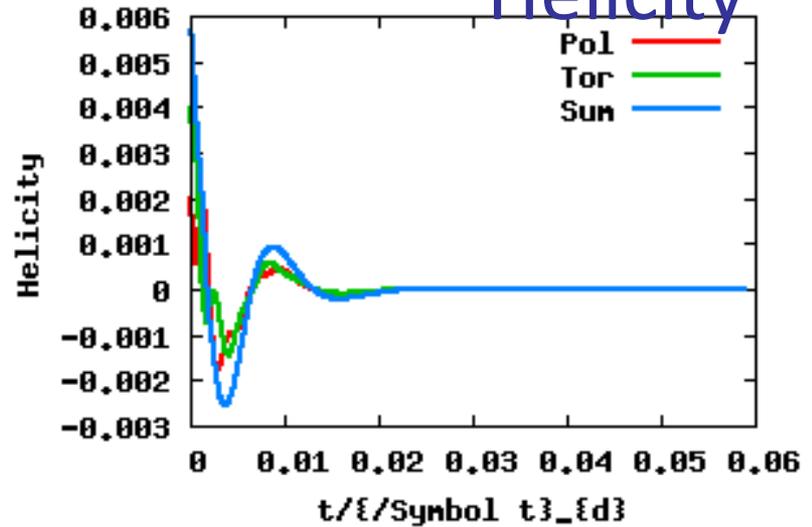
Poloidal – Toroidal 4

磁気エネルギー

Seppd1



Helicity



多重極子

Poloidal 支配的
強い磁場 (100)
Toroidal 変動

Kojima+Kisaka 2012

Critique ? Prejudice?

個人的主観

✓ Toroidal が主でも $l=1$ を増幅するのは難？

→ 計算が悪い？

→ 特殊な初期配置？ Helicity $\int GS / R^2 dV$

✓ 指数関数より緩やかな結果

10^4 年で1.5倍 (数値計算例)

PRS → Magnetar にならない

(その必要もないが)

移流であり、減少する場合も観測にもあるのでは？

+a (=ダイナモ)

一つの試み

+ differential rotation + Hall drift

Kinematical dynamo

$$\partial_t G = \frac{1}{\tau_D} D(G) + \frac{1}{\tau_H R} (\vec{\nabla} G \times \vec{\nabla} S) \cdot \vec{e}_\phi$$

$$\partial_t S = \frac{1}{\tau_D} D(S) + \frac{R}{\tau_H} \vec{\nabla} \times \left(\frac{1}{R^2} (D(G) \vec{\nabla} G + S \vec{\nabla} S) \right) \cdot \vec{e}_\phi$$

$$+ R (\vec{\nabla} \Omega \times \vec{\nabla} G) \cdot \vec{e}_\phi \approx -\varepsilon \Omega_0 \vec{\nabla}_z G$$

$$\vec{\nabla} \Omega = \vec{\nabla} (\Omega_0 / (1 + \varepsilon (R/R_s)^2)^{1/2}) \approx -\varepsilon \Omega_0 R / R_s^2$$

-参考- 比較 : Axi-sym. Kinetic Dynamo

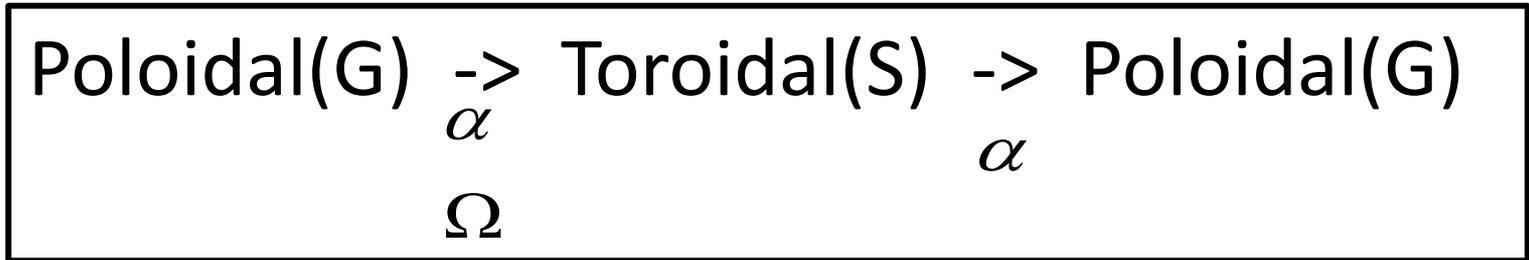
Mean fields + Turbulences

$$\vec{B} = \vec{B}_0 + \underline{\vec{b}}, \vec{v} = (\vec{V}_p + R\Omega\vec{e}_\phi) + \underline{\vec{u}}$$

