The Revised Criterion for Self-Gravitational Fragmentation of Protoplanetary Disks

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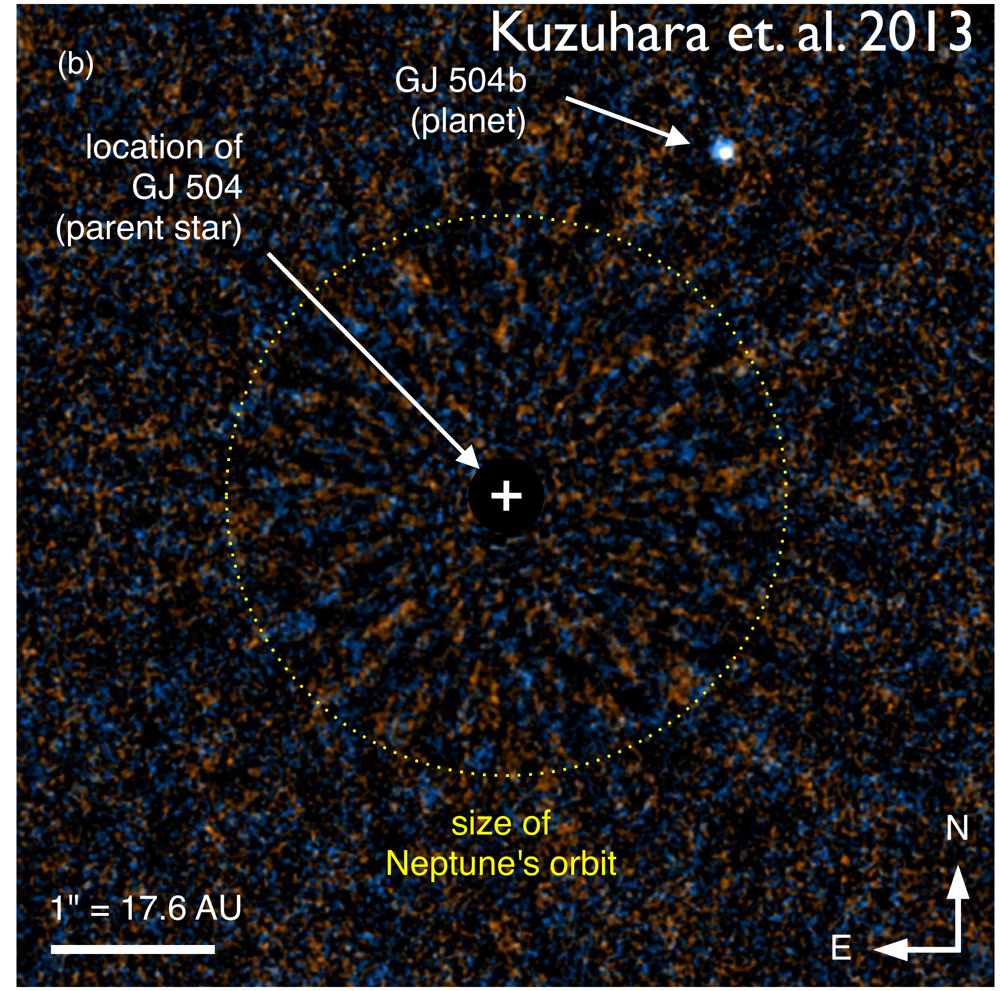
Gas giant planets

○Direct imaging of the exoplanet

Gas giant planets are observed far from central stars.

It is difficult to explain the formation of such gas giant planets by the **core accretion model.**



○Numerical simulations

Protopranetary disks are **massive** during its early formation

$$(M_{\rm disk} \gtrsim M_*)$$

Fragmentation of the disks due to gravitational instability is candidate for the formation process of gas giant planets.

(cf. Inutsuka et .al. 2010, Vorobyov and Basu2010)

When fragmentation occurs?

Toomre's Q parameter $Q \equiv \frac{\Omega_{\rm epi} c_{\rm s}}{\pi G \Sigma}$

- Local Stability Analysis (Toomre 1969)

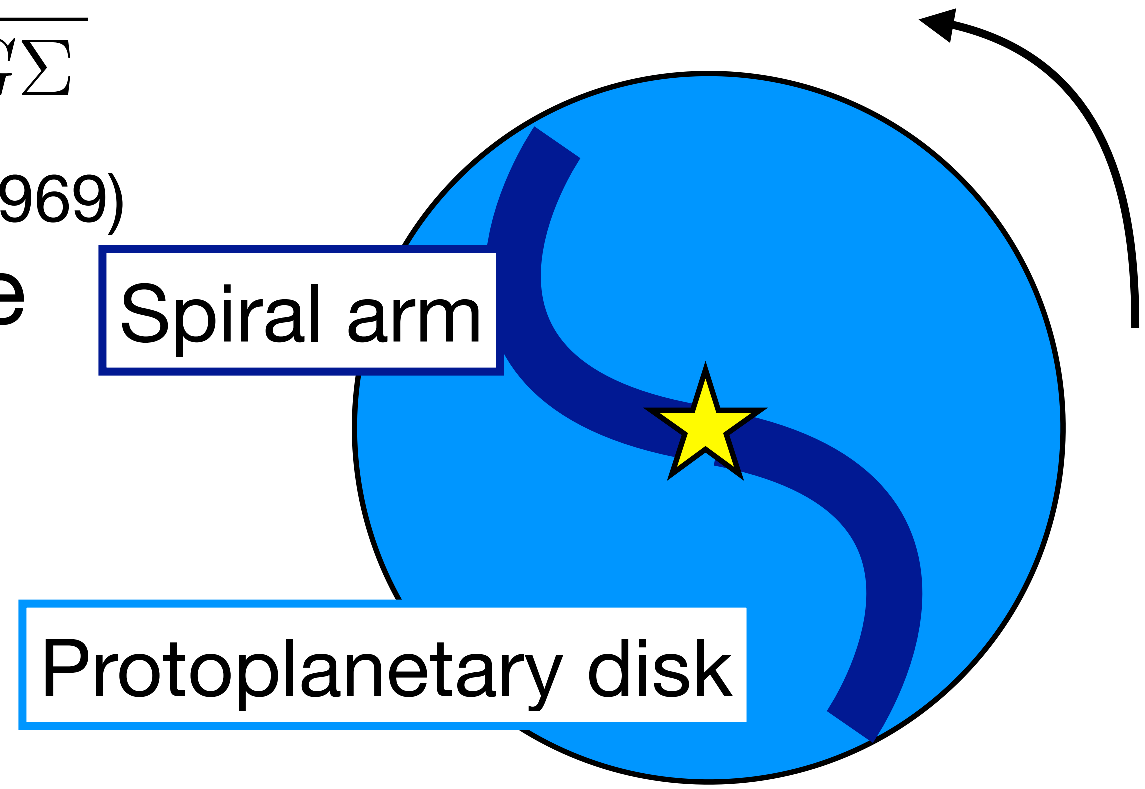
  $Q < 1 \Rightarrow$ gravitationally unstable

- Global Stability Analysis (Takahara 1976, 1978, Iye 1978)

  $1 \lesssim Q \lesssim 2$ $\Rightarrow$ **Spiral arms**

  - Angular momentum transfer due to the gravitational torque $\Rightarrow$ gas accretion

    Stabilization



We need the condition for the fragmentation due to the gravitational instability to overtake the stabilizing effect.

