Neutrino diffraction : finite-size correction to Fermi's golden rule

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- Is neutrino interference (diffraction) observable?

(1) Neutrino is a quantum mechanical wave and interacts with matter extremely weakly.

(2) Neutrino should show a phenomenon of interference or diffraction, in a double slit-like experiment.

(3)However, it is very hard to control the neutrino , from the above (1).
Hence a new method must be considered.

(4)Our answer and proposal "
Neutrino diffraction is observable . Use a pion (and other particle) decay process !"

Contents of this talk

- References

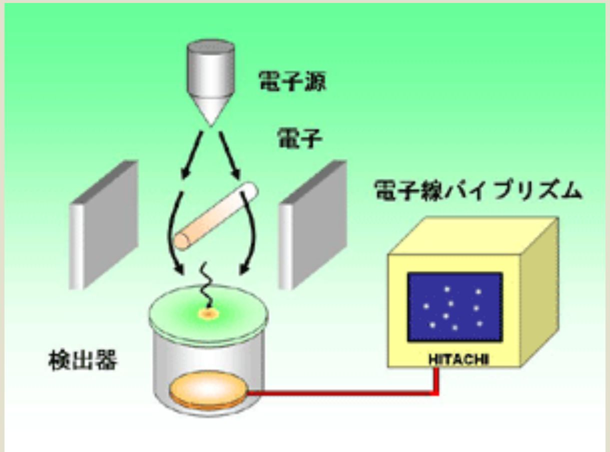
  K.Ishikawa and Y. Tobita arXiv: 1206.2593,1209.5586 1109.3105, 1109.4968 ,others [hep-ph], prog.theo.phy.(2009)

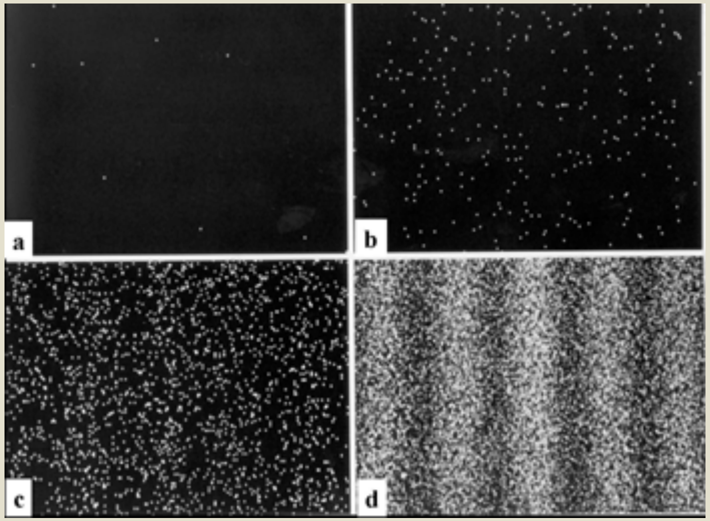
  K.I and Yabuki, progress of theo.physics(2002),

  K.I and T.Shimomura,progress of theo.physics(2005)

1. Introduction
2. Fermi's golden rule : finite-size correction
3. Probability to detect a neutrino in pion decay
4. Light-cone singularity and neutrino diffraction
5. Implications

I. Single electron interference (Tonomura)





Number of events
N_a<N_b<N_c<N_d

interference pattern becomes clear and visible in d

We study single neutrino interference

Unknown facts on neutrinos

- Absolute neutrino mass

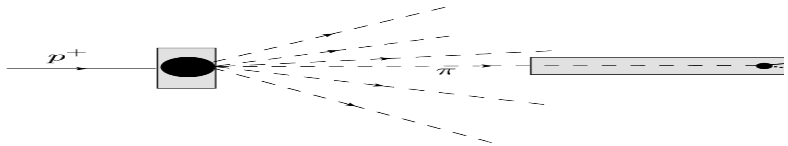
  Tritium decay electron spectrum

  ν - less double Beta decay

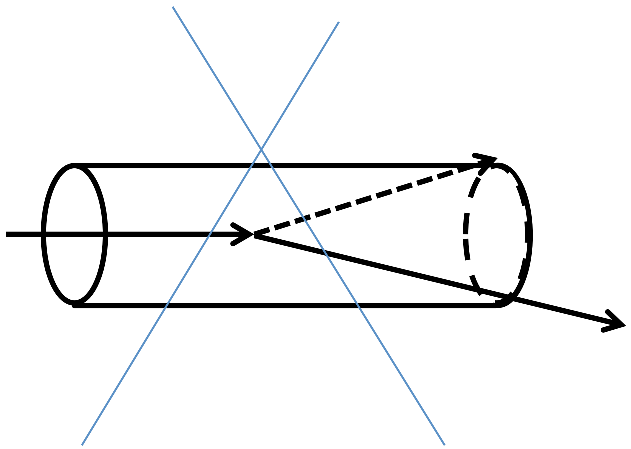
  Cosmology gives bound.

  Unclear now !

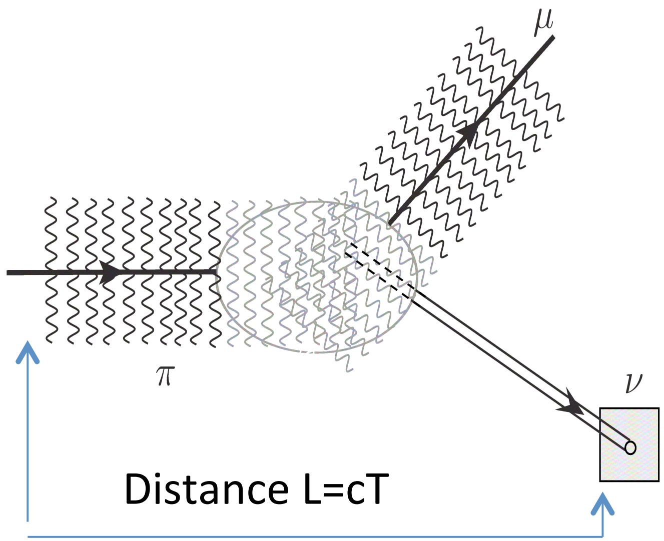
Real pion decays



experiment



Particle decay
(traditional ?)