Averaging + turbulence model (b,u from B,V)

$$\{\partial_t + \vec{V}_p \cdot \vec{\nabla}\}G = \eta D(G) + \underline{\alpha S}$$

$$\begin{aligned} \{\partial_t + \vec{V}_p \cdot \vec{\nabla}\}S + \overset{\dots\dots}{SR^2\vec{\nabla} \cdot (\vec{V}_p R^{-2})} &= \eta D(S) + \overset{\dots\dots}{\vec{\nabla}\eta \cdot \vec{\nabla}S} \\ &+ \underline{R(\vec{\nabla}\Omega \times \vec{\nabla}G)} - \underline{\alpha D(G)} - \underline{\vec{\nabla}\alpha \cdot \vec{\nabla}G} \end{aligned}$$



Hall Attractor

Gourgouliatos+ PRL (2014)

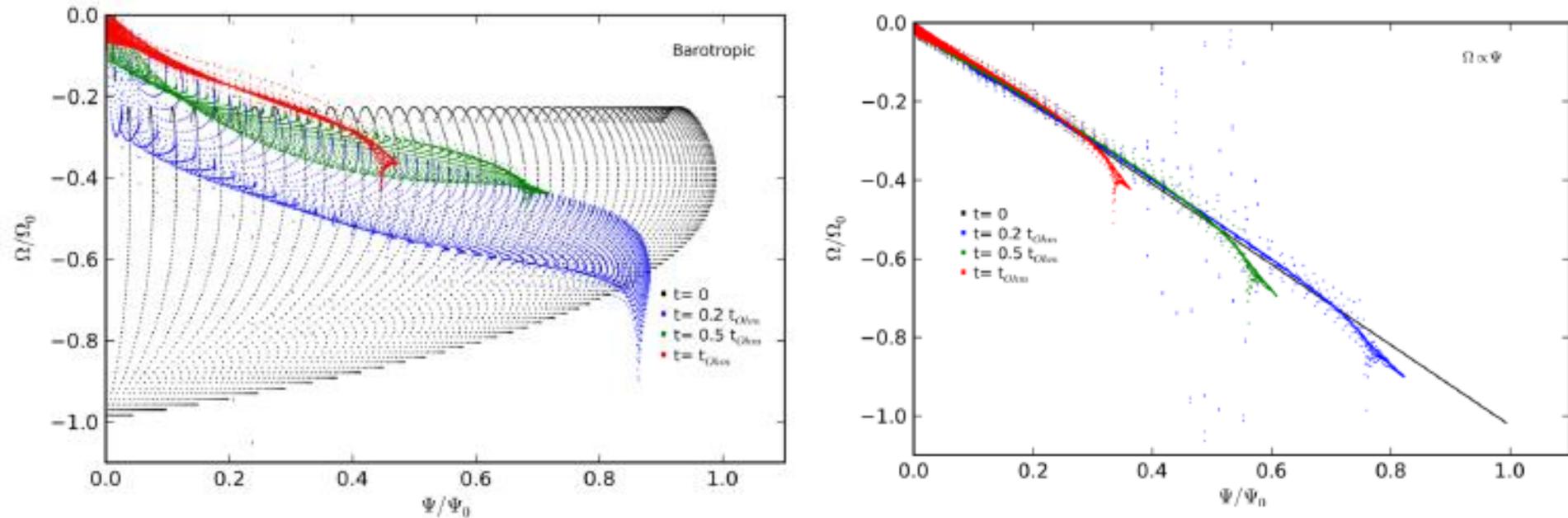


FIG. 2 (color online). Scatter plot of Ω and Ψ , each point corresponding to a grid point of the simulation. Top panel: The scatter plot for the case shown in Fig. 1, top panel. Initially, there is differential rotation of the electron fluid along the field lines (black points), with multiple values of Ω for given Ψ , which after some Hall evolution tend to concentrate in a narrower region (blue and green points), and eventually the system saturates to isorotation (red points). Bottom panel: The scatter plot for a system starting with $\Omega = \alpha\Psi$ (black points). The initial structure is very close to the attractor state; thus, the system changes its structure only slightly, as shown with Ω and Ψ deviating from linearity (blue and green points, extending to smaller Ψ 's). However, the system maintains isorotation, even after a significant part of the field has been dissipated (red points) (see Ref. [20]).