Previous works

Gammie's criterion (Gammie 2001, Rice et al. 2005, 2014, Meru and Bate2012)

Numerical simulation of the self-gravitating disks

Radiative cooling is modeled with parameter β

$$\left.\frac{dE}{dt}\right|_{\mathrm{rad}} = -\frac{E}{t_{\mathrm{cool}}} \qquad t_{\mathrm{cool}} = \beta\Omega^{-1}$$

The fragmentation occurs when
$\beta < \beta_{\mathrm{crit}} \sim 30$ (Meru and Bate 2012)

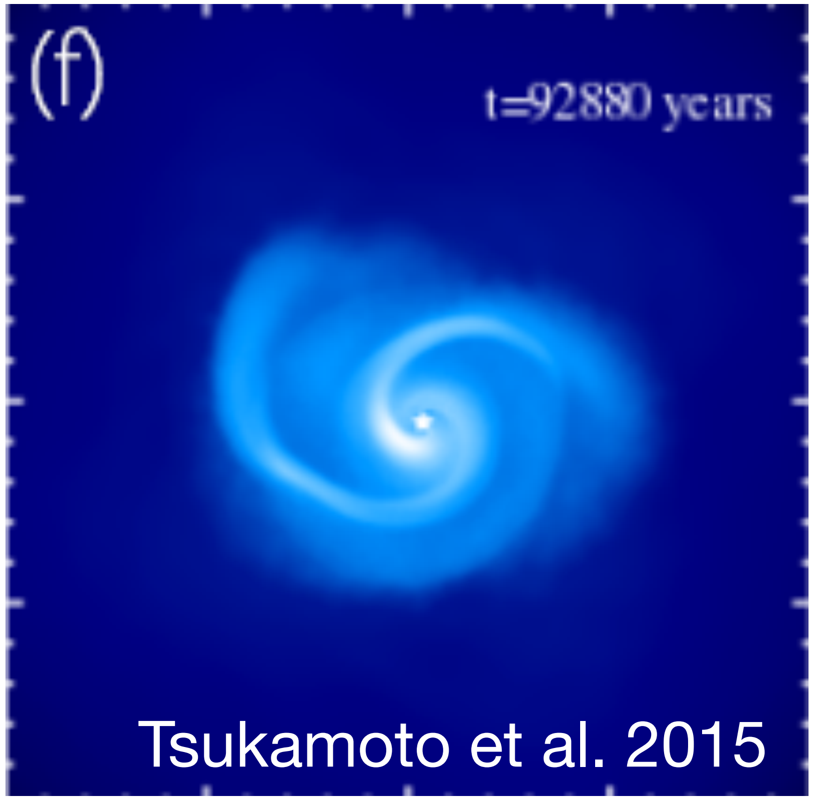
Conditions for the fragmentation given by previous works are

$Q \sim 1$ and **cooling is fast** enough.

Validity of Gammie's criterion

The results of some numerical simulations are inconsistent with Gammie's criterion.

- Tsukamoto et al. 2015 :
  $\beta < \beta_{\mathrm{crit}}$, Fragmentation does not occur.
- Machida et al. 2010 :
  adiabatic ($\beta = \infty$), Fragmentation occurs



The criterion of the cooling time is not always correct.

In this work, we perform the numerical simulations of the self-gravitating protoplanetary disks to investigate the **realistic** criterion for the fragmentation.

Setup (FARGO)

Basic equations

$$\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\Sigma v) = 0$$

$$\Sigma \left(\frac{\partial v}{\partial t} + v \cdot \nabla v \right) = -\nabla P - \Sigma \nabla \Phi$$

$$P = (\gamma - 1) E$$

$$\frac{\partial E}{\partial t} + \nabla \cdot (E v) = -P \nabla \cdot v$$

$$-\frac{8}{3}\sigma (T^4 - T_{\mathrm{ext}}^4) \frac{\tau}{\frac{1}{4}\tau^2 + \frac{1}{\sqrt{3}}\tau + \frac{2}{3}}$$

radiative cooling, external heating

(Hubeny 1990)

Optical depth

$$\tau = \kappa_{\mathrm{R}} \Sigma$$

$$\kappa_{\mathrm{R}} = \kappa_{10} \left(\frac{T}{10[\mathrm{K}]} \right)^2 [\mathrm{cm}^2 \mathrm{g}^{-1}]$$

κ_{10} =0.05 (Semenov et al. 2003)

Open boundary

r = 20, 1000AU

$$M_* = 0.5 M_\odot$$

$$M_{\mathrm{disk}} = 0.34\, M_\odot, 0.38\, M_\odot$$

$$T_{\mathrm{ext}} = 150[\mathrm{K}] \left(\frac{\mathrm{r}}{1[\mathrm{AU}]} \right)^{-3/7}$$

(Chiang and Goldreich 1997)