Decay product of wave is
measured in wave packet

2. Fermi's Golden rule

2-1. Transition rate at a large T is:

$$f_{\alpha,\beta} = g\int_0^T dte^{i\omega t}T_{\alpha,\beta} \approx g2\pi\delta(\omega)T_{\alpha,\beta}, \quad \omega = E_\beta - E_\alpha \qquad (1)$$

$$P = \int d\omega|f_{\alpha,\beta}|^2 = g^2 2\pi T\int d\omega\delta(\omega)|T_{\alpha,\beta}|^2 \qquad (2)$$

$$P/T = g^2 2\pi\int d\omega\delta(\omega)|f_{\alpha,\beta}|^2 \qquad (3)$$

Dirac(1927), Fermi Golden rule (1949),

2-1.T is large but finite. Compute a finite-T correction to above Eq. (3).

$$\int_0^T dte^{i\omega t} = \frac{1}{i\omega}(e^{i\omega T}-1) = e^{i\omega T/2}\frac{2\sin\omega T}{\omega}$$

$$P/T = g^2\int d\omega(\frac{2\sin\omega T}{\omega})^2|f_{\alpha,\beta}|^2$$

$$= g^2 T\int dx(\frac{2\sin x}{x})^2[g(0)+\frac{x}{T}g'(0)+\cdots] \qquad (g(\omega)=|f_{\alpha,\beta}|^2)$$

$$= g^2 2\pi Tg(0)[1+\frac{1}{T}\times\infty+\cdots],$$

$$x=\omega T, g=|f_{\alpha,\beta}|^2$$

Correction diverges

To avoid divergence, include a boundary condition at T !

- Compute transition probability with a boundary condition of an experiment.
- A rigorous treatment is made with out-going (in-coming) state expressed by a wave packet that is localized in space and time. (LSZ,55)
- Compute the finite-size (time) correction with the transition amplitude defined by wave packets. Then a unique value is obtained.

Wave function at a finite t.

$$i\hbar\frac{\partial}{\partial t}\Psi = (H_0 + H_1)\Psi \tag{1}$$

$$\Psi(t) = e^{\frac{E_0 t}{\hbar}}(|\psi^{(0)}\rangle + \int d\beta D(\omega, t)|\beta\rangle\langle\beta|H_{int}|\psi^{(0)}\rangle)$$

$$\omega = E_\beta - E_0, H_0|\beta\rangle = E_\beta|\beta\rangle, H_0|\psi^{(0)}\rangle = E_0|\psi^{(0)}\rangle$$

$$|\psi^{(0)}\rangle = |\vec{p}_\pi\rangle, |\beta\rangle = |\vec{p}_l, \vec{p}_\nu\rangle$$

$$D(\omega, t) = \frac{e^{-i\omega t} - 1}{\omega} \to -2\pi\delta(\omega); t \to \infty (\hbar = 1)$$

$$\neq -2\pi\delta(\omega);\ finite\ t$$

At a finite t, the kinetic energy is not constant.

$$E_l + E_\nu \neq E_\pi \qquad \langle\vec{p}_\pi|H_1|\vec{p}_l, \vec{p}_\nu\rangle \neq 0$$

Transition in a finite time-interval

- S[T] ; S matrix of a finite time interval T.

$$H = H_0 + H_1 \quad (1)$$

$$U(t) = e^{-iHt}, U_0 = e^{-iH_0 t} \quad (2)$$

$$\Omega_{\pm}(T) = lim_{t \to -\pm T/2} U(t)^{\dagger} U^{(0)}(t) \quad (3)$$

$$S - matrix : S[T] = \Omega_{-}^{\dagger}(T) \Omega_{+}(T) \quad (1)$$

$$[S[T], H_0] = i((\frac{\partial}{\partial T} \Omega_{-}(T))^{\dagger} \Omega_{+}(T) - i\Omega_{-}^{\dagger} \frac{\partial}{\partial T} \Omega_{+}(T) \quad (2)$$

Kinetic energy is not conserved

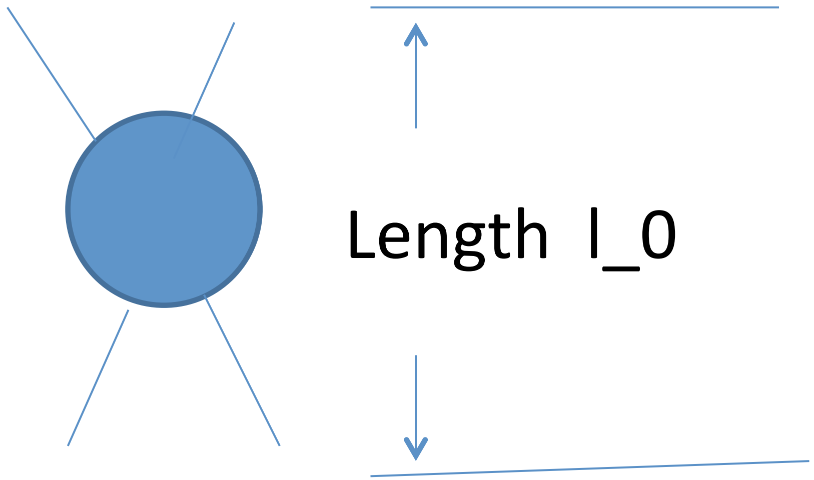
$$transition\ amplitude;\ \langle \beta | S[T] | \alpha \rangle = \delta(E_{\alpha} - E_{\beta}) f_{\alpha,\beta} + \delta f \quad (1)$$

$$probability; \quad (2)$$

$$P = P_{normal} + P_{non-conserving}, P_{non-conserving} = \sum |\delta f|^2 \geq 0$$

Finite-size correction

Asymptotic region



How large is l_0 ?

Wave packets dynamics

K.I and T.shimomura(prog.theor.phys:2005)

$$\langle \vec{x}|\vec{P},\vec{X}\rangle = (\pi\sigma)^{-3/2} e^{i\vec{P}(\vec{x}-\vec{X})-\frac{1}{2\sigma}(\vec{x}-\vec{X})^2} \quad (1)$$

$$\langle \vec{p}|\vec{P},\vec{X}\rangle = N e^{-i\vec{p}\vec{X}-\frac{\sigma}{2}(\vec{p}-\vec{P})^2} \quad (2)$$

$$\int \frac{d\vec{P}d\vec{X}}{(2\pi)^3} |\vec{X},\vec{P},\sigma\rangle\langle\vec{X},\vec{P},\sigma| = 1$$

$$\langle t,\vec{x}|\vec{P},\vec{X},T_0\rangle = N e^{-\frac{1}{2\sigma}(\vec{x}-\vec{X}-\vec{v}(t-T_0)^2 - iE(\vec{P})(t-T_0)+i\vec{P}(\vec{x}-\vec{X})}$$

