Three-body calculations of the triple-alpha reaction rate at low temperatures

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1. INTRODUCTION

Triple-alpha reaction

$$^{4}\mathrm{He} + {}^{4}\mathrm{He} + {}^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C}$$

-Resonant process ($T>10^8$ K)

^{8}Be, ^{12}C*

Resonance formula

-Non-resonant process ($T<10^8$ K)

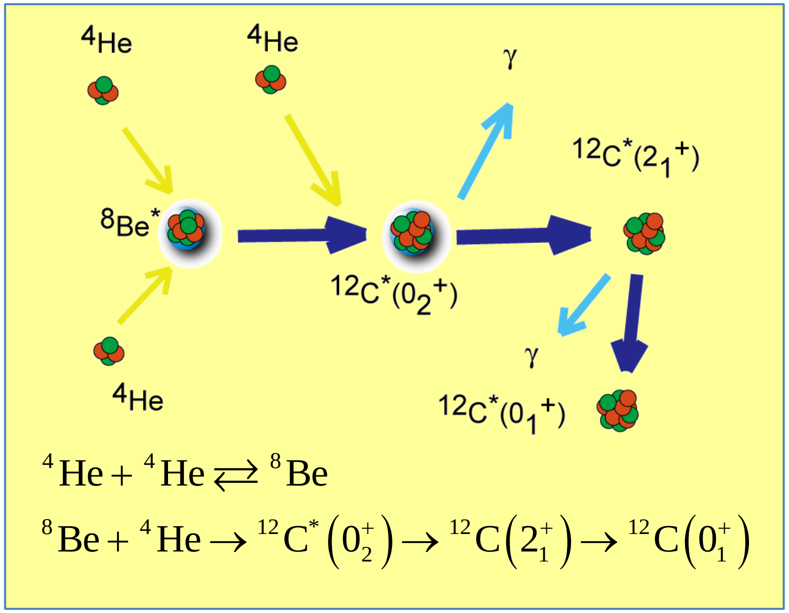
[A] Extension of the resonance formula with energy dependent widths.

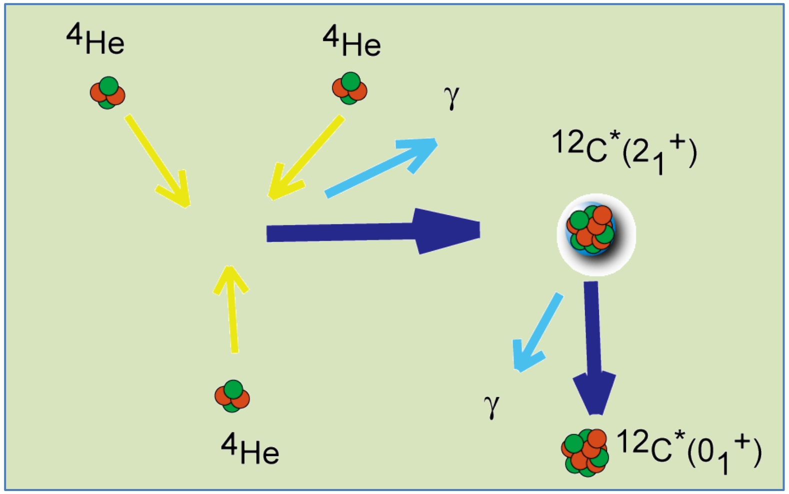
-NACRE [1]

[B] Quantum mechanical 3-body calculations

-OKK: CDCC calculations (Ogata et al.[2])

Significant effects at low temperature



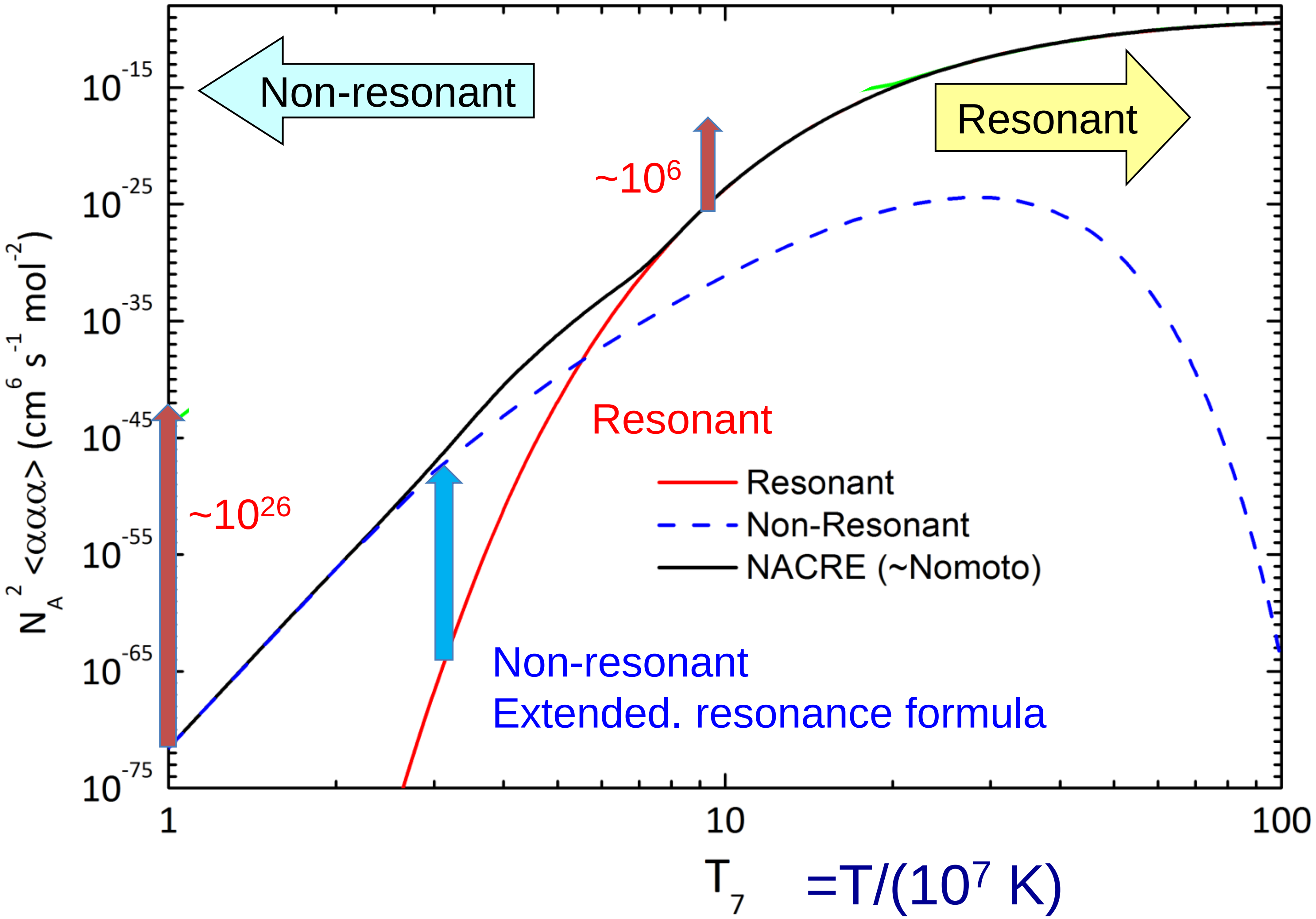


[1] C. Angulo et al., NPA**656** (1999) 3. [2] K. Ogata et al., PTP**122** (2009) 1055.