$$\gamma = 5/3$$

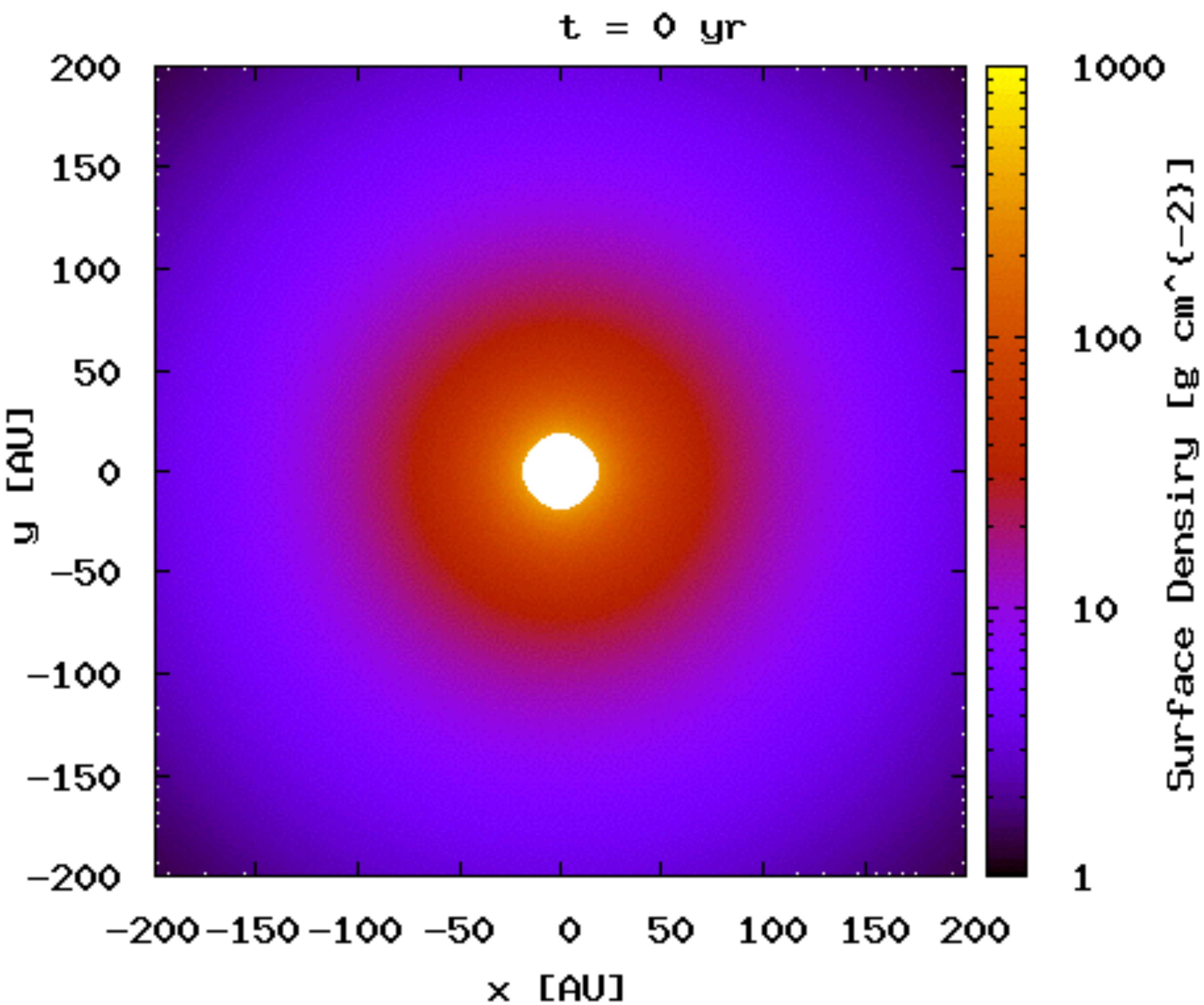
Initial conditions

$$\Sigma \propto r^{-12/7} \exp \left(-\frac{r}{r_{\mathrm{disk}}} \right)$$

$$r_{\mathrm{disk}} = 250[\mathrm{AU}]$$

$$Q \sim 2 \qquad (Q \equiv c_{\mathrm{s}} \Omega / (\pi G \Sigma))$$

Result: No fragmentation



$$M_{\rm disk} = 0.34\, M_{\odot}$$

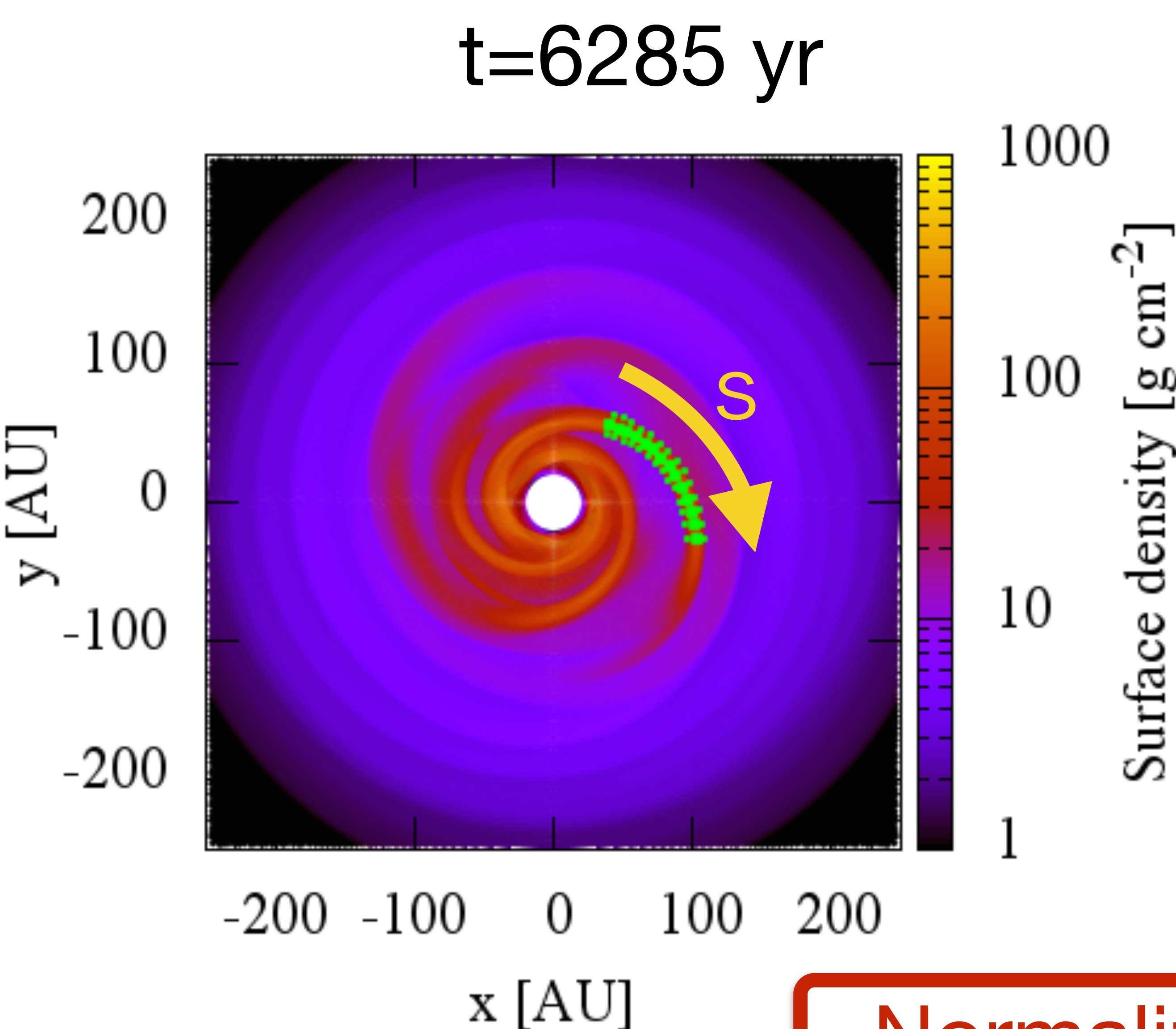
Spiral arms are formed due to gravitational instability.

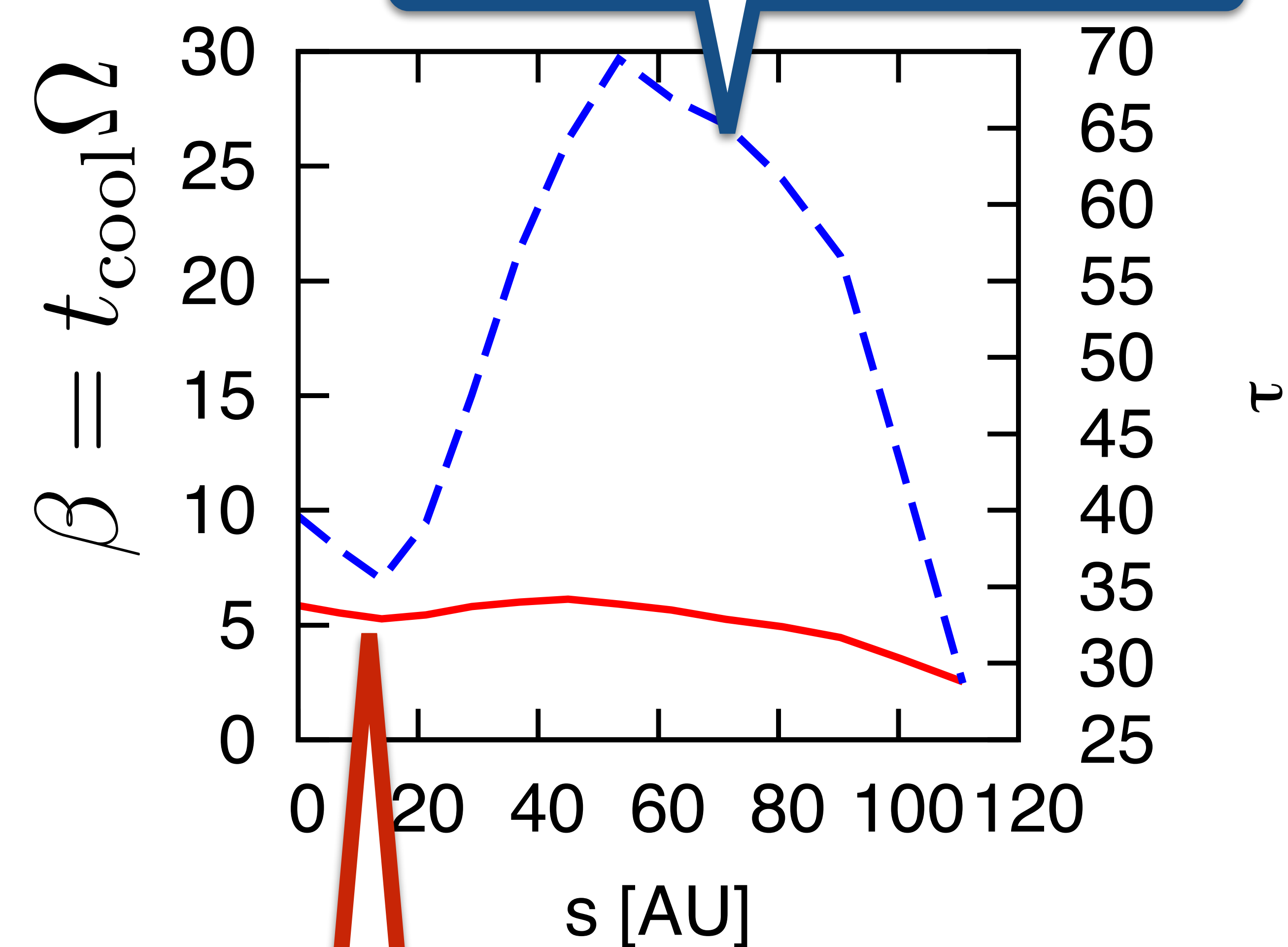
The disk does not fragment.

