Hydrodynamic Instability in the Similarity Solution of Viscous Accretion Disks

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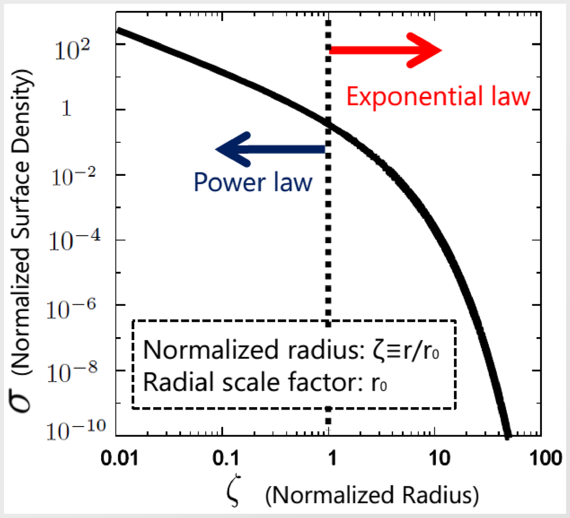
Abstract

Protoplanetary disks are the birthplaces of planets. Studying structures and evolutions of protoplanetary disks is important for planetary science. It is generally thought that the magneto-hydrodynamic instability is a dominant mechanism in disk evolution. However, when we consider the region with steep pressure gradient such as a density bump or gap in disks, or an edge of disks, the hydrodynamic instability can be important. Protoplanetary disks evolve via turbulent viscosity, and their evolution is well represented by the model of Lynden-Bell & Pringle (1974). In this model, the density profile of disks is analytically derived as the similarity solution. The similarity solution has an exponential cutoff in surface density at the outermost region, and pressure gradient becomes steep in this region. Therefore, hydrodynamic instability may occur. We first examine the stability of the similarity solution for Rotational instability. Rotational instability is local hydrodynamic axisymmetric instability. We find that the similarity solution is unstable against rotational instability (Ono et al., 2014). We also aim to investigate the Rossby wave instability of the similarity solution. Rossby wave instability is global hydrodynamic non-axisymmetric instability. As a first step, we perform linear perturbation analysis using the method similar to that in Li et al. (2000), and find that the disk is unstable against Rossby wave instability when its surface density profile has a cut-off in the outer region.

Introduction

The Similarity Solution

- The similarity solution, which is derived analytically by Lynden-Bell & Pringle (1974; LBP74), well represents evolution of surface density profile of ProtoPlanetary Disks (PPDs) via turbulent viscosity.
- The surface density decreases in power law in the inner region, and exponential law in the outer region.
- The similarity solution is utilized widely in many works.
- Some observations have suggested that gas surface density profiles of PPDs are well fitted by the similarity solution (e.g., Hughes et al.2008).



The similarity solution of LBP74

Hydrodynamic Instability

- In the region where the radial pressure gradient force is strong, such as the outer region of similarity solution, the disk is unstable for hydrodynamic instability.
- In this work, we investigate the stability of the similarity solution for hydrodynamic instability.
- We focus on two kinds of hydrodynamic instability as below.
  1. Rotational Instability (Ono et al., 2014)
  2. Rossby wave Instability

Normalization & Parameter

we define non-dimensional parameters as follows.

・radius: $r = r_0\zeta$ ・specific angular momentum: $J = J_0 j$

We assume a power-law temperature profile with respect to radius.

・temperature: $T = T_0\zeta^{-\beta}$ (β is power law index.)

(subscript "0" means the value at r0)

In this work, we consider the aspect ratio at r0 : H_0/r_0 =0.1, 0.2, 0.3.

A large value of H_0/r_0 means PPD is hot or mass of central star is low.

H_0/r_0 ~0.1-0.18: Typical T Tauri disks (e.g., Andrews et al. 2009)

H_0/r_0 ~0.3 : Disks irradiated by nearby massive stars in young clusters (e.g., Hillenbrand et al. 1998)

For simplicity, we consider isothermal disks (β=0) in this poster.

Rotational Instability

Rotational Instability is a local axisymmetric instability.