Astrophysical input: 3α reaction rate <ααα> [cm^6/s]

$$\dot{n}_{12} \equiv \frac{(n_4)^3}{6}\langle\alpha\alpha\alpha\rangle$$

n_{12} (n_4): Number density of ^{12}C (^{4}He)



3-body calculations for 3α reaction

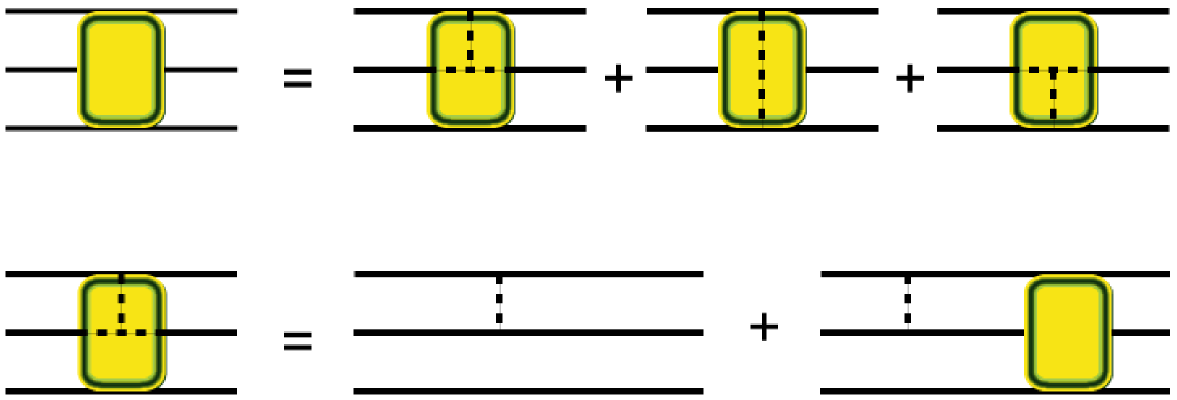
- Ccontinuum Discretized Coupled Channel (CDCC)
  K. Ogata et al., PTP**122** (2009) 1055. [OKK]
- Hyperspherical Harmonics basis + R-matrix (HHR)
  N.B. Nguyen et al. arXiv:1112.2136, arXiv:1209.4999
- This workshop:
  K. Yabana

  Ref: "Imaginary-time method for the radiative capture reaction rate"
  K. Yabana and Y.Funaki, PRC **85**, 055803 (2012)
- Faddeev
  S. Ishikawa, INPC2010, APFB2011, OMEG11
  (paper in preparation)

In the present talk:

- Calculation of 3α reaction based on the Faddeev 3-body theory.
- Discussion about the difference from the OKK rate

CONTENTS



2. FORMALISM

3α reaction $\alpha+\alpha+\alpha \rightarrow {}^{12}\mathrm{C}(2^+)+\gamma$

1. Inverse reaction: Photo induced 3α breakup of ^{12}C(2$^+$)

$$ {}^{12}\mathrm{C}(2^+)+\gamma \rightarrow \alpha+\alpha+\alpha $$

2. Define a wave function for the breakup process

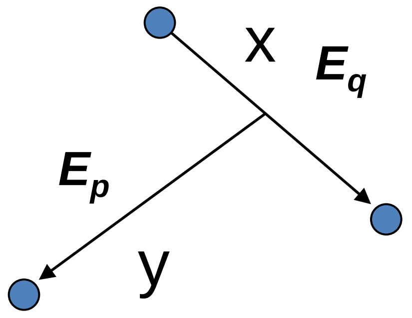
$$ |\Psi\rangle \equiv \frac{1}{E+i\varepsilon-H_0-V}H_\gamma|\Psi_b\rangle \underset{R\to\infty}{\rightarrow} \frac{e^{iKR}}{R^{5/2}} f^{(B)}\left(E_q,x,y\right) $$

$$ R=\sqrt{x^2+\frac{4}{3}y^2} $$

3. Photodisintegration cross section

$$ E=E_q+E_p $$

$$ \sigma_\gamma(E) \propto \iint d\hat{x}d\hat{y}\int_{E_q>0} dE_q\sqrt{E_qE_p}\left|f^{(B)}\left(E_q;\hat{x},\hat{y}\right)\right|^2 $$



4. Reaction rate

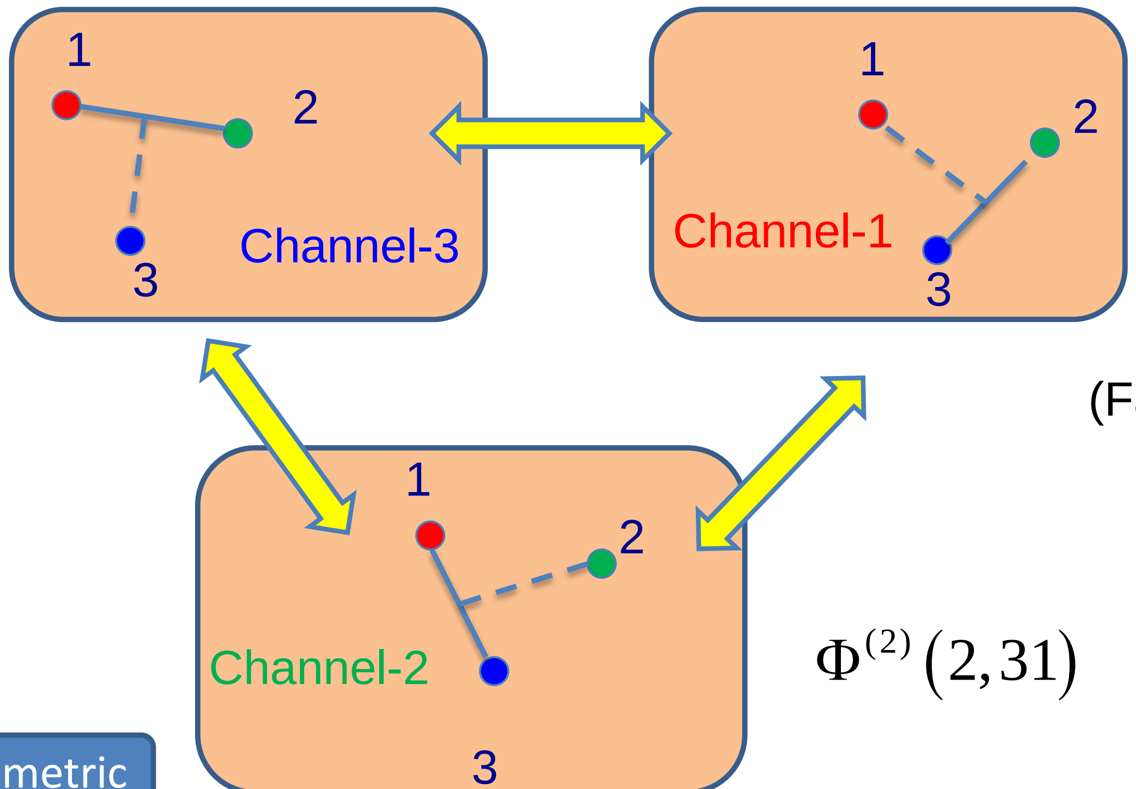
$$\langle \alpha\alpha\alpha \rangle = 240(3)^{3/2}\pi\left(\frac{\hbar}{mc}\right)^3 c \int_0^\infty \frac{dEE_\gamma^2}{(kT)^3} e^{-E/k_B T} \sigma_{^{12}C(2_1^2)+\gamma\to 3\alpha}(E_\gamma) \qquad \left(E_\gamma = E - E_{^{12}C(2^+)}\right)$$

5. Apply the Faddeev formalism [1] to solve the equation for the 3-body disintegration process.
6. Apply the Sasakawa-Sawada method [2] to accommodate the long-range Coulomb interaction.
7. An approximation is made to treat a long-range contribution

[1] L.D. Faddeev, Soviet Phys. JETP **12** (1961) 1041.
[2] T. Sasakawa and T. Sawada, PRC 20 (1979) 1954.