We test the criterion given by the previous works

$$t_{\rm cool} = \beta\Omega^{-1} < 30\Omega^{-1}$$

Cooling time in the spiral arm



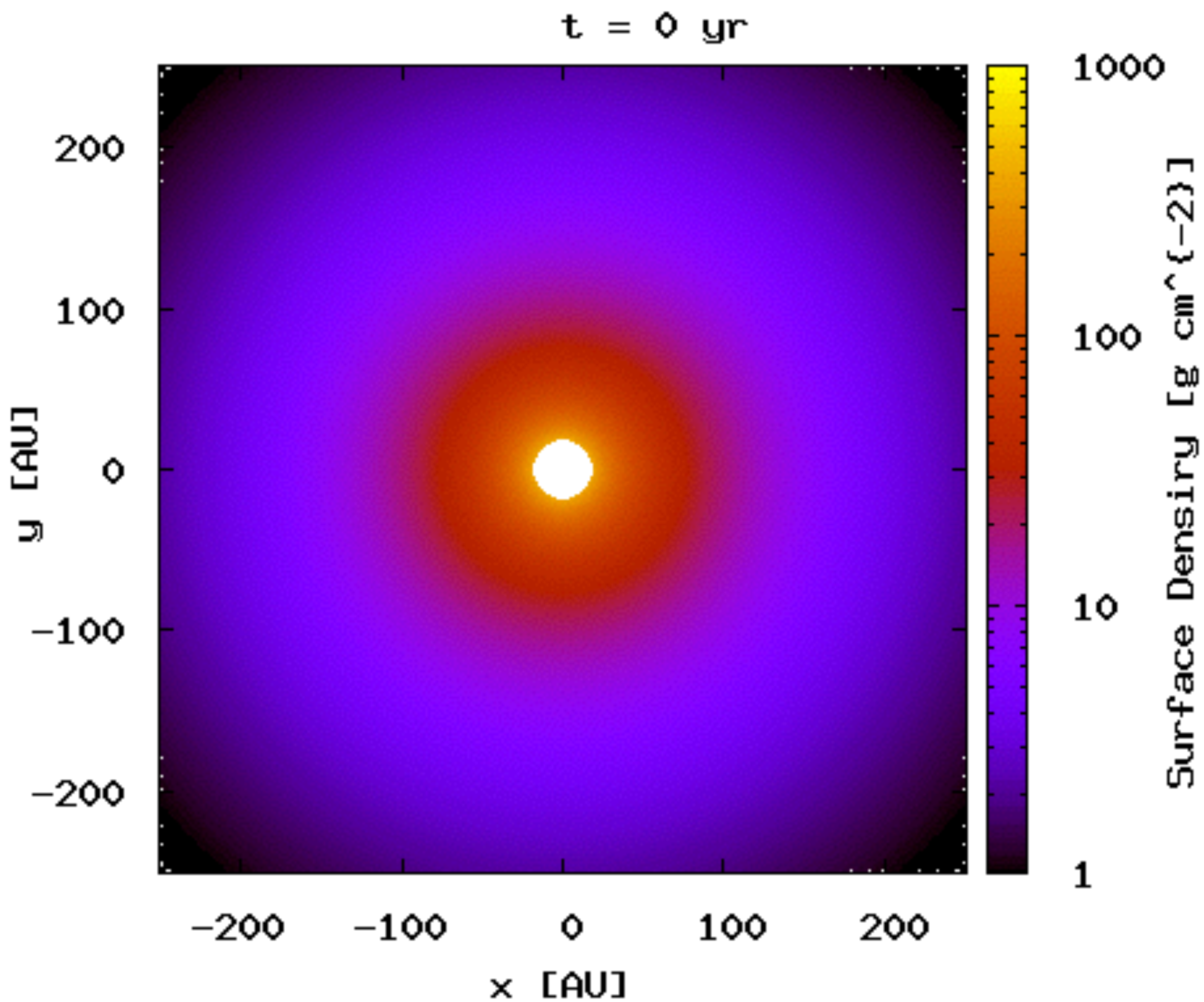


Normalized Cooling Time

$$\beta = E\Omega \frac{3\left(\frac{1}{4}\tau^2 + \frac{1}{\sqrt{3}}\tau + \frac{2}{3}\right)}{8\sigma T^4 \tau}.$$

In the spiral arm, $\beta < \beta_{\rm crit} \sim 30$ is satisfied, but the disk does not fragment.

Result: Fragmentation



$$M_{\rm disk} = 0.38\, M_\odot$$

Spiral arms are formed due to gravitational instability.

Spiral arms **fragment.**

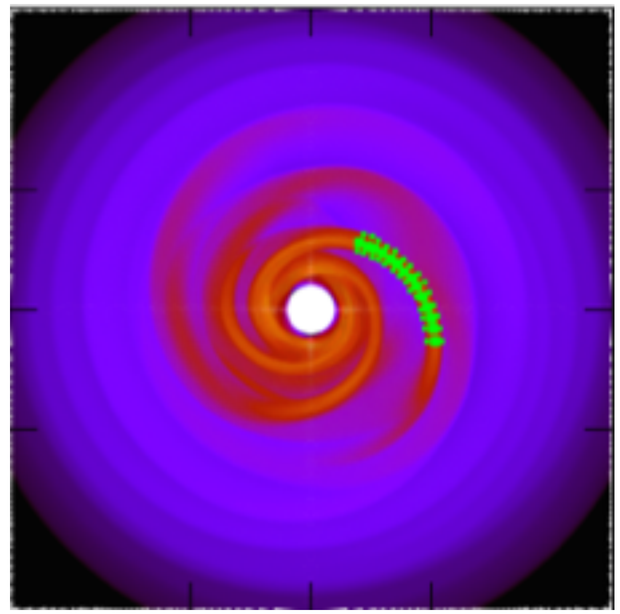
What is the difference between these two results?

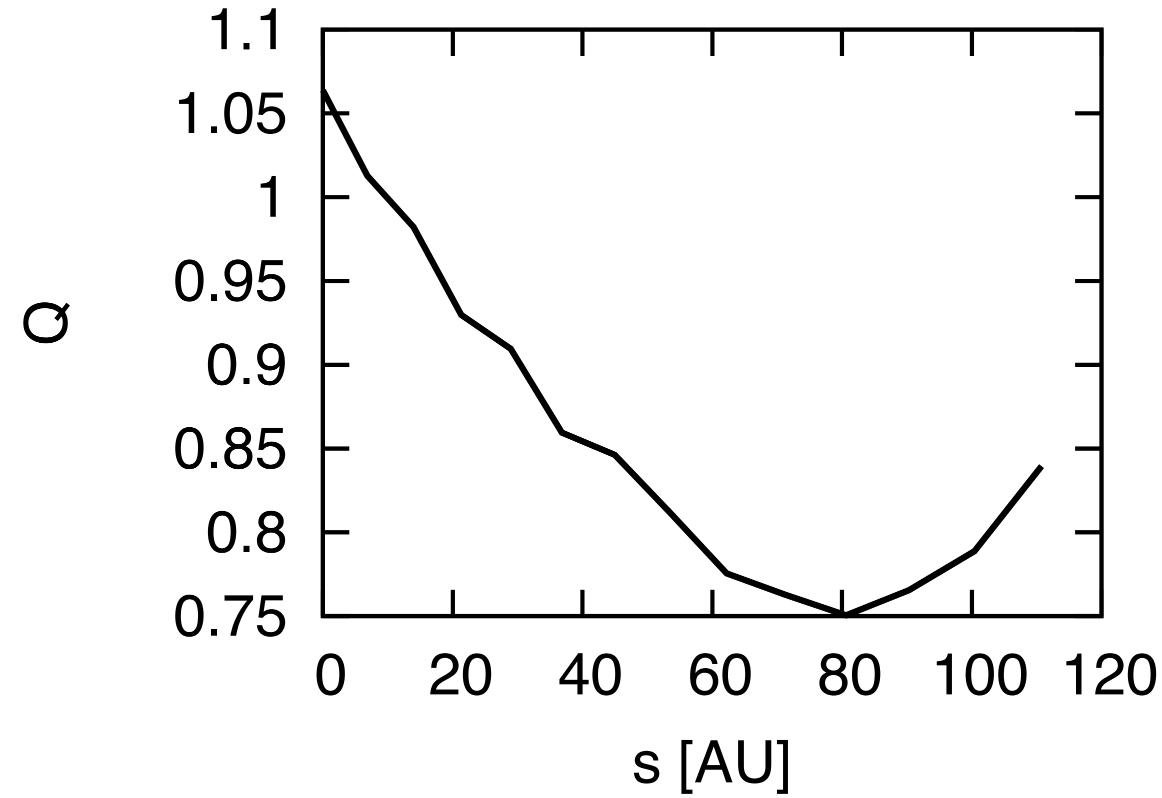
We focus on the **Q parameter in the spiral arms.**