$$(-j_\phi / R =) D(G) / R^2 = \alpha G$$

この部のまとめ

- NS (Magnetar) の磁場は時間変化？
観測的には？
PRS → Magnetar に進化するか？

現状の理論モデルでは初期の配置などモデルの不定性が大きいのか？

- 強磁場ではHall 効果は重要であるが。
非線形項であり、小から大スケール、又は逆の方向になるか。
→ 理論的課題 有

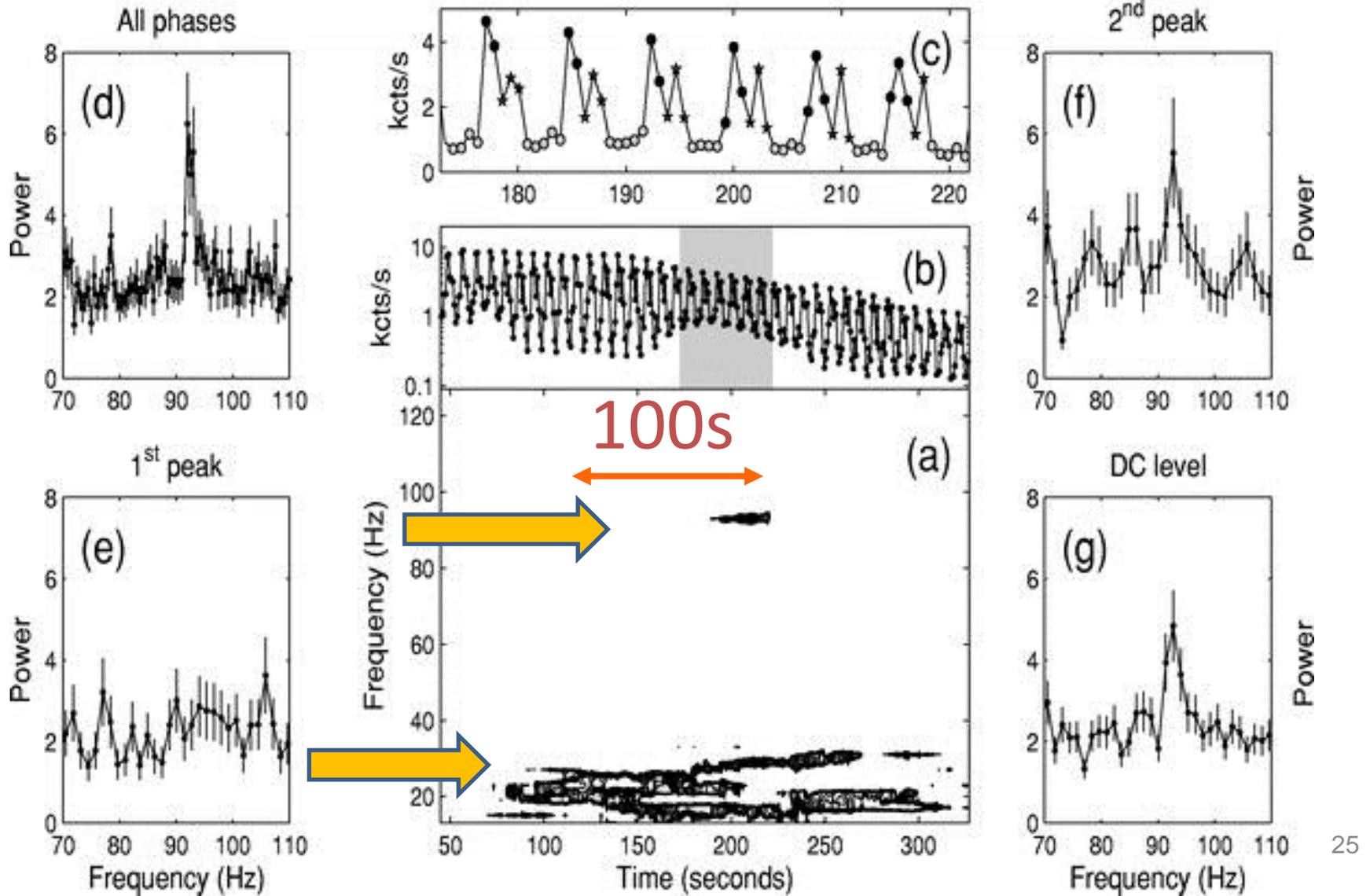
Magnetar Oscillation

Observed in Giant Flares (QPOs)