$$\vec{v} = \frac{\partial}{\partial p_i} E(\vec{p})$$

Wave packet moves with the velocity, $\vec{x} = \vec{v}t + \vec{X}$

A phase $(E - \vec{v}\vec{p})(t - T_\nu) = \frac{m^2}{E}(t - T_\nu)$

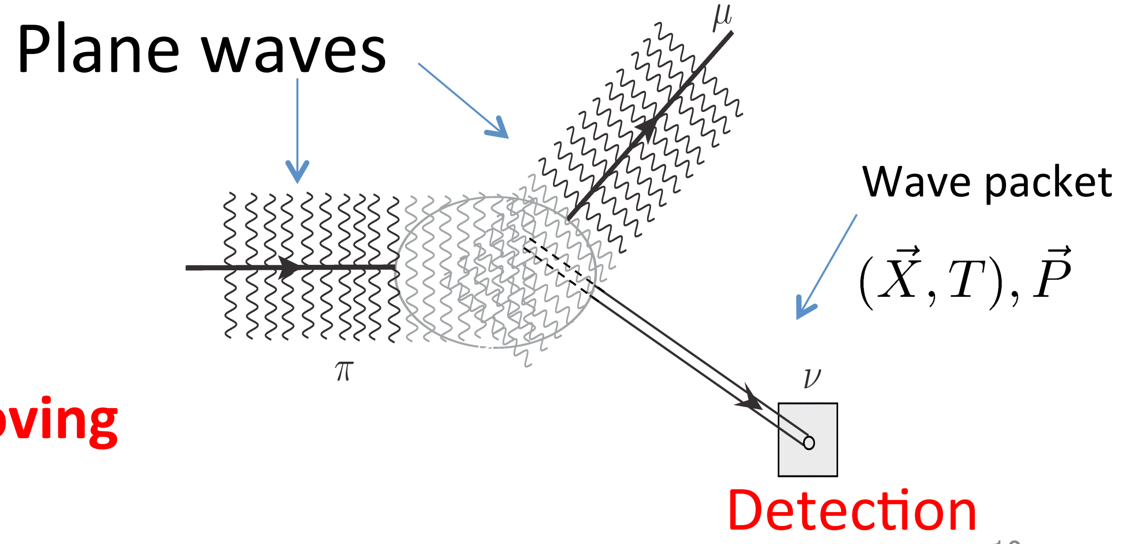
3. Amplitude to detect a neutrino

Neutrino's momentum and position are measured

$$f = \int d^4x \langle 0 | J^{\mu}_{V-A} | \pi \rangle \bar{u}(p_l) \gamma_\mu (1-\gamma_5) \nu(p_\nu) e^{ip_l x + ip_\nu (x - X_\nu) - \frac{1}{2\sigma_\nu}(\vec{x} - \vec{X}_\nu - \vec{v}_\nu (t - T_\nu))^2}$$

$$= N \frac{2 \sin \omega T}{\omega} e^{-\frac{\sigma}{2}(\delta \vec{p})^2} \bar{u}(p_l) \gamma (1-\gamma_5) \nu(p_\nu),$$

$$\omega = \delta E - \vec{v} \delta \vec{p}$$



Integration over x and t = interaction region is in a moving frame

Wave packet effects

1 . Approximate momentum conservation .

Energy conservation in the moving frame. (Pseudo-Doppler shift)

2 . Finite-T correction emerges for light particles.

$$\omega = \delta E - \vec{v}\delta\vec{p} = 0, |\vec{v}| \approx c$$

$$\delta E \neq 0, \delta\vec{p} \neq 0$$

$$f = N\frac{2\sin\omega T}{\omega}e^{-\frac{\sigma}{2}(\delta\vec{p})^2}\bar{u}(p_l)\gamma(1-\gamma_5)\nu(p_\nu),$$

$$\omega = \delta E - \vec{v}\delta\vec{p}$$

4. Calculation with a correlation function

decay amplitude

$$f = \int d^4x d\vec{k}_\nu \langle 0 | J^\mu_{V-A} | \pi \rangle \bar{u}(p_l) \gamma_\mu (1-\gamma_5) \nu(k_\nu) e^{ip_l x + ik_\nu (x - X_\nu) - \frac{\sigma_\nu}{2}(k_\nu - p_\nu)^2}$$

Probability is expressed with a correlation function

$$\int \frac{d\vec{p}_l}{(2\pi)^3} \sum_{s_1,s_2} |f|^2 = \frac{N_2}{E_\nu} \int d^4x_1 d^4x_2 \Delta_{\pi,l}(\delta x) e^{i\phi(\delta x)}$$

Correlation function

$$\Delta_{\pi,l}(\delta x) = \frac{1}{(2\pi)^3} \int \frac{d^3 p_l}{E_l} (2p_\pi p_\nu p_\pi p_l - m_\pi^2 p_l p_\nu) e^{-i(p_\pi - p_l)\delta x}$$

$$= m_l^2 p_l p_\nu$$ (Only If energy-momentum is conserved)

1.Correlation function has a light-cone singularity which is generated by a superposition of relativistic waves.
2.The light-cone singularity is real and long-range and gives the finite-size correction.
Since the energy conservation is violated, this has anomalous properties.
3. Wave packet ensures asymptotic boundary condition（LSZ）and leads new effects .

Correlation function

- Integration variable is change to $q = p_l - p_\pi$

$$\Delta_{\pi,\mu}(\delta x = x_1 - x_2)$$
$$= \frac{1}{4\pi^4} \int d^4q \,\mathrm{Im}\left[\frac{1}{q^2 + 2p_\pi \cdot q + \tilde{m}^2 - i\epsilon}\right] \theta(p_\pi^0 + q^0)$$
$$\times \{(2p_\pi \cdot p_\nu) p_\pi \cdot (p_\pi + q) - m_\pi^2 (q + p_\pi) \cdot p_\nu\} e^{-iq\cdot\delta x} \qquad \tilde{m}^2 = m_\pi^2 - m_l^2$$

- Integral region is separated in two parts

$$\Delta_{\pi,\mu}(\delta x) = I_1 + I_2$$

$q^0 > 0 \qquad 0 > q^0 > -p_\pi^0$

Finite size correction (energy non-conserving) → I_1 ($q^0 > 0$)

Normal term → I_2 ($0 > q^0 > -p_\pi^0$)

I_1: Extract the light-cone singularity

About I_1

$$I_1 = \left[p_\pi \cdot p_\nu - i p_\nu \cdot \left(\frac{\partial}{\partial \delta x} \right) \right] \tilde{I}_1$$

To extract light-cone singularity expanding in $2p_\pi \cdot q$

convergence condition: $2p_\pi \cdot p_\nu \le m_\pi^2 - m_\mu^2$

$$\tilde{I}_1 = \int d^4q \frac{\theta(q^0)}{4\pi^4} \mathrm{Im}\left[\frac{1}{q^2 + 2p_\pi \cdot q + \tilde{m}^2 - i\epsilon} \right] e^{iq\cdot\delta x}$$

$$= \int d^4q \frac{\theta(q^0)}{4\pi^4} \mathrm{Im}\left[\frac{1}{q^2 + \tilde{m}^2 - i\epsilon} \right] e^{iq\cdot\delta x}$$

$$+ \sum_{n=1} \frac{1}{n!} \left(2p_\pi \cdot \left(-i \frac{\partial}{\partial \delta x} \right) \frac{\partial}{\partial \tilde{m}^2} \right)^n \int d^4q \frac{\theta(q^0)}{4\pi^4} \mathrm{Im}\left[\frac{1}{q^2 + \tilde{m}^2 - i\epsilon} \right] e^{iq\cdot\delta x}$$