Q parameter in the spiral arms

No fragmentation

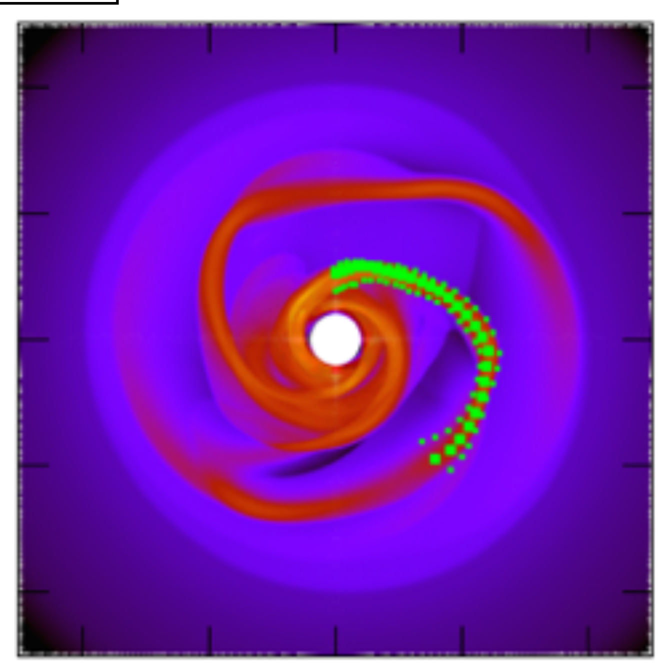
$M_{\rm disk} = 0.34\, M_{\odot}$

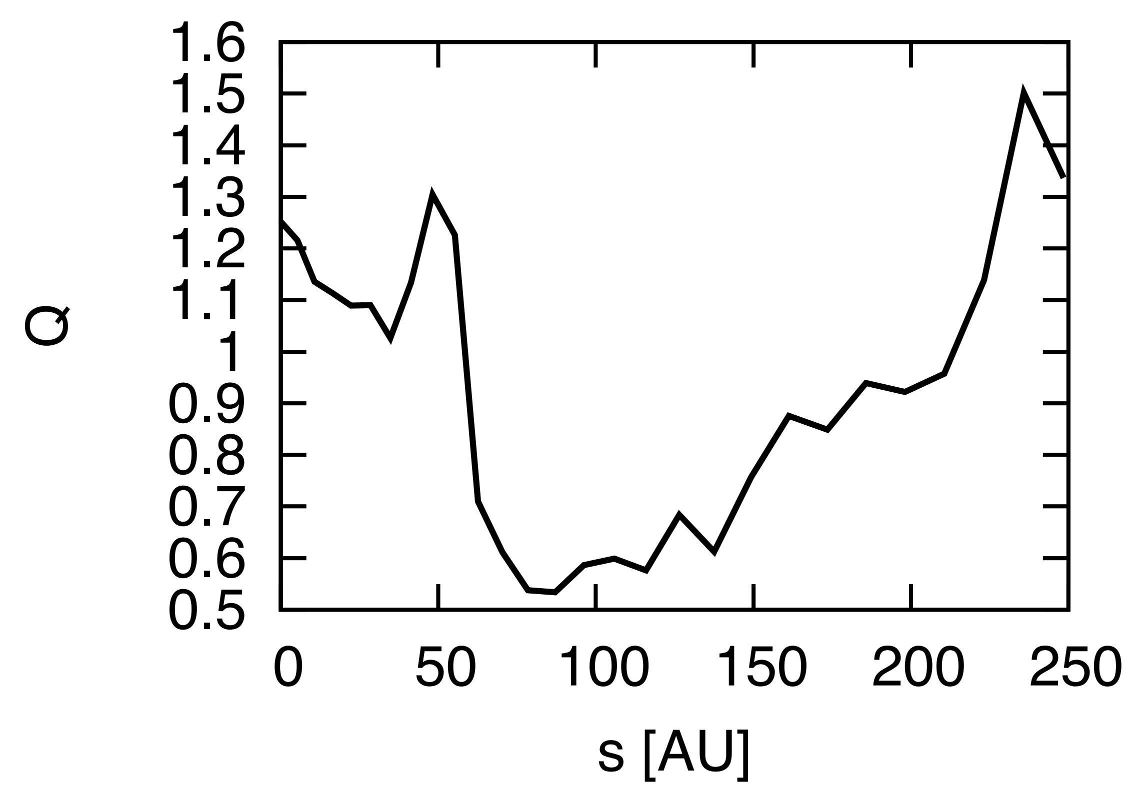




Fragmentation

$M_{\rm disk} = 0.38\, M_{\odot}$

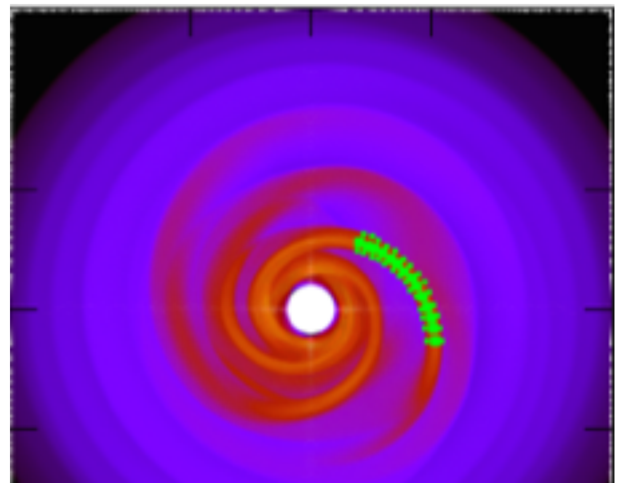


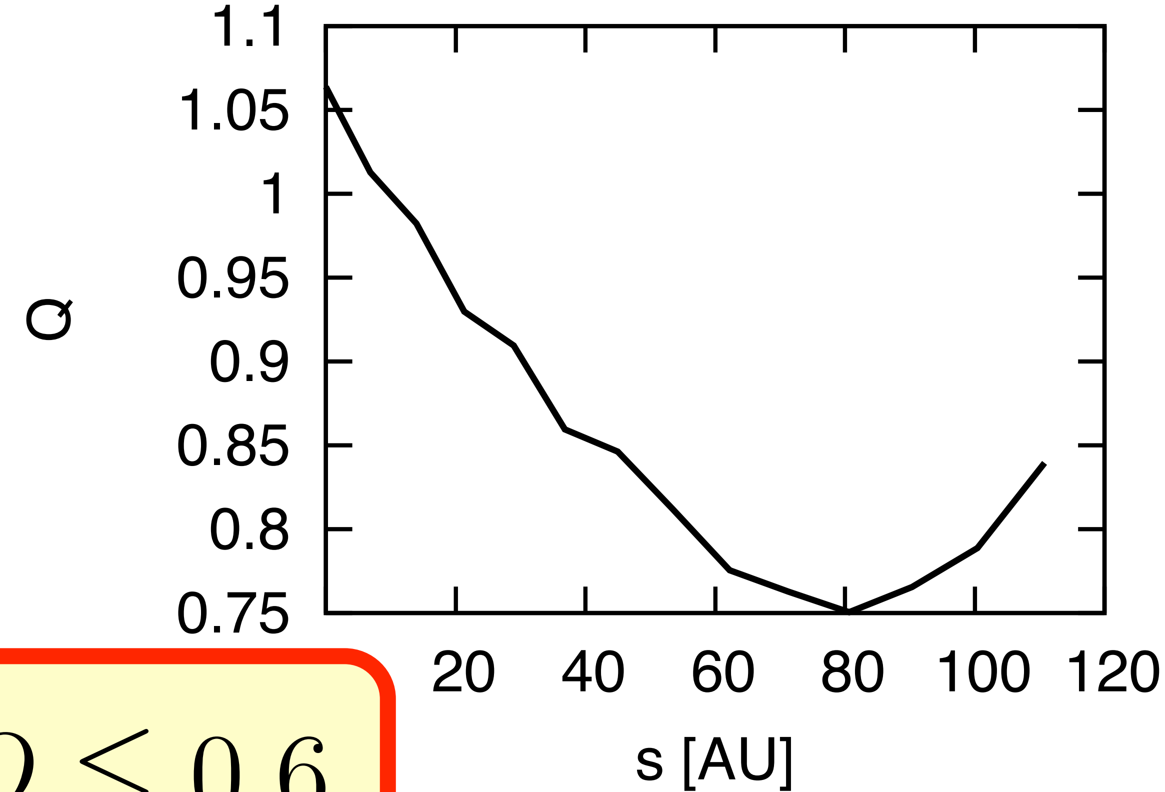


Q parameter in the spiral arms

No fragmentation