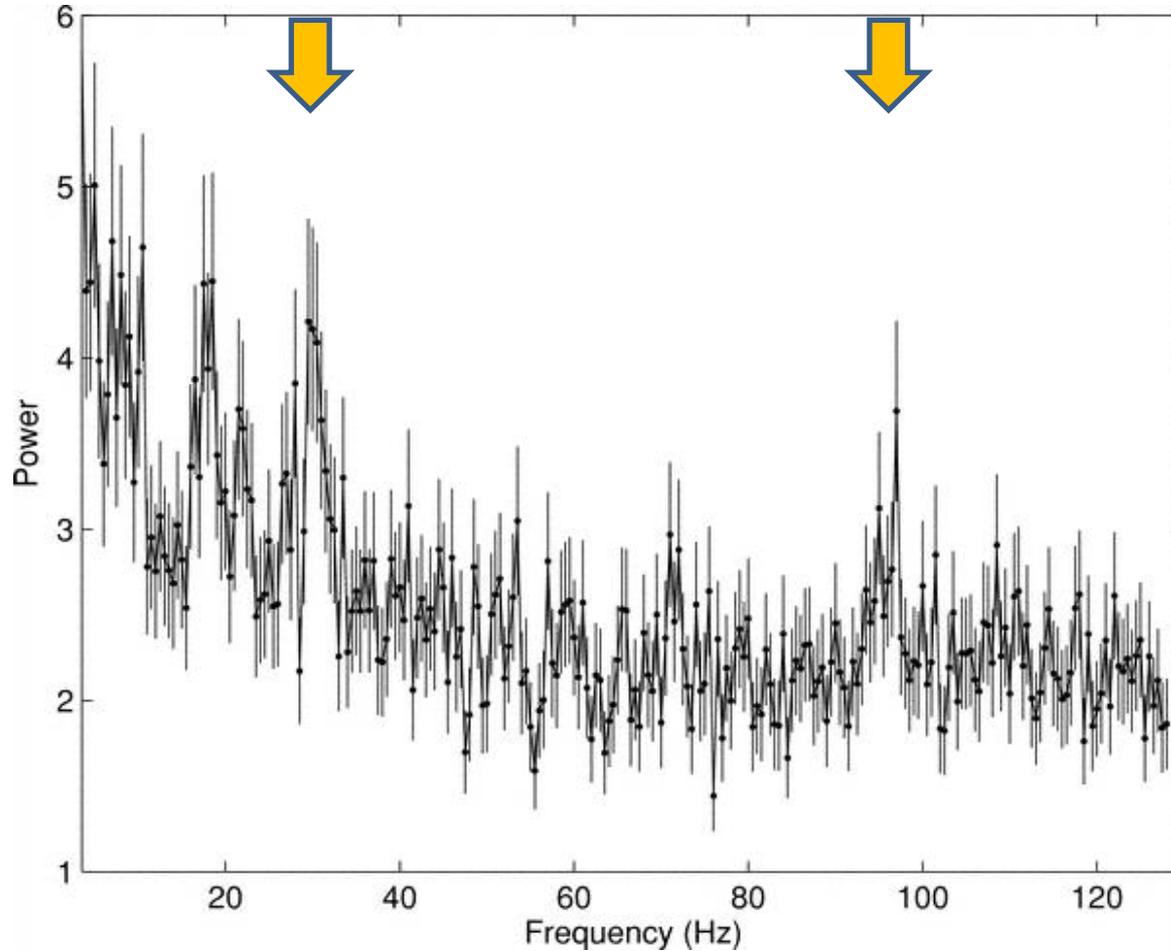
- SGR1806-20(2004)
30Hz(2,0) 92Hz(6,0) 150Hz(10,0),...
are observed in hard X-rays (<200keV)
- SGR1900+14 (1998)
28Hz(2,0) 53Hz(3,0) 84Hz(6,0) ,...
- SGR0526-66 (1979)
... ..
-> Seismology /Gravitational astrophysics

✓ Upper limit of GW for SGR1806 (LIGO PR07)