Green's function

$$\int d^4q \frac{\theta(q^0)}{4\pi^4} \mathrm{Im}\left[\frac{1}{q^2 + \tilde{m}^2 - i\epsilon} \right] e^{iq\cdot\delta x} = \frac{2}{(2\pi)^3} \int d^4q\, \theta(q^0) \delta(q^2 + \tilde{m}^2) e^{iq\cdot\delta x}$$

$$= 2i \left[\frac{\epsilon(\delta t)}{4\pi} \delta(\lambda) + f_{\mathrm{short}} \right]$$

Light-cone singularity

$$f_{\mathrm{short}} = -\frac{i\tilde{m}^2}{8\pi\sqrt{-\lambda}} \theta(-\lambda) \left\{ N_1\left(\tilde{m}\sqrt{-\lambda}\right) - i\epsilon(\delta t) J_1\left(\tilde{m}\sqrt{\lambda}\right) \right\} - \theta(\lambda) \frac{i\tilde{m}}{4\pi^2\sqrt{\lambda}} K_1\left(\tilde{m}\sqrt{\lambda}\right), \quad \lambda = \delta t^2 - \delta\vec{x}^2$$

Integration over space-time coordinates

$$\int \frac{d^3p_l}{(2\pi)^3}\sum_{spin}|T|^2 = \frac{N_2}{E_\nu}\int d^4x_1 d^4x_2 e^{-\frac{1}{2\sigma_\nu}\sum_i(\vec{x}_i-\vec{\mathrm{X}}_\nu-\vec{v}_\nu(t_i-\mathrm{T}_\nu))^2}$$

$$\times \Delta_{\pi,l}(x_1-x_2)e^{ip_\nu\cdot(x_1-x_2)}$$

$$= \frac{N_3}{E_\nu}\int d^4x_1 d^4x_2 e^{-\frac{1}{2\sigma_\nu}\sum_i(\vec{x}_i-\vec{\mathrm{X}}_\nu-\vec{v}_\nu(t_i-\mathrm{T}_\nu))^2}\left[i\frac{\epsilon(\delta t)}{4\pi}\delta(\lambda)+f'_{\mathrm{short}}+I_2\right]e^{ip_\nu\cdot\delta x}$$

$$N_3 = 8g^2\left\{p_\pi\cdot p_\nu(m_\pi^2-2p_\pi\cdot p_\nu)\right\}(\pi^2/\sigma_\nu)^{\frac{3}{2}}$$

I_1: Long-range term

$$\int d^4x_1 d^4x_2 e^{-\frac{1}{2\sigma_\nu}\sum_i(\vec{x}_i-\vec{\mathrm{X}}_\nu-\vec{v}_\nu(t_i-\mathrm{T}_\nu))^2}e^{ip_\nu\cdot\delta x}\times i\frac{\epsilon(\delta t)}{4\pi}\delta(\lambda)$$

$$= (\sigma_\nu\pi)^{\frac{3}{2}}\frac{\sigma_\nu}{2}i\int_0^{\mathrm{T}}dt_1dt_2\frac{\epsilon(\delta t)}{|\delta t|}e^{i\frac{m_\nu^2}{2E_\nu}\delta t} \leftarrow \tilde{g}(\omega_\nu T)$$

$\mathrm{L} = c\mathrm{T}$ is length of decay volume

I_2 short range term

Probability at a finite distance

- Probability is composed of the normal term and a T-dependent diffraction term.

$$P = P^{normal} + P^{diffraction}$$

$$P^{normal}/T = \quad constant$$

$$P^{diffraction} = C(\sigma)\tilde{g}(T\omega_\nu);\ universal\ \ function\ ,$$

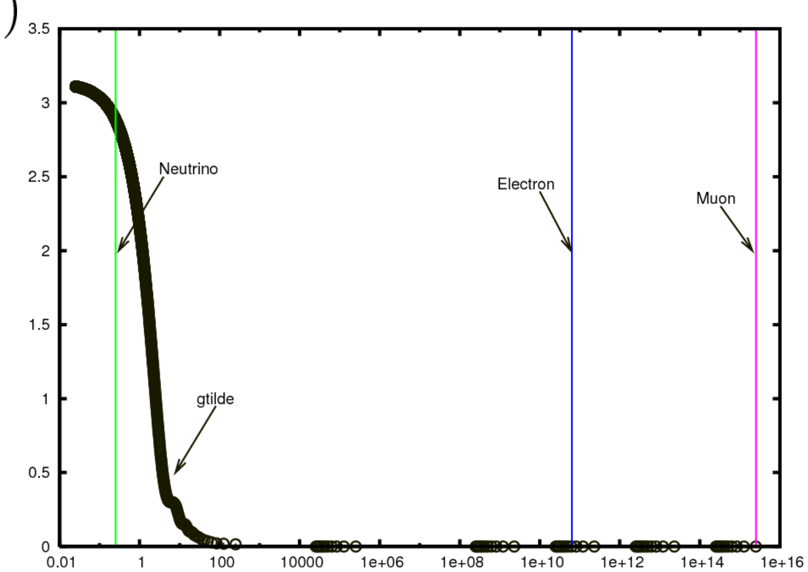
$$\omega_\nu = E_\nu - \vec{n}_\nu \cdot \vec{p}_\nu = \frac{m_\nu^2}{2E_\nu}$$

Diffraction term + normal term (muon neutrino)

$$P = P_{normal} + P_{diffraction}$$

$$P_{diffraction} = C(\sigma)\tilde{g}(T\omega), \omega = \frac{m_\nu^2}{2E_\nu}$$

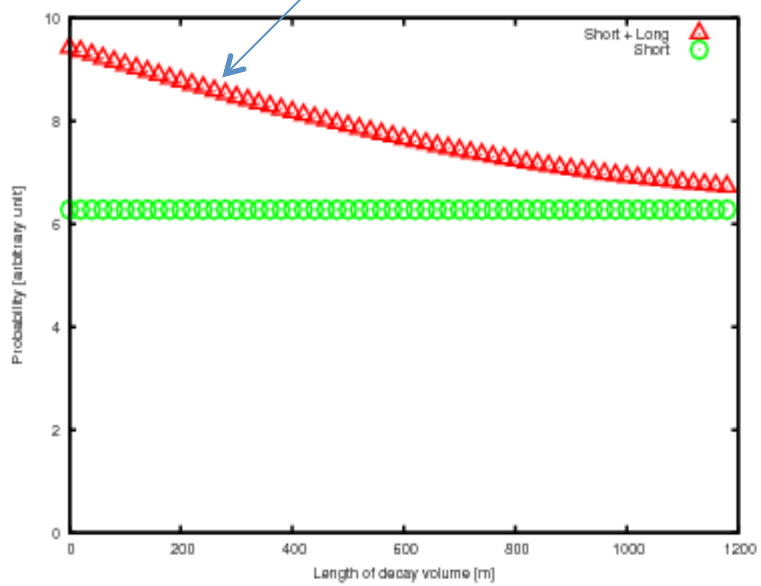
$\tilde{g}(T\omega)$



$T\omega$

P_diffraction

P/T



Length(M)