Faddeev eq. (1961)
Multiple scattering with rearrangements

$\Phi^{(3)}(3,12)$



$\Phi^{(1)}(1,23)$

(Faddeev component)

$\Phi^{(2)}(2,31)$

Totally symmetric

$$\Psi(123) = \Phi^{(1)}(1,23) + \Phi^{(2)}(2,31) + \Phi^{(3)}(3,12)$$

Symmetric for 2<->3

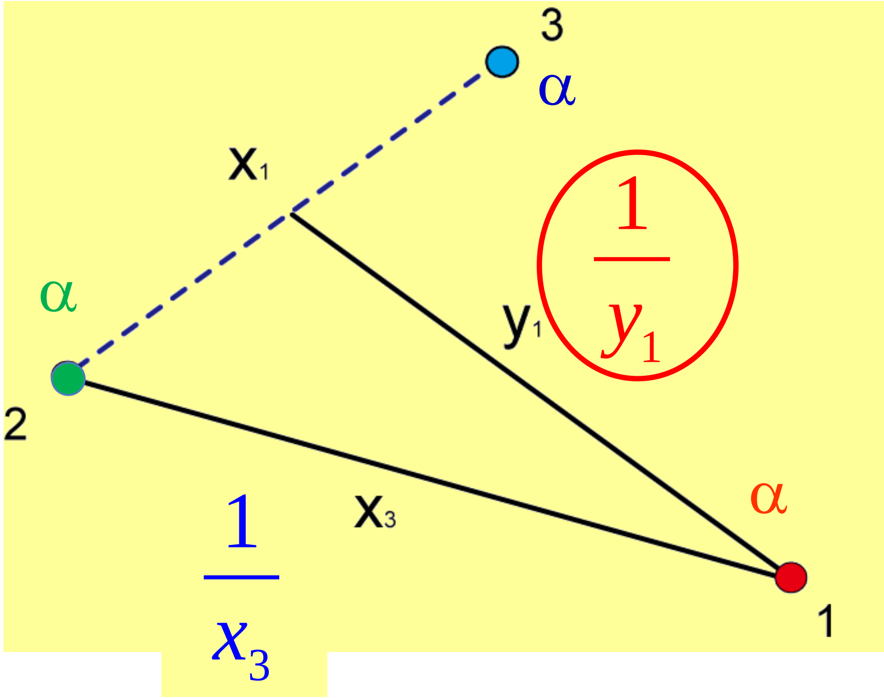
An approximation

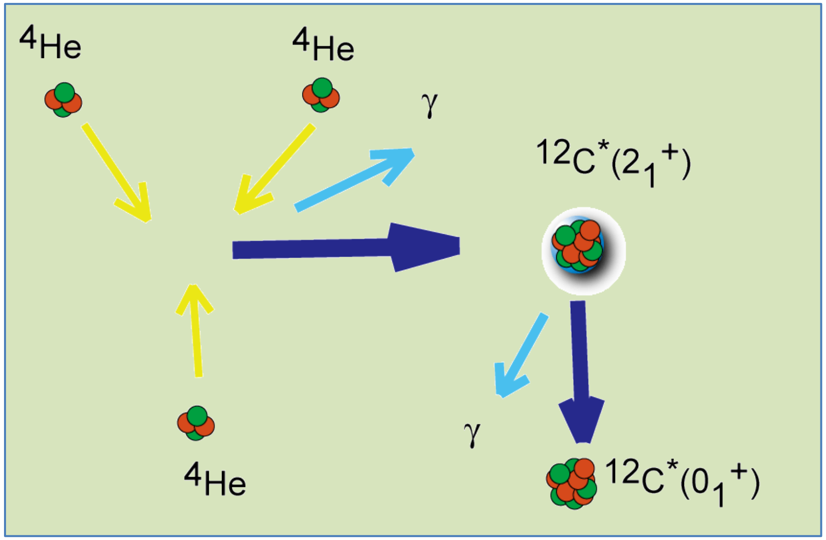
- A term $\left(\frac{1}{x_3} - \frac{1}{y_1}\right)$ appeared in the integral kernel, which is expected to be short range because of a cancellation. But, the cancellation is not perfect for breakup channels.

→ treat this problem approximately by a (mandatory) cutoff procedure

$$\left(\frac{1}{x_3} - \frac{1}{y_1}\right) \times e^{-(x_3/R_{\text{cut}})^4}$$

$R_{\text{cut}} = 20 \text{ fm} - 35 \text{ fm}$





3. CALCULATIONS AND RESULTS

3α model

- αα-potential

  Ali-Bodmer type (2-range Gaussian)

$$V_{\alpha\alpha}(x) = \left(V_R^{(0)}\hat{P}_{L=0} + V_A^{(2)}\hat{P}_{L=2}\right)e^{-(x/a_R)^2} + V_A e^{-(x/a_A)^2}$$