$M_{\rm disk} = 0.34\, M_\odot$

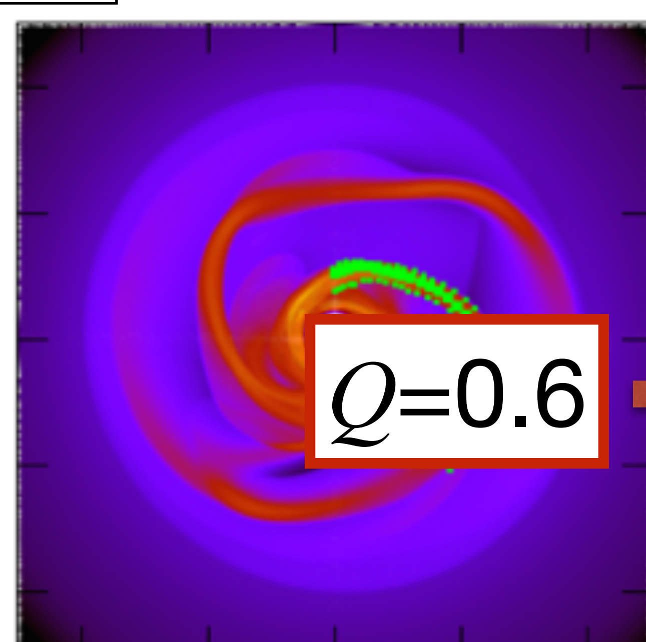


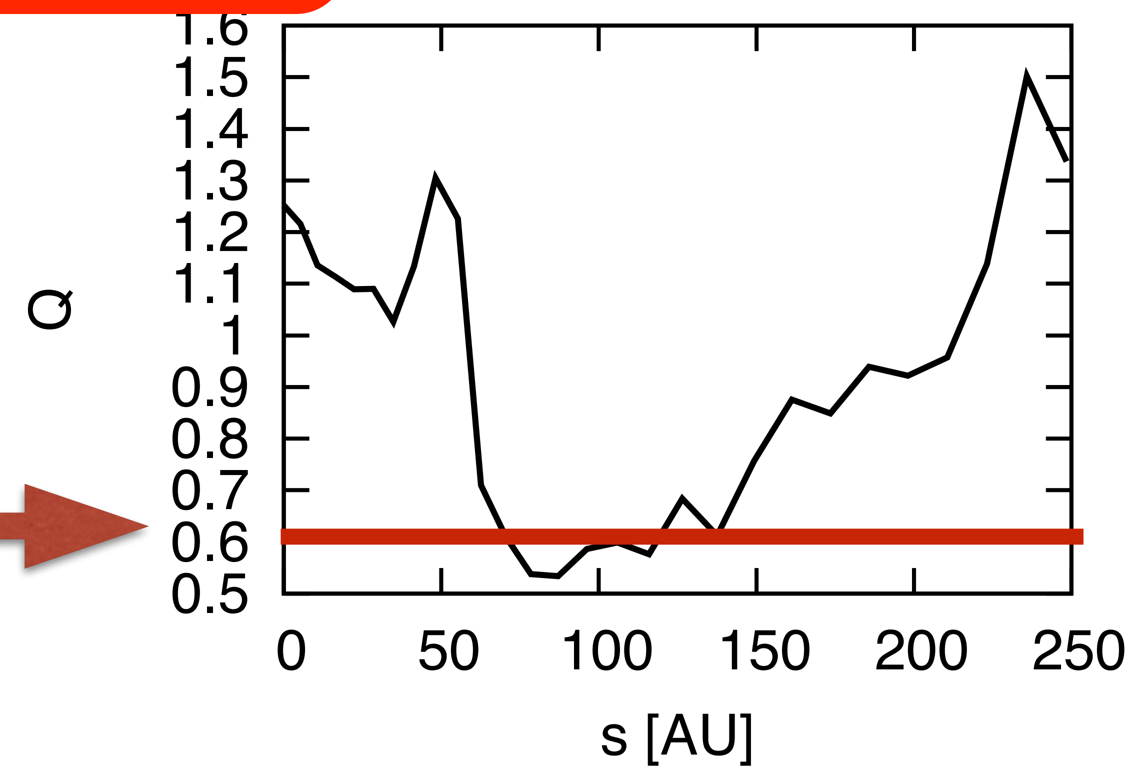


Condition for fragmentation: $Q \lesssim 0.6$

Fragmentation

$M_{\rm disk} = 0.38\, M_\odot$



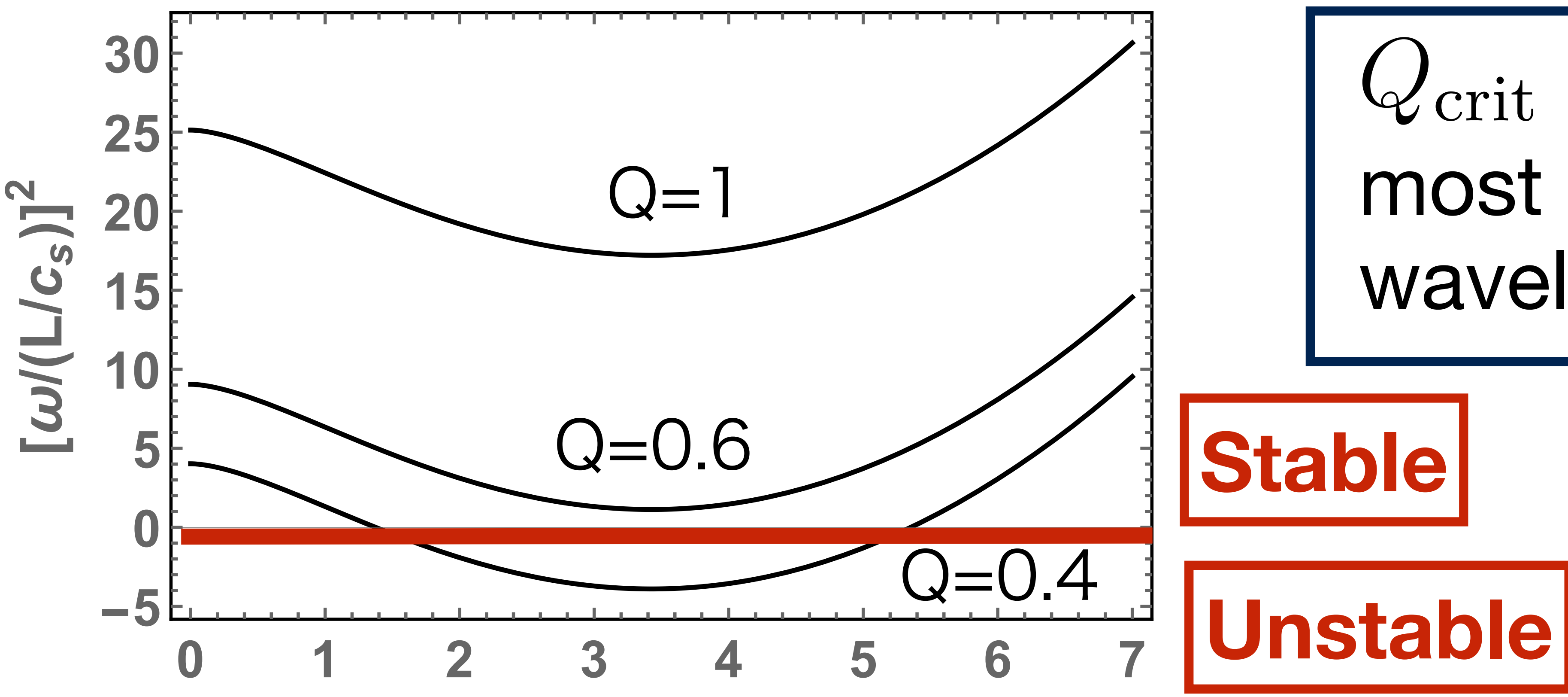


Condition for disk fragmentation

For simplicity, we treat the spiral arms as rotating thin rings.
(cf. Inutsuka-san's talk)