Discriminant for this instability

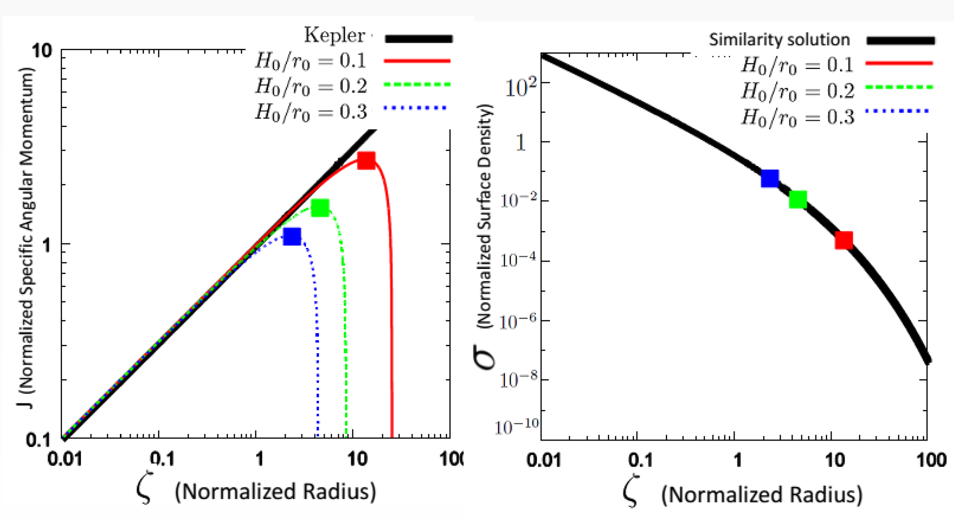
- The Rayleigh criterion $\frac{\partial j^2}{\partial \zeta} < 0$ is the discriminant for this instability.

Method

- We assume the radial equilibrium and calculate the rotation profile.
- We examine the stability of the similarity solution with Rayleigh criterion.

Result

- The outer region of the similarity Solution is unstable against Rotational instability.
- The marginally stable point approaches central star with increase of H_0/r_0.



Rossby Wave Instability

Rossby wave instability is a global non-axisymmetric instability.

This instability is proposed by Lovelace+99 and Li+00.

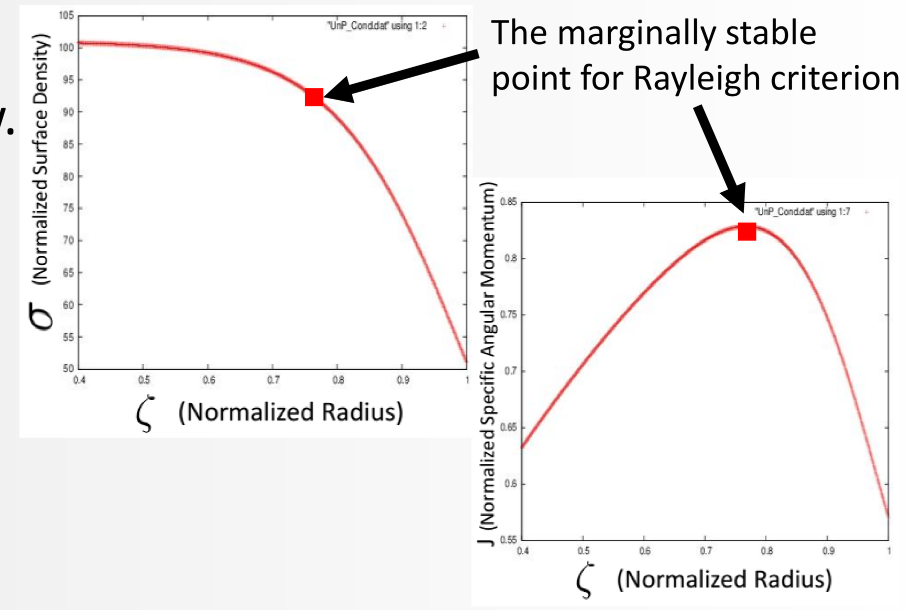
In this poster, we show the results of some test calculations.

Settings

We set the density profile as below.

$\Sigma/\Sigma_0 = 1 - 50[\tanh\{\zeta - 1)/0.2\} - 1]$

This surface density profile has the marginally stable point for Rayleigh criterion and a cut-off in the outer region, and resembles the similarity solution.



Method and Results

linear perturbation analysis using the method similar to that in Li+00.