For three flavors

$$P = P^{(0)}(long) + P^{diffraction}(short), \bar{m}_i^2 \gg \delta m^2$$

$$P^{(0)}(long) = flavour\ oscillation(\delta m^2)$$

standard

Kayser,-,-,-Akhmedov, et al, Smirnov,--

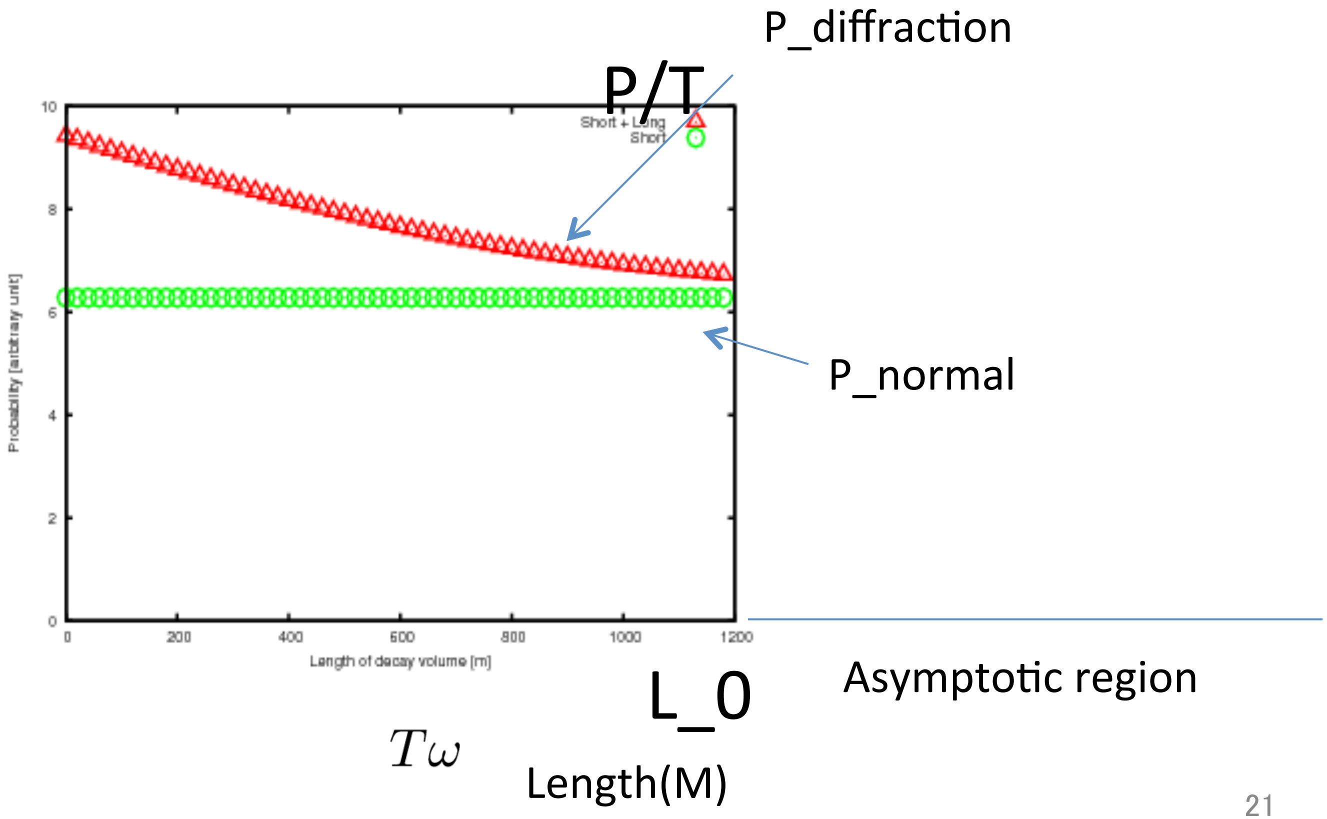
$$P^{diffraction}(short) = neutrino\ diffraction(\bar{m}^2)$$

New term

5. Diffraction term in a muon neutrino

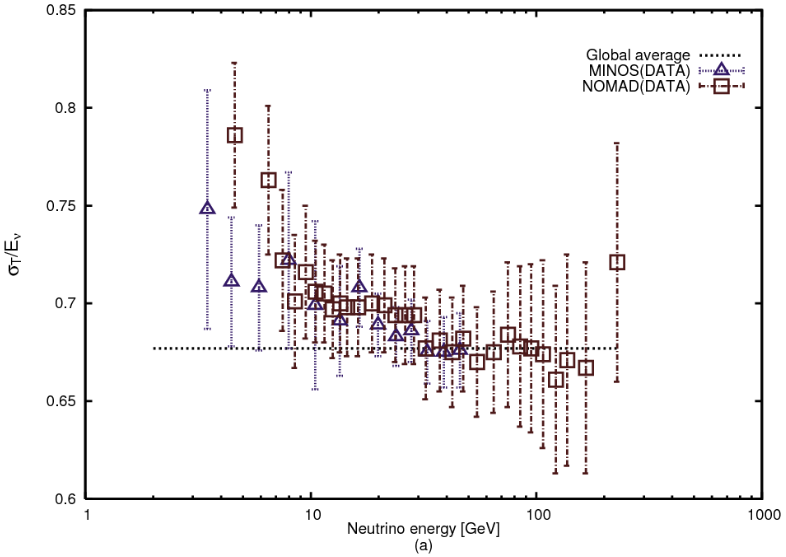
$$P = P_{normal} + P_{diffraction}(new\ term)$$

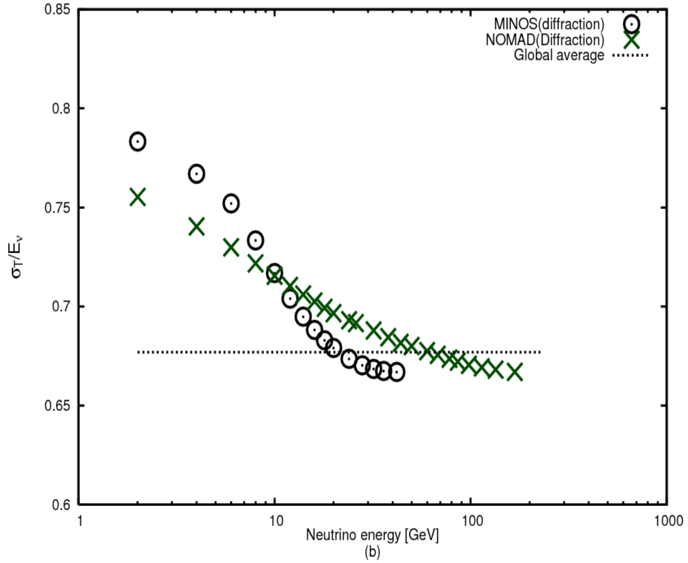
$$P_{diffraction} = C\tilde{g}(T\omega), \omega = \frac{m_\nu^2}{2E_\nu}$$