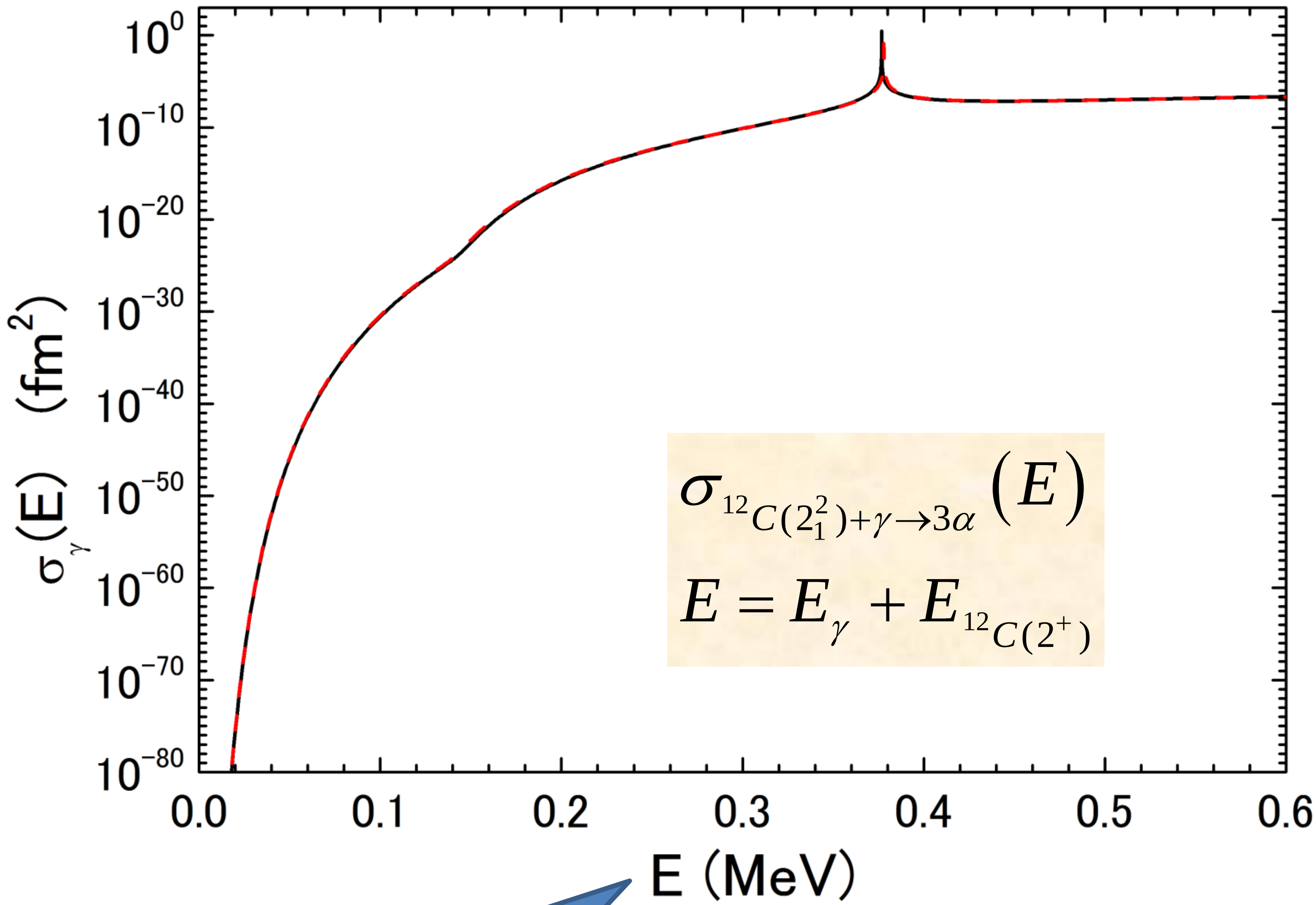
	a_R (fm)	$V_R^{(0)}$ (MeV)	$V_R^{(2)}$ (MeV)	a_A (fm)	V_A(MeV)
AB(A')	1.53	125.0	20.0	2.85	-30.18
AB(D)	1.40	500.0	320.0	2.11	-130.0

- 3-body potential [1] to reproduce the binding energy and resonance energy

$$V_{\alpha\alpha\alpha} = \left(W_0\hat{P}_{L=0} + W_2\hat{P}_{L=2}\right)e^{-(\rho/3.9)^2} \qquad \rho^2 = 3.97\sum_{i=1}^{3} r_i^2$$

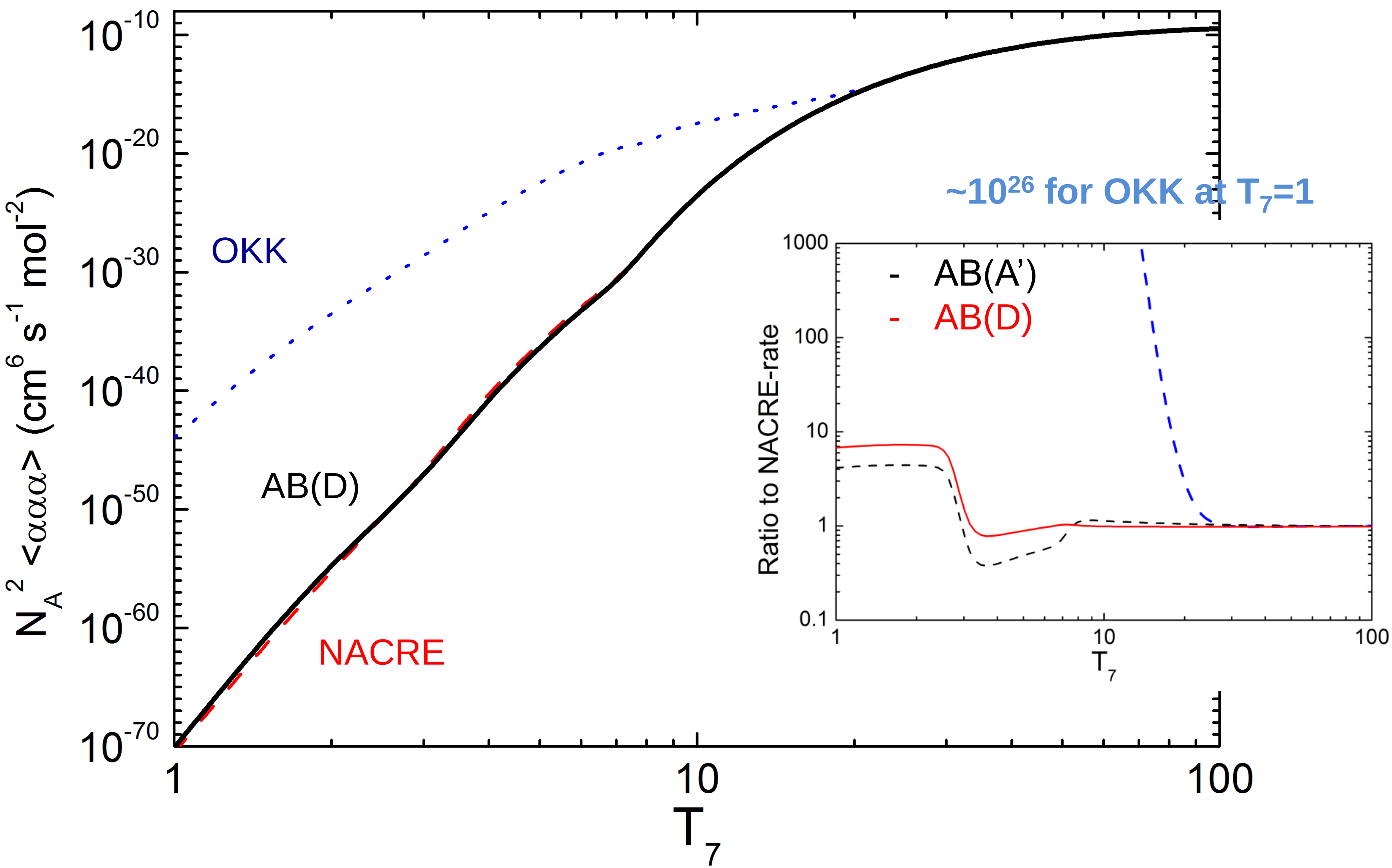
[1] D.V. Fedorov and A. S. Jensen, PLB **389** (1996) 631

Photodisintegration cross section (AB-A’ & AB-D)



3α energy in the cm system

ααα reaction rate (AB-A', AB-D)