Basic Equaiton

EOC: $\frac{D\Sigma}{Dt} + \Sigma\nabla\cdot\vec{v} = 0,$

EOM: $\frac{D\vec{v}}{Dt} = -\frac{1}{\Sigma}\nabla\tilde{P} - \nabla\Phi,$

EOE: $\frac{D}{Dt}\left(\frac{\tilde{P}}{\Sigma^{\Gamma}}\right) = 0,$

Linearize these equations.
We consider perturbations,

$$\propto f(\zeta)\exp(im\phi - i\omega t).$$

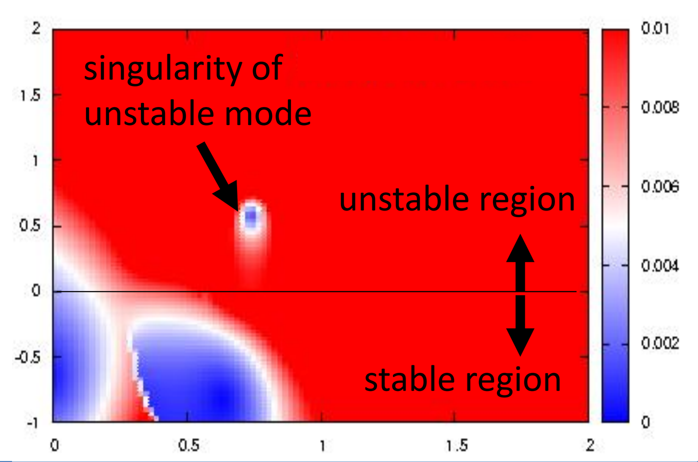
m is the azimuthal mode number.
ω is the mode frequency.

We can obtain the perturbation equation,

$$\Psi'' + B(\zeta,\omega)\Psi' + C(\zeta,\omega)\Psi = 0$$

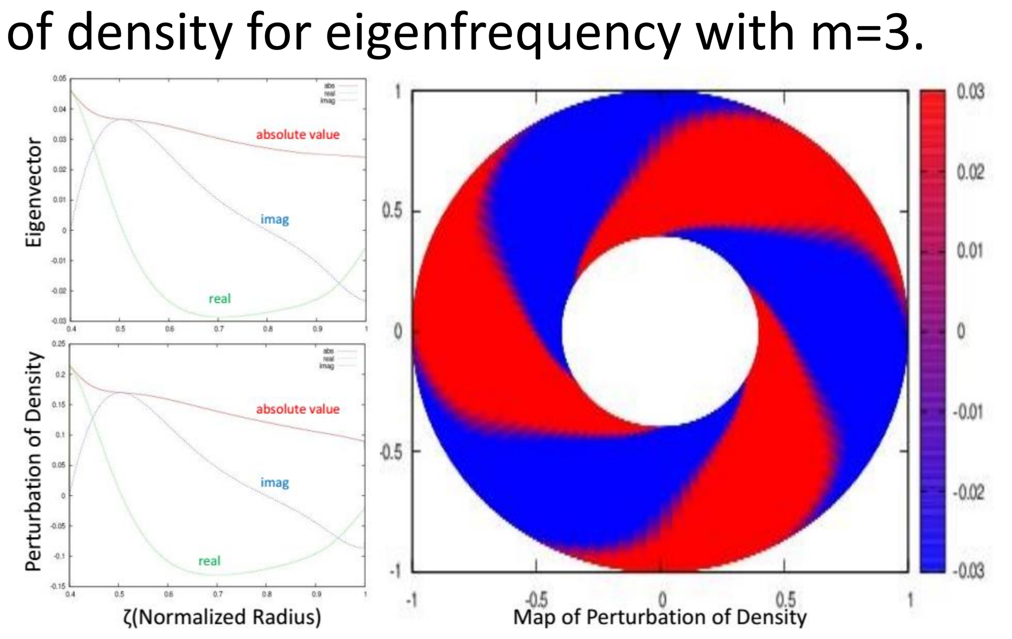
where $\Psi \equiv \delta P/\Sigma$ is the enthalpy.
B and C are the non-linear functions of ω.
The eigenfrequency ω is in general complex.

To solve this perturbation equation, we need to obtain the complex roots of the determinant of an tridiagonal matrix.
We calculate the map of determinant.
(color map: low "blue", high "red")



We look for the regions where roots exist in the map, and then use the Muller method to obtain exact values of the roots.

We calculate eigenvector and perturbation of density for eigenfrequency with m=3.



- This method well reproduces the results of Li+00.
- This profile has unstable modes against Rossby wave instability (m=3).
- We can infer that the disk with a cut-ff in the outer region is unstable against Rossby wave instability .

Future Work

- Upgrade the method in order to investigate Rossby wave instability of the similarity solution.
- Investigate which azimuthal mode has the fastest growth rate of Rossby wave instability.