\nu_mu-nucleon total cross section

$\frac{\sigma_{total}}{E_\nu}$





E_{neutrino}

Experiments.:
NOMAD & MINOS

theory : normal
+diffraction

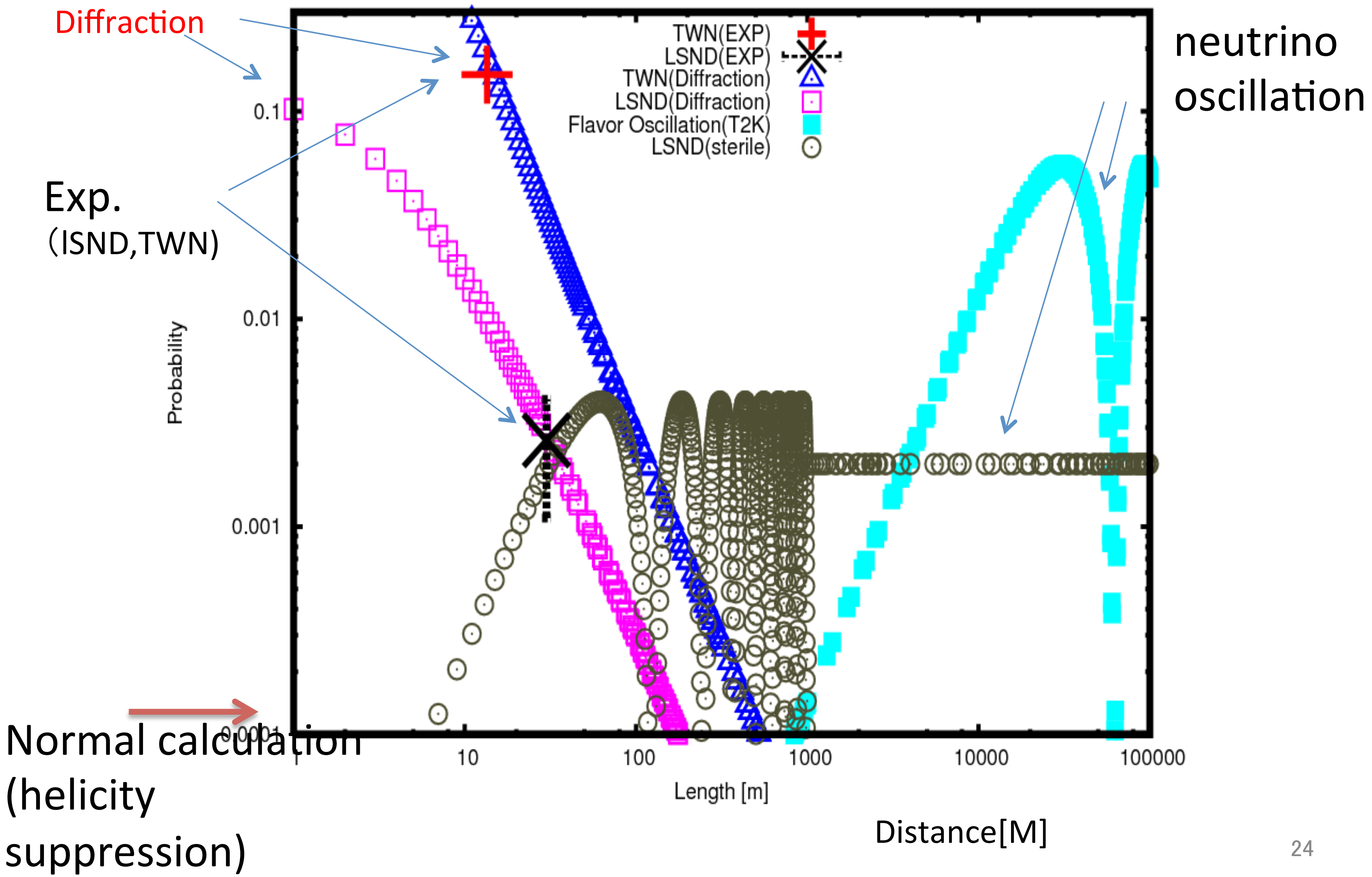
Enhancement of electron mode in the diffraction

The normal term in the electron mode is suppressed by the angular momentum and energy-momentum conservation. (Steinberger,Rudermann-Finkelstein,Sasaki-Oneda-Ozaki,)

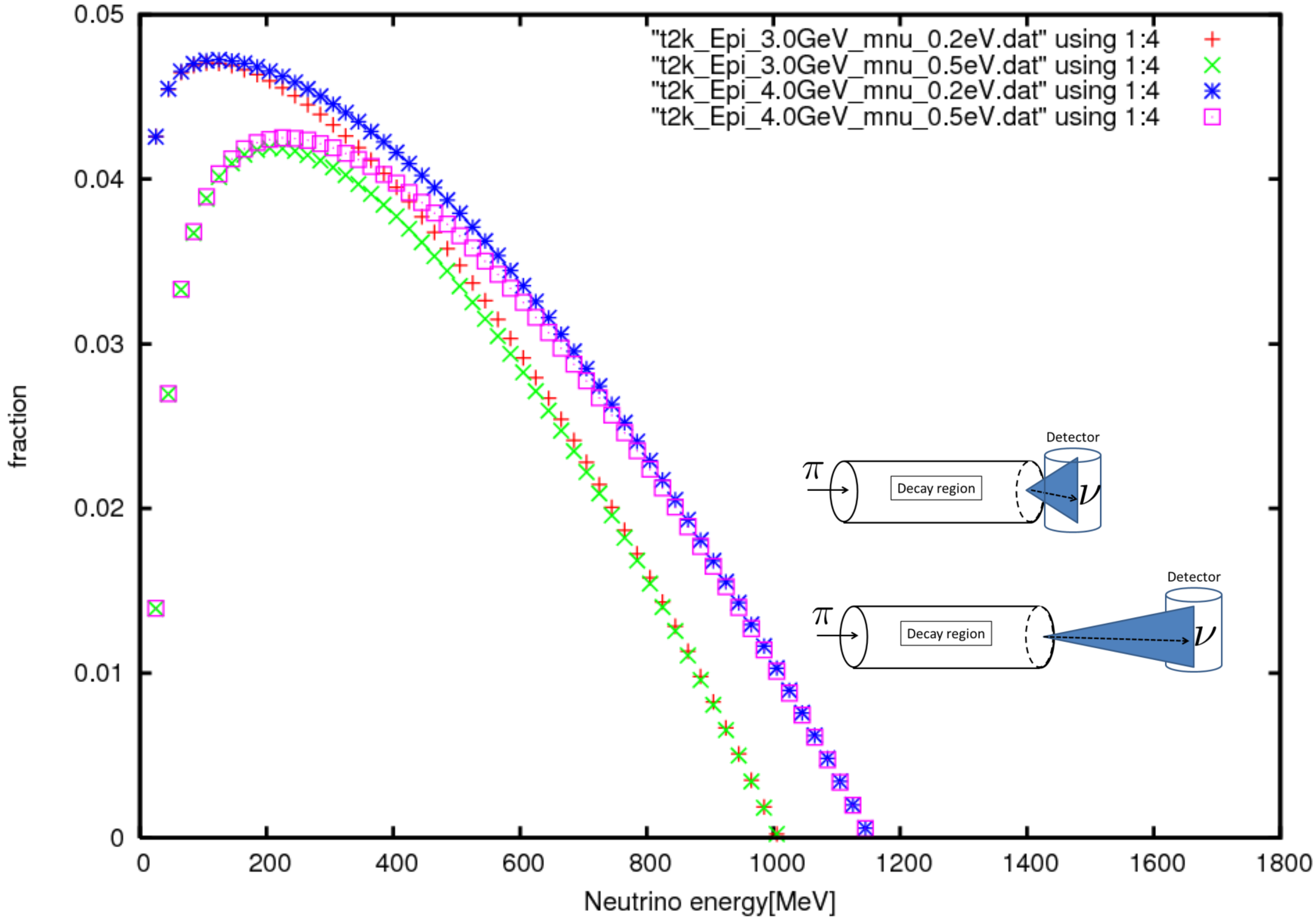
Since the finite-size correction does not conserve the energy, it violates the helicity suppression .