SGR1806-20 Israel+ 2005



SGR1806-20 Israel+ 2005



Peaks at 20, 30, 95Hz during 200-300s

QPOs in Magnetar Giant Flare

○ SGR1806-20 (Galactic 6-15kpc) Event in 2004
P=7.5s, B=2.1x10¹⁵G

E(EM)~10⁴⁶ ergs

[>>Brightening AXP E~10⁴¹ erg]

$$E_g \approx 10^{53}, E_R \approx 10^{45}, \underline{E_B \approx 10^{48}}, E_T \approx 10^{43}, \dots$$

- ✓ Magnetic Energy Source ~ 10¹⁵ G
- ✓ QPO observed in hard X-rays (<200keV)
30Hz(2,0) 92Hz(6,0) 150Hz(10,0),...

Torsional Shear Oscillation in Crust (l,n)

半径 → EOS

既に多くの理論研究

残る疑問：

- QPOの幅
→ 短寿命の重ね合わせ？
+ 磁場
- 減衰（速すぎる？） $\text{obs}(\text{min}) \ll \text{theo}(\text{yr})$
100 秒程度しか観測できていない

本日の焦点：磁気的な結合
(他の可能性：磁気圏あるが)

Magnetic coupling

General consideration

Poloidal magnetic field only

Axial shear **oscillation** is decoupled

No density perturbation

+Toroidal magnetic field $B_\phi \neq 0$

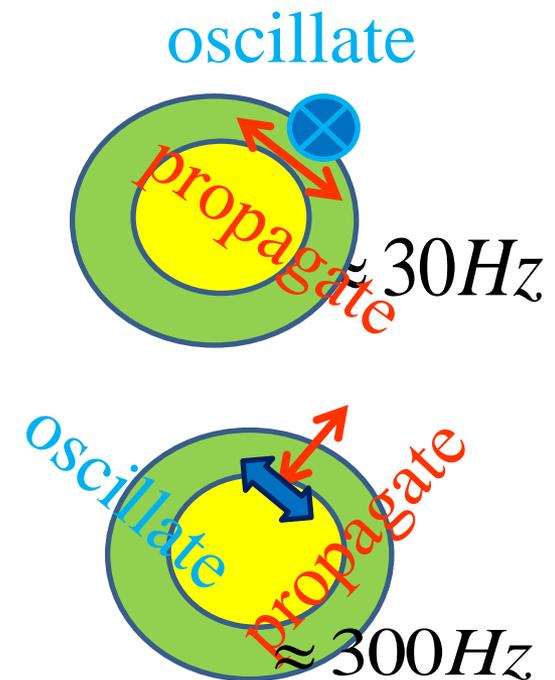
-> Coupling to polar density pert.

-> Globally enhanced density pert.

p(/f)-mode osc. is excited.

-> Damping through GW emission

Damping is determined by the coupling strength



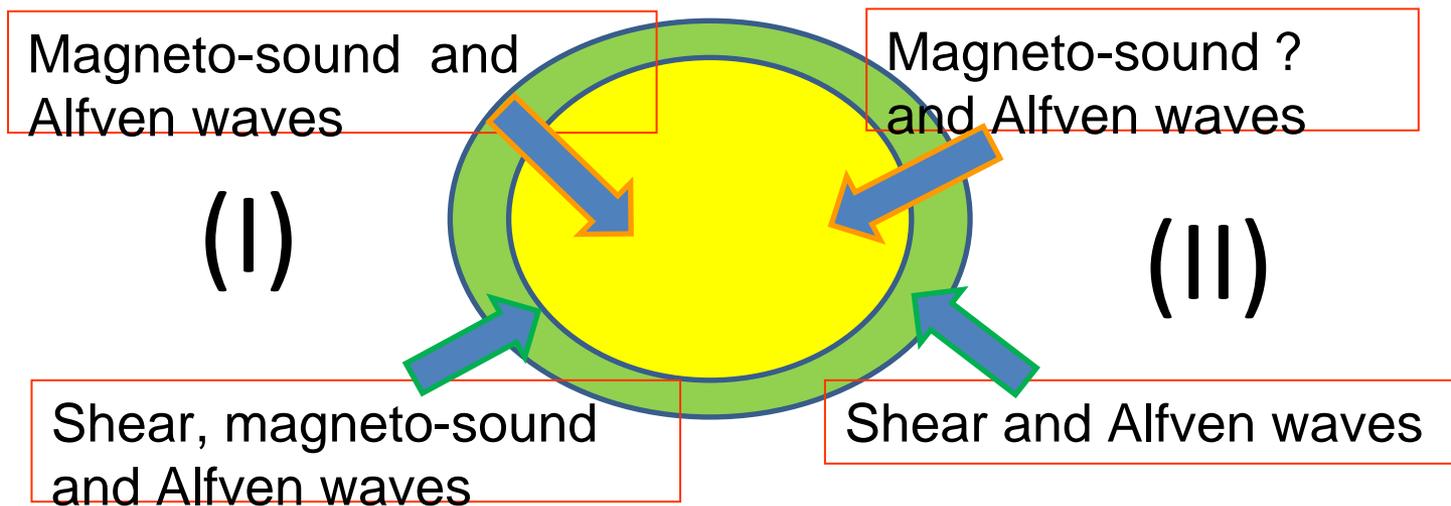
Specific initial cond.