Dispersion relation

(Line mass $M_{\rm L} = c_{\rm s}^2/G \Leftrightarrow f = 1$)

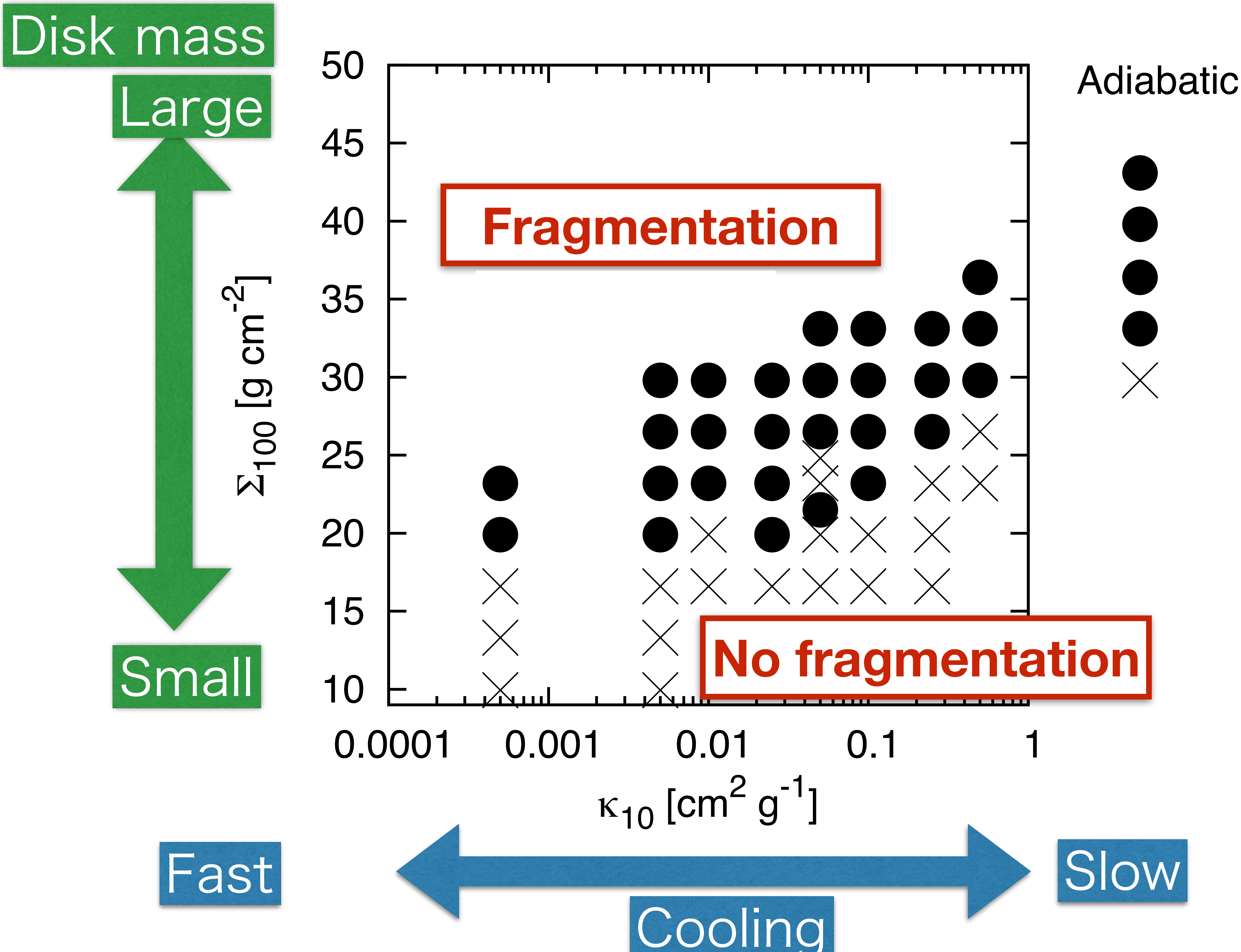


wavenumber kL, L: ring width

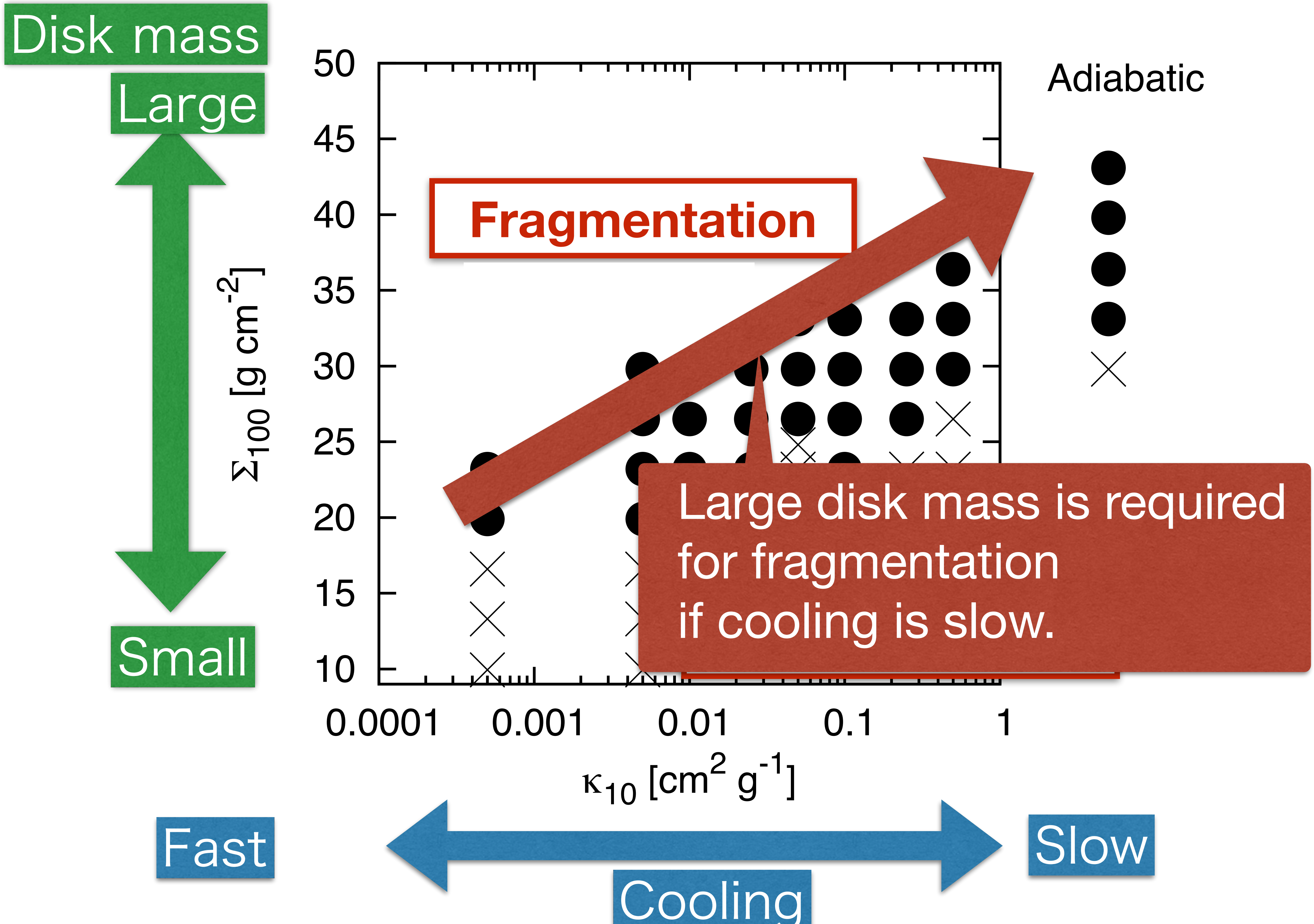
$Q_{\rm crit} \sim 0.6$
most unstable wavelength ~$2L$

Condition for fragmentation of disks
⇔ Condition for the gravitational instability of spiral arms

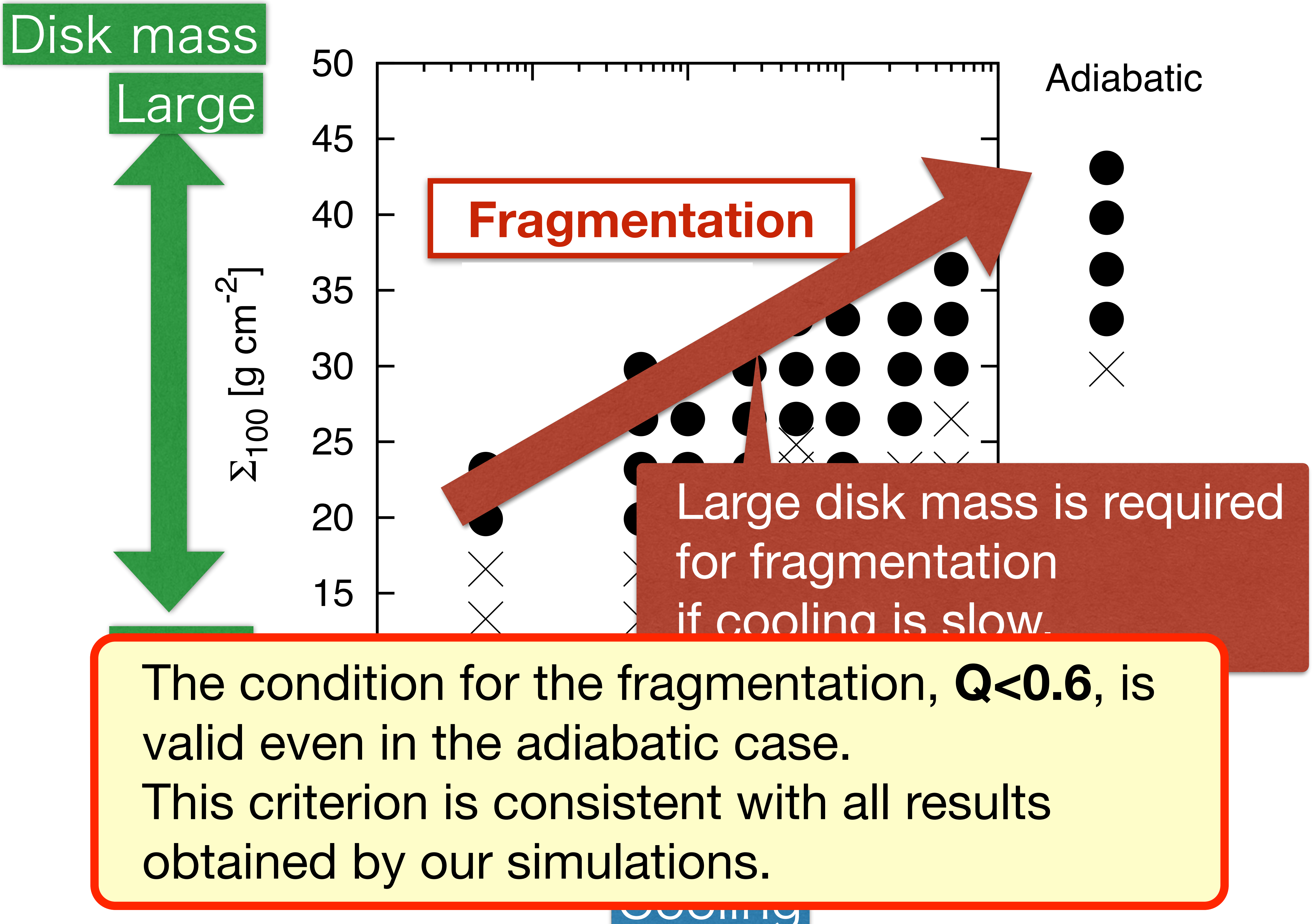
Effect of Cooling



Effect of Cooling



Effect of Cooling

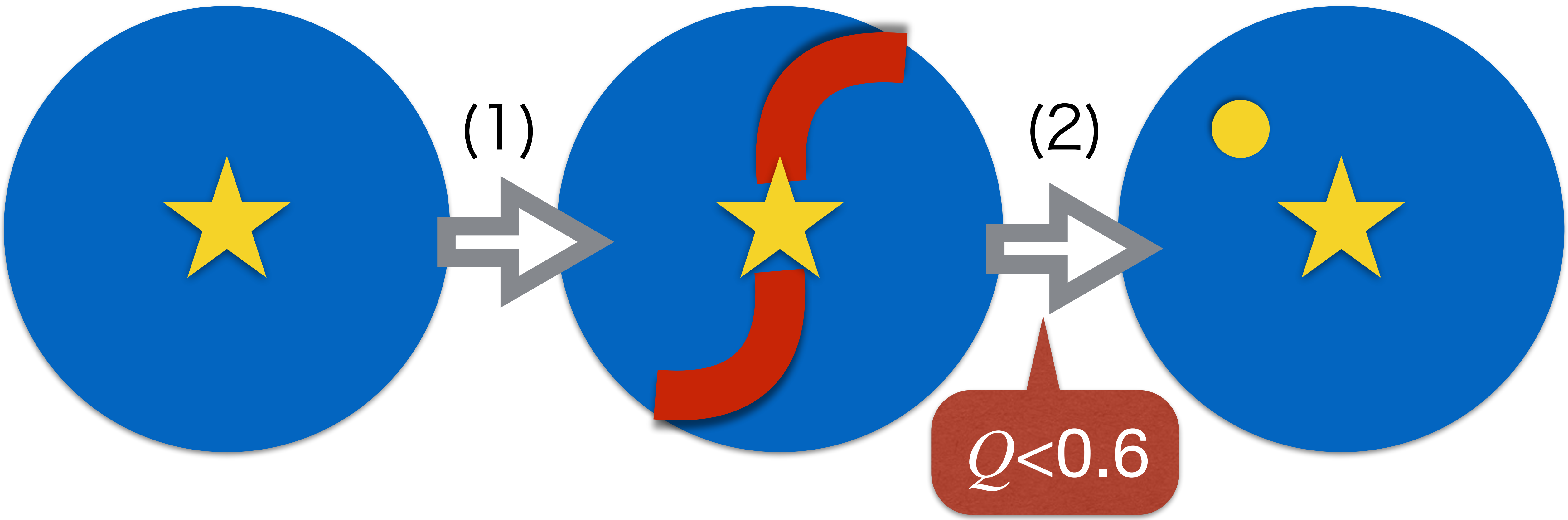


The condition for the fragmentation, **Q<0.6**, is valid even in the adiabatic case.
This criterion is consistent with all results obtained by our simulations.

Conclusion

Fragmentation process is divided into two steps

(1) Spiral arm formation (2) Fragmentation of spiral arm



The problem is reduced to the formation process of the spiral arms that satisfy the condition Q<0.6.

Summary

- Fragmentation of the protoplanetary disks due to the gravitational instability is important for the formation of gas giant planets.
- Previous works suggest that the condition of the fragmentation of the disks is that the cooling is fast enough. However, the criterion of cooling time is inconsistent with some results of numerical simulations of disk formation.
- We performed the numerical simulations of self gravitating disks, and found that the condition of the fragmentation of the disks is that the $Q<0.6$ is satisfied in the spiral arms. This criterion is the same as the condition of the gravitational instability of the thin rings obtained by the linear stability analysis.
- Large disk mass is required for fragmentation when the opacity is large and the cooling is slow. The criterion of the fragmentation obtained from this work is valid even in the adiabatic case.