- When the neutrino is detected, the electron mode is enhanced.

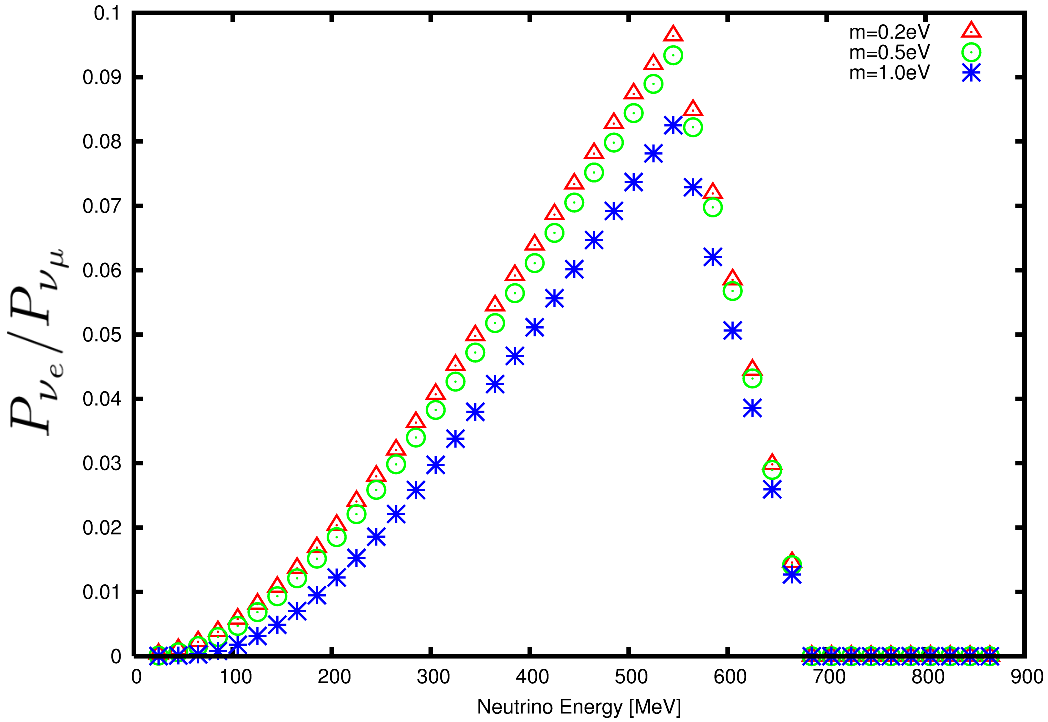
Diffraction Events(enhances electron neutrino)



Diffraction prediction of the electron neutrino diffraction(T2K、on axis)



ν_e appearance at near detector

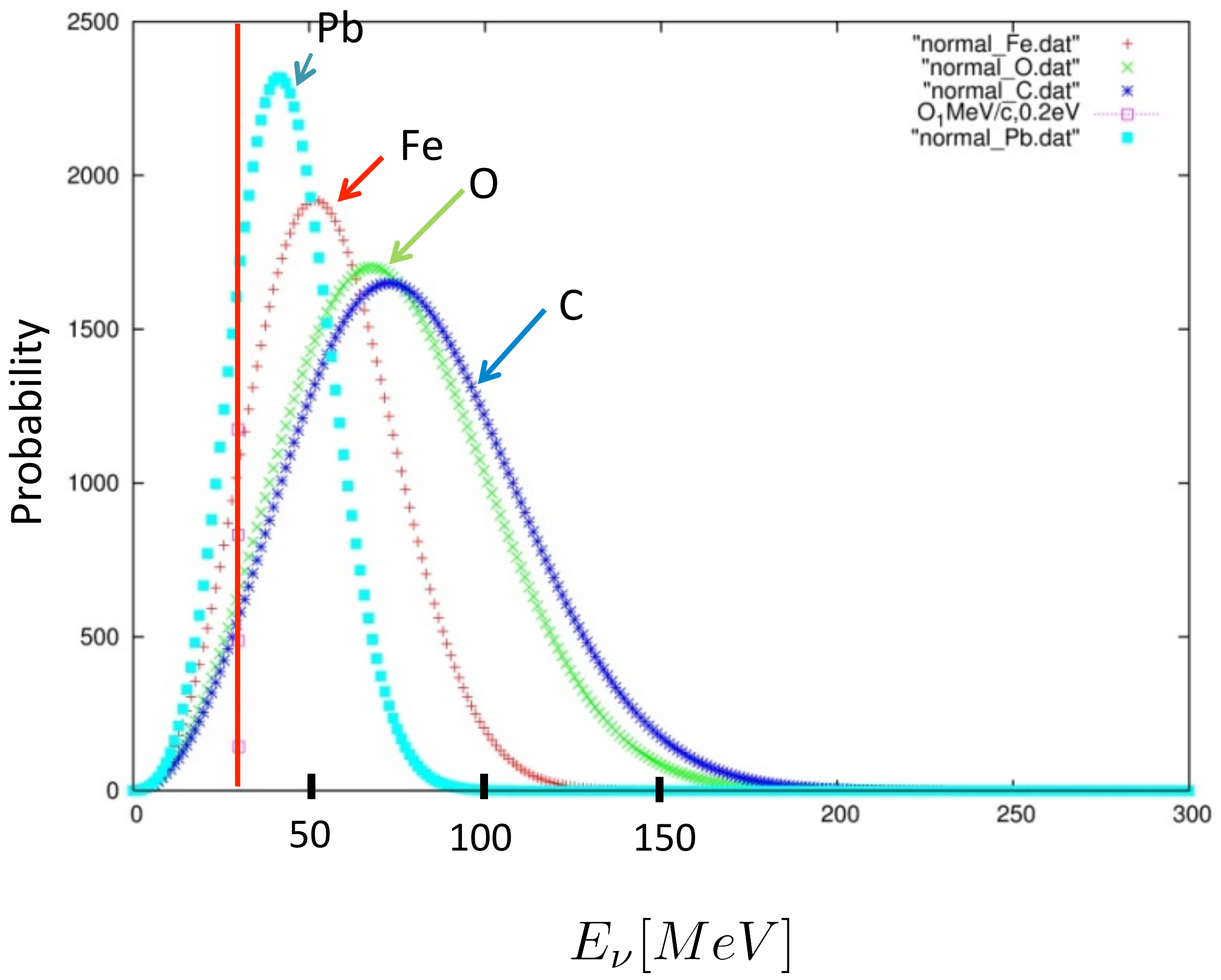


$$|\vec{p}_\pi| = 2[\mathrm{GeV}],\ \mathrm{L} = 110[\mathrm{m}]$$

L_{decay-detector} = 170[m], axis = 2.5 degree, Detector 3[m]x3[m]