Comparison with CDCC results

4. DISCUSSION

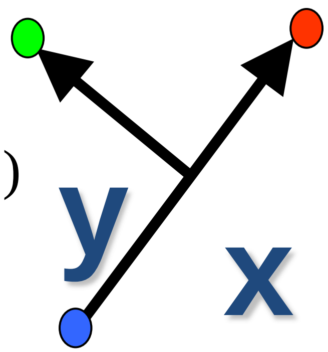
CDCC calculation of photo induced 3α breakup of ^{12}C(2^+)

$$^{12}\mathrm{C}(2^+) + \gamma \to \alpha + \alpha + \alpha$$

Wave function for (photo-) disintegration process

$$\left|\Psi\right\rangle \equiv \frac{1}{E + i\varepsilon - H_0 - V} H_\gamma \left|\Psi_b\right\rangle \underset{R\to\infty}{\to} \frac{e^{iKR}}{R^{5/2}} f^{(B)}\left(E_q, x, y\right)$$

$$\left(E - H_0 - V\right)\left|\Psi\right\rangle = H_\gamma \left|\Psi_b\right\rangle$$



Discretized α-α functions $u_n(x):\quad [T_x + V_{\alpha\alpha}(x)]u_n(x) = E_{q_n} u_n(x)$

$$\Psi(x,y) = \sum_n u_n(x)\varphi_n(y)$$

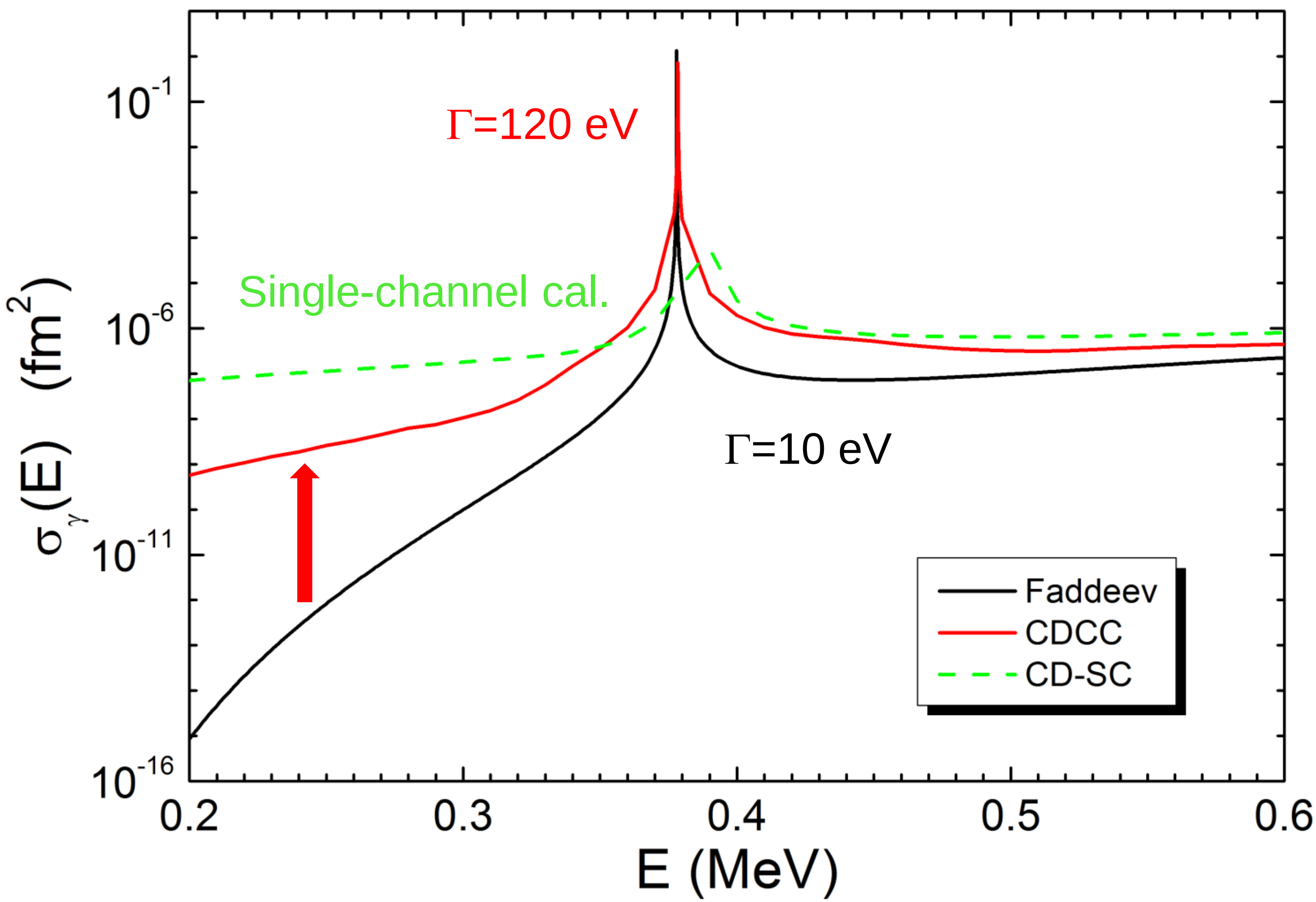
$$\varphi_n(y) \to [\text{Outgoing wave}] \times T_n$$

$$\sum_{n'}\left[\left(E_p - T_y\right)\delta_{n,n'} - V_{n,n'}(y)\right]\varphi_{n'}(y) = \left\langle u_n \middle| H_\gamma \middle| \Psi_b\right\rangle$$