‘Rotational’ perturbation only in crust is given
Kinematically divergence-free cond.

$$(I) \quad \vec{\xi}_p = \vec{\nabla} \times (f \vec{e}_\phi), \quad \delta\rho = -\vec{\nabla} \cdot (\rho \vec{\xi}_p) \neq 0$$

Or (II) $\rho \vec{\xi}_p = \vec{\nabla} \times (f \vec{e}_\phi), \quad \delta\rho = 0$

Testing the propagation w/w.o. magnetic field



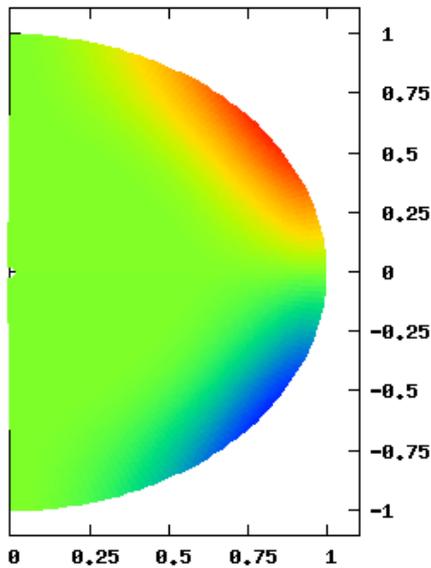
ξ_ϕ

トロイダル方向の振動

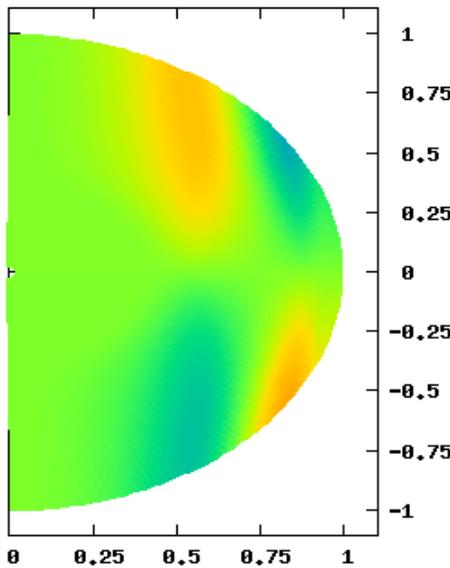
→ 時間進化

ゆっくり変動する部分が顕著

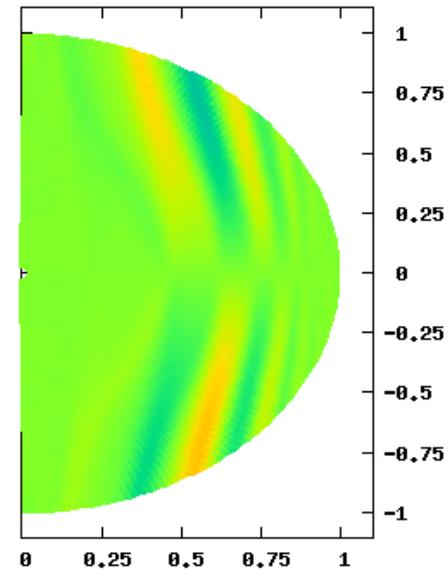
Preliminary