～a few % excess

Neutrino from decay of pion at rest



赤は通常の保存則での計算値(～30MeV)です

Anomalous properties of the diffraction

１ neutrino diffraction is easily observed once the statistics becomes large. Single quantum interference

２ kinetic-energy conservation is violated : finite-size effect

３ lepton number appears to be non-conserved.

P(L) decreases with L , so unitarity appears to be violated. But they are not. : finite-size effect and retarded effect.

４ pion life time varies due to the measurement, quantum (anti) Zeno effect . However the majority of the pion are unchanged because the neutrino interacts with matter so weakly.

Comparisons with previous experiments

- Diffraction effect has been observed but that has not been recognized. So, unusual events have been regarded as anomalous events . They are explained with the neutrino diffraction.

- High energy neutrino nucleon scattering cross section decreases with the energy slowly. This is understood by the diffraction effect of the neutrino process.

- High precision experiment may provide the neutrino absolute mass.

Other channels on the neutrino processes

- 1. muon decay $\mu \to e + \nu_\mu + \bar{\nu}_e$
- 2. neutron decay $n \to p + e + \bar{\nu}_e$
- 3.nucleus decays $A \to A' + e + \bar{\nu}_e$

  $A + e \to A' + \nu$
- 4.neutrino scattering $\nu + A \to \nu + A$

(In progress)

New phenomena caused by finite-size effects

１. Emission of light particles .

Kinetic energy non-conserving transition lead background noises that has universal properties.

"theory of universal noises "

２. Interference and diffraction.

interference of a new scale that is very different from wave length "physics of a new scale "

３. Energy shift : "pseudo-Doppler shift "

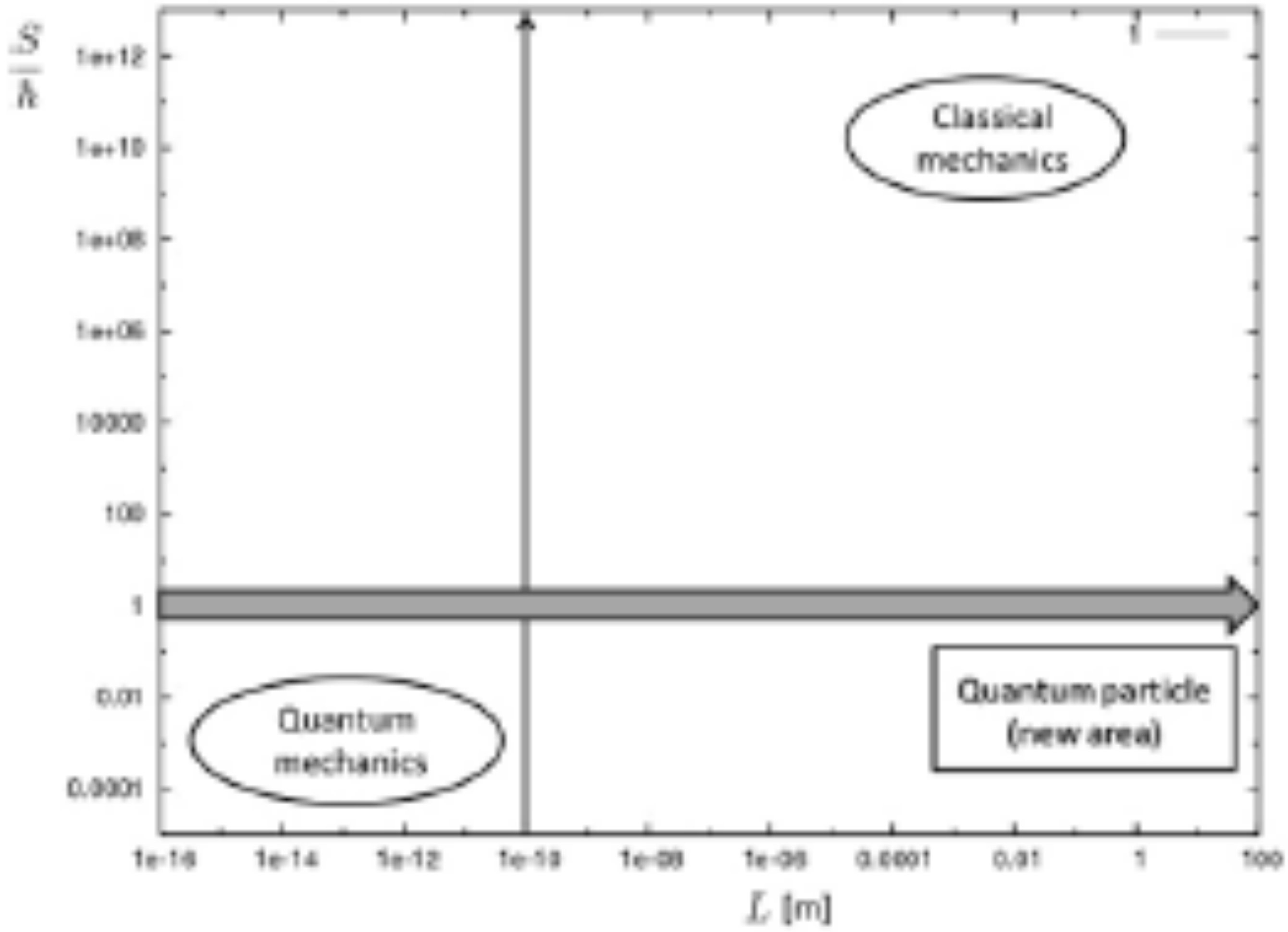
6. Summary

- Finite-size correction to the probability to detect the neutrino is large and macroscopic.

  S[\infty] is applied in l_0 >few 100 meters

The neutrino diffraction, which is the main part of the finite-size correction, is easily observed and may provide the absolute neutrino mass. with enough number of neutrino events.

Macroscopic quantum phenomenon



Other particle or processes?

- N’ >N+ Gamma

$$m_{\gamma.eff} = Plasma\ oscillation$$