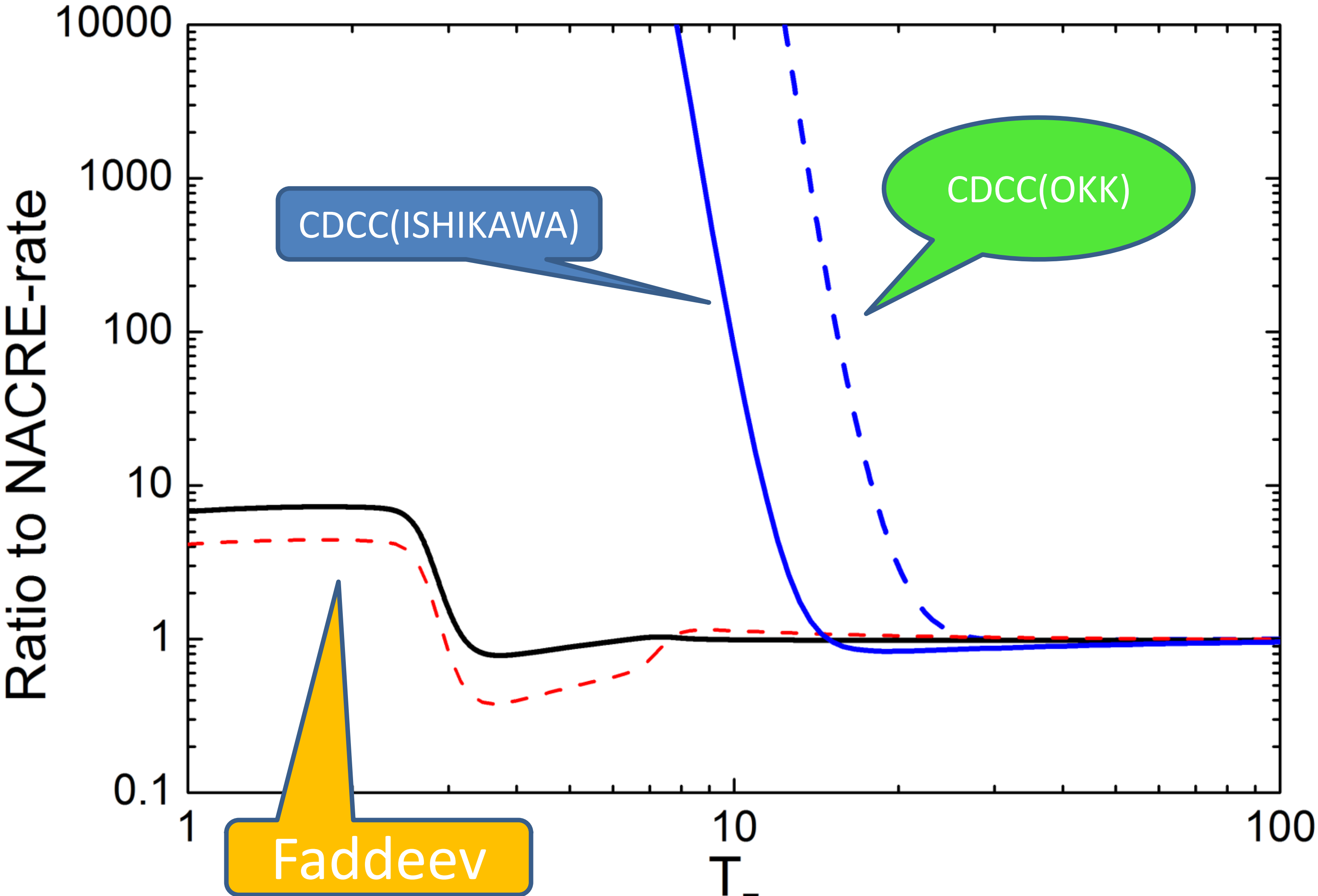
$$\sigma_\gamma(E) \propto \sum_{n<n_0} \frac{\left|T_n\right|^2}{p_n}$$

# of base functions = 120 (~OKK)

Photodisntegration cross section (AB-A')
(CDCC calculations by S.I.)



ααα reaction rate (Ratio to NACRE rate)

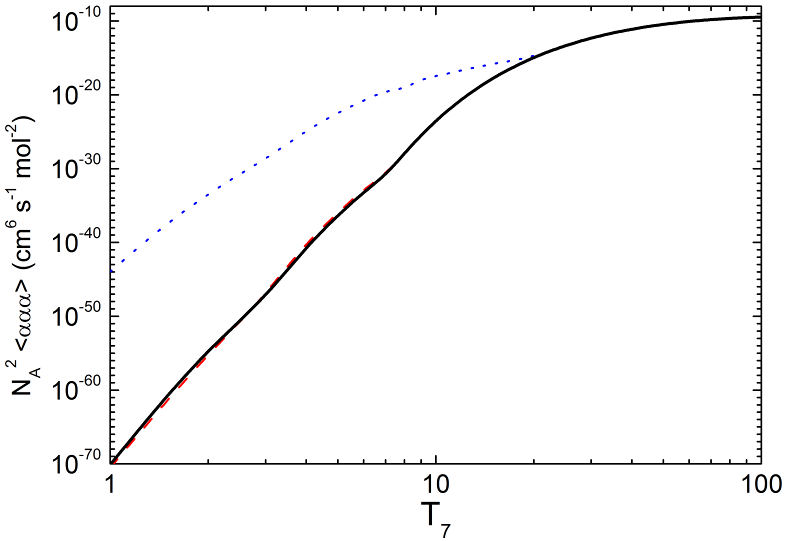


- At low temperatures ($T<10^8$ K):

  $<\alpha\alpha\alpha>_{NACRE}$ ~ $<\alpha\alpha\alpha>_{Faddeev}$ << $<\alpha\alpha\alpha>_{CDCC}$

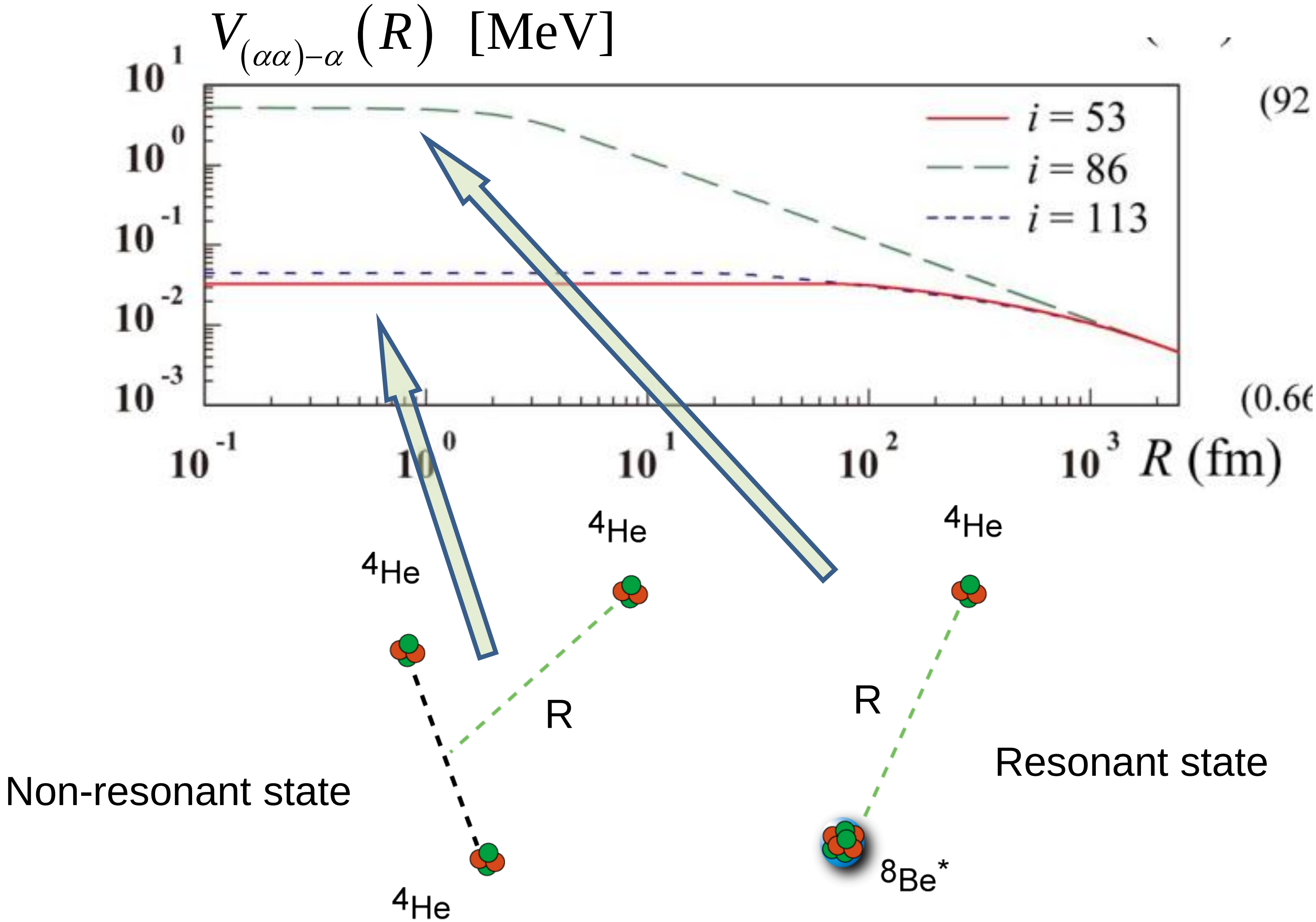
- Explanation of this enhancement by Ogata: Coulomb barrier between $\alpha\alpha$-pair and α:

  non-resonant pair vs. resonant pair



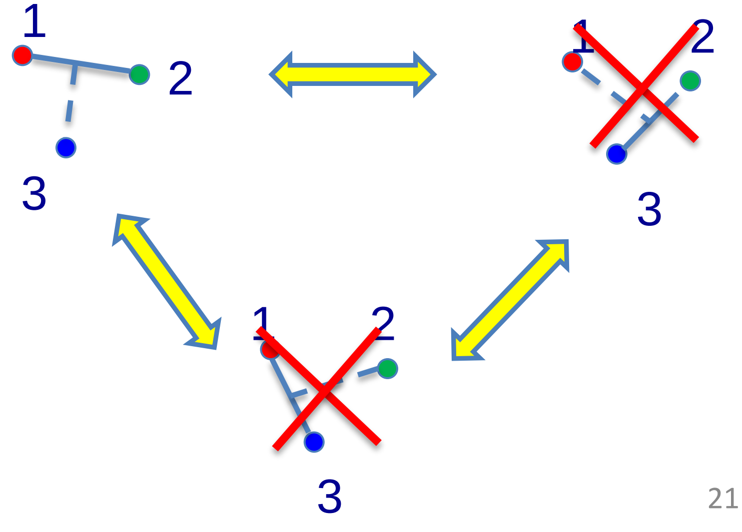
Reason for the enhancement (Ogata)

- Coulomb potential between αα-pair and α-particle

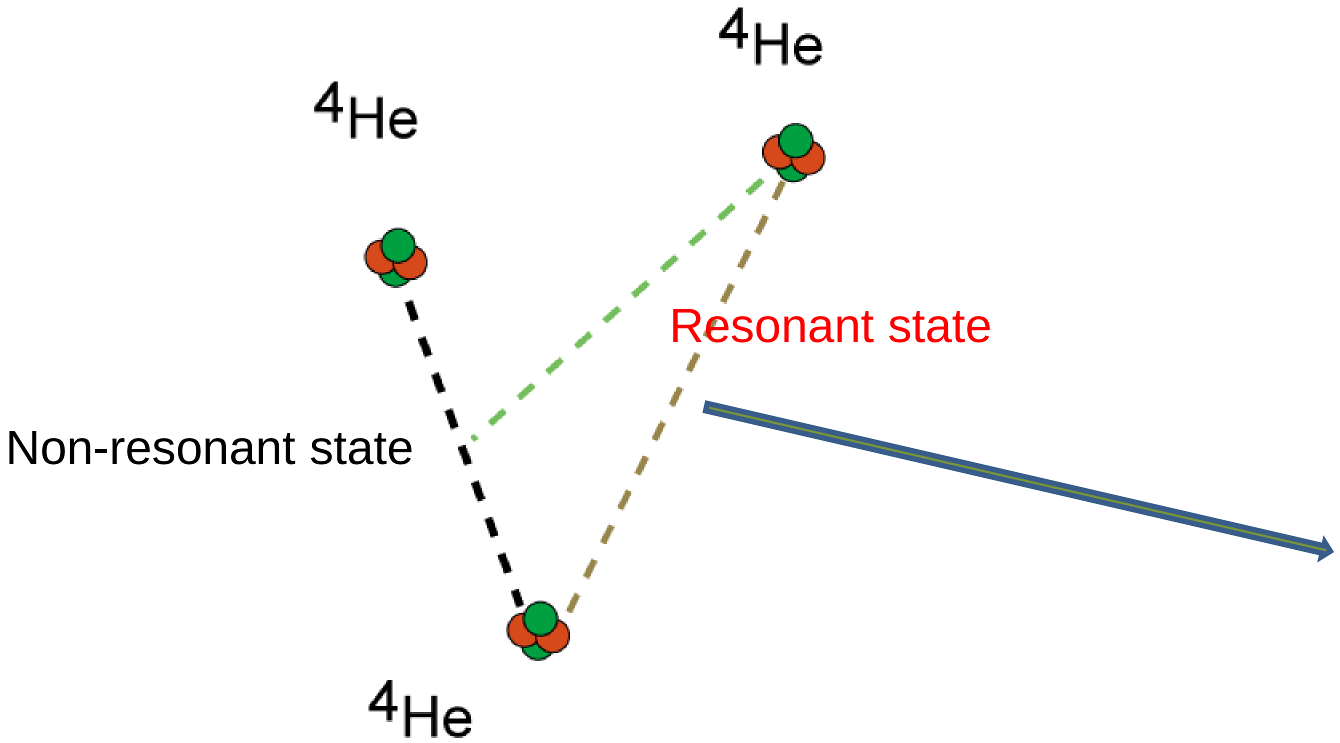


Model space of CDCC calculation

- Only one set of Jacobi coordinate is used:
  Neglects of rearrangement channels as well as symmetrization of the wave functions



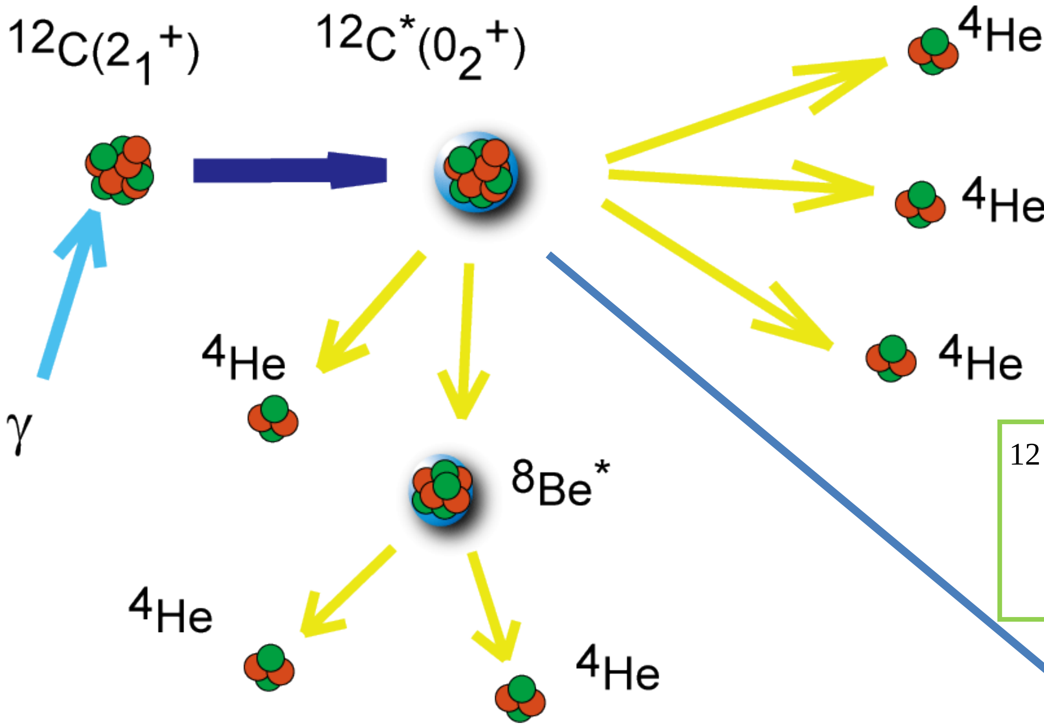
Rearrangement effect



3α decay mechanism of the Hoyle state

- The enhancement of $\sigma_{\gamma}(E)$ by the CDCC calculation at low energies is due to the reduction of Coulomb barrier between α and non-resonant $\alpha\alpha$-pair.

- This reduction may cause an enhancement of non-resonant (direct) process of 3α-decay of the Hoyle state.



Direct decay

$$^{12}C(2_1^+)+\gamma \rightarrow {}^{12}C^*(0_2^+)$$
$$\mapsto {}^4He + {}^4He + {}^4He$$

Sequential decay

$$^{12}C(2_1^+)+\gamma \rightarrow {}^{12}C^*(0_2^+) \rightarrow {}^8Be + {}^4He$$
$$\mapsto {}^4He + {}^4He$$

3α decay of the Hoyle state

- Direct decay or Sequential two-step process

- Ad.R. Raduta et al., PLB **705**, 65 (2011).
  $^{40}Ca + ^{12}C$ at 25MeV/nucleon
  Direct-decay contribution: 7.5 ± 4.0 %

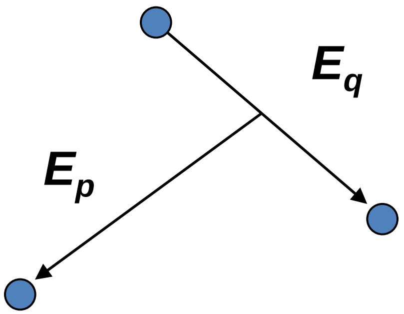
- O. S. Kirsebom et al. PRL **108**, 202501 (2012).
  $^{11}B(^{3}He,d)$
  "no evidence for direct-decay branches"
- J. Manfredi et al. PRC **85**, 037603 (2012).
  $^{10}C + ^{12}C$ "An upper limit of 0.45%"

Decomposition of the cross section

$$\sigma_\gamma(E) \propto \iint d\hat{x}d\hat{y} \int_{E_q>0} dE_q \sqrt{E_q E_p} \left| f\left(E_q;\hat{x},\hat{y}\right) \right|^2$$

$$\sigma_\gamma^{\mathrm{R}}(E) \propto \iint d\hat{x}d\hat{y} \int_{E_q=E_r\pm\Delta E} dE_q \sqrt{E_q E_p} \left| f\left(E_q;\hat{x},\hat{y}\right) \right|^2$$

$$\sigma_\gamma^{\mathrm{NR}}(E) = \sigma_\gamma(E) - \sigma_\gamma^{\mathrm{R}}(E)$$



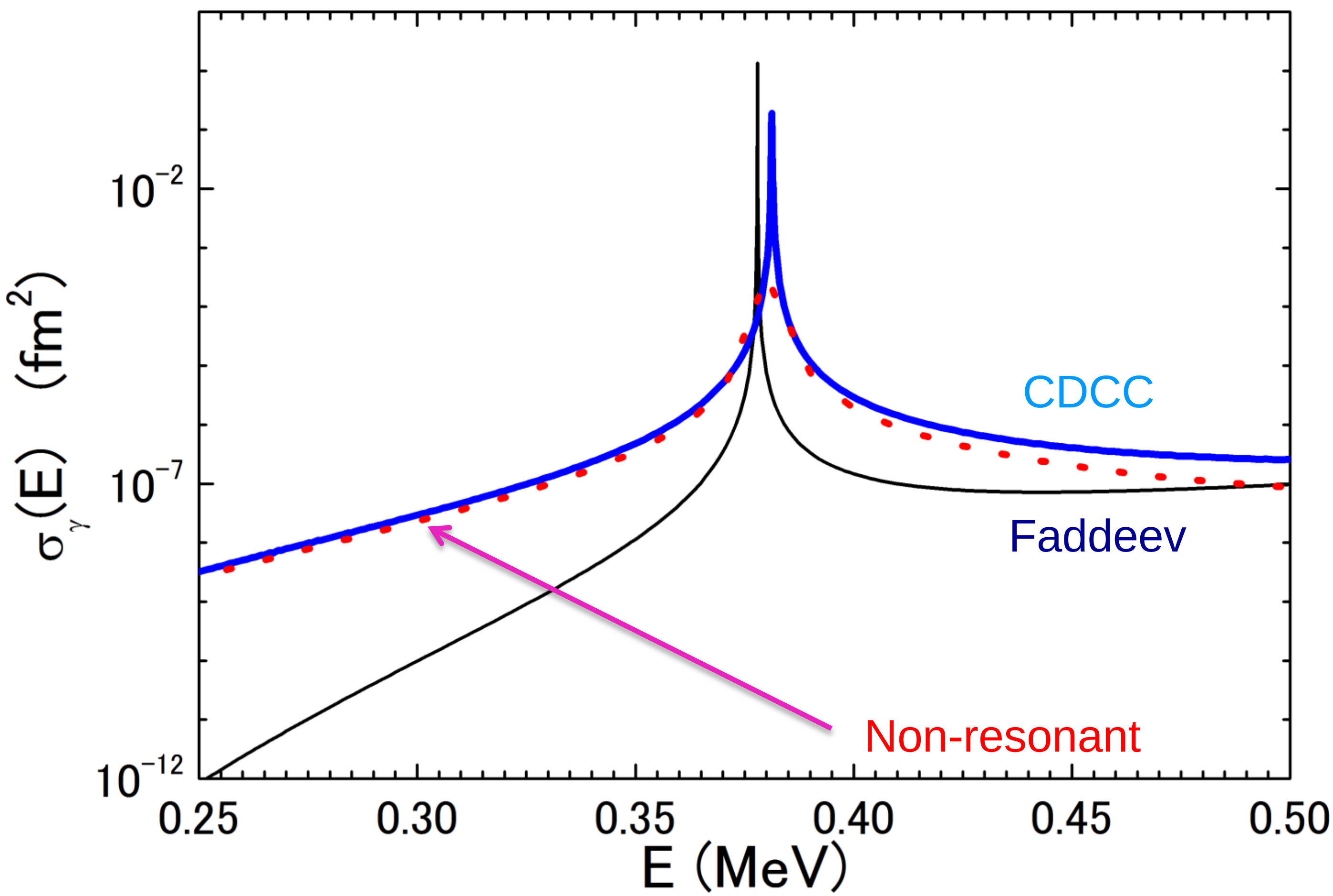
Faddeev vs. CDCC (SI)

- 3α-decay of the Hoyle state
  Sequential decay vs. Direct decay