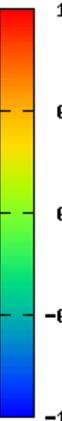
→ T



4T



16T

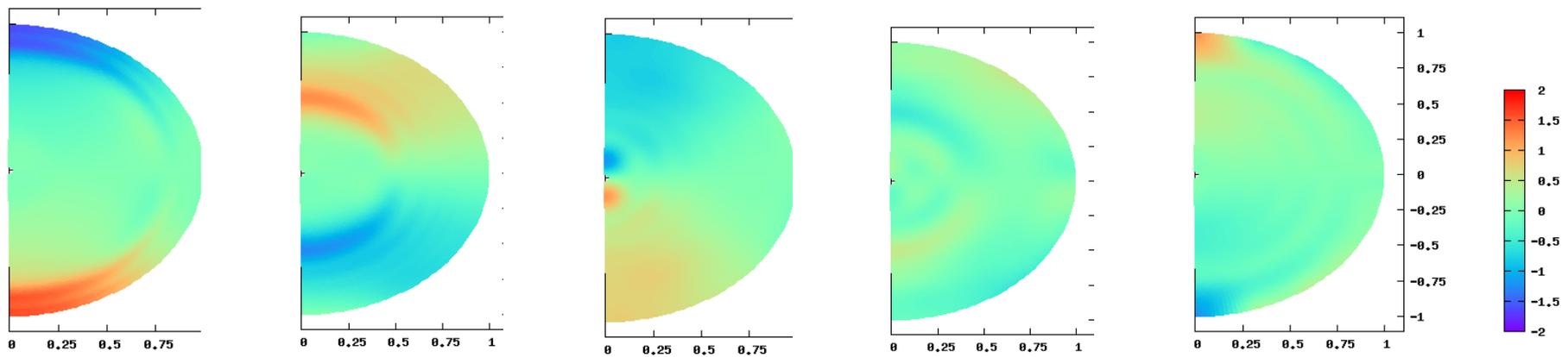


ξ_r, ξ_θ ポロイダル方向の振動

→ 密度の時間進化

殻部分のみに変動を加えた

Preliminary



→ T

2T

3T

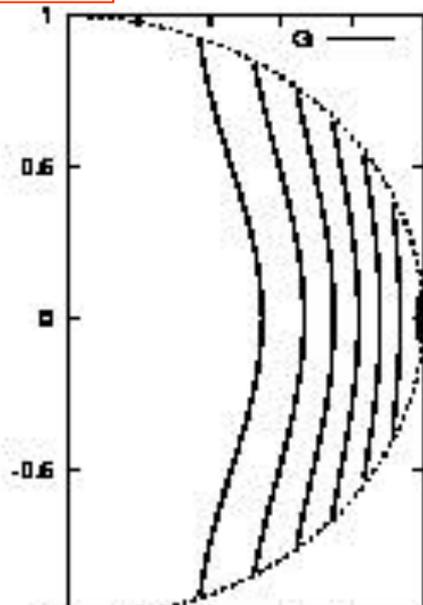
4T

5T

$B_\phi = 0$ 磁力線と速度分布

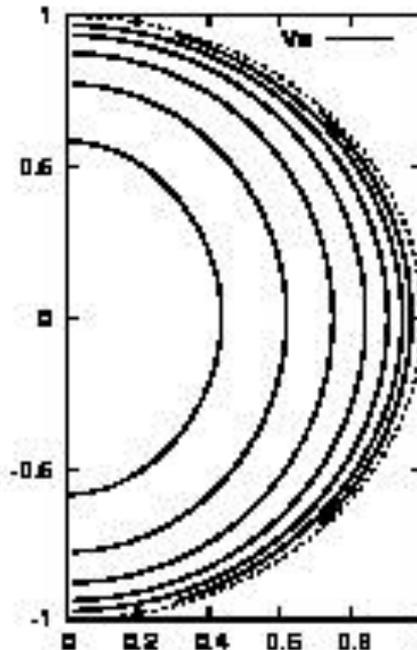
MHD平衡解

$$V_a / C_s = B_p / (C_s \sqrt{4\pi\rho})$$



磁気関数G

表面で双極子・中心で磁場がゼロとなる解



中心の音速との比

表面で速い 0.3程度

この部のまとめ(課題)

現在進行中で以下の課題

? 境界条件と波の反射

表面のAlfven 波 (速い)

$$V_a = B / \sqrt{4\pi\rho} \uparrow \infty$$

? より現実的な速度比

現状10の比 → 100程度(場所にもよる)

→ 長時間の計算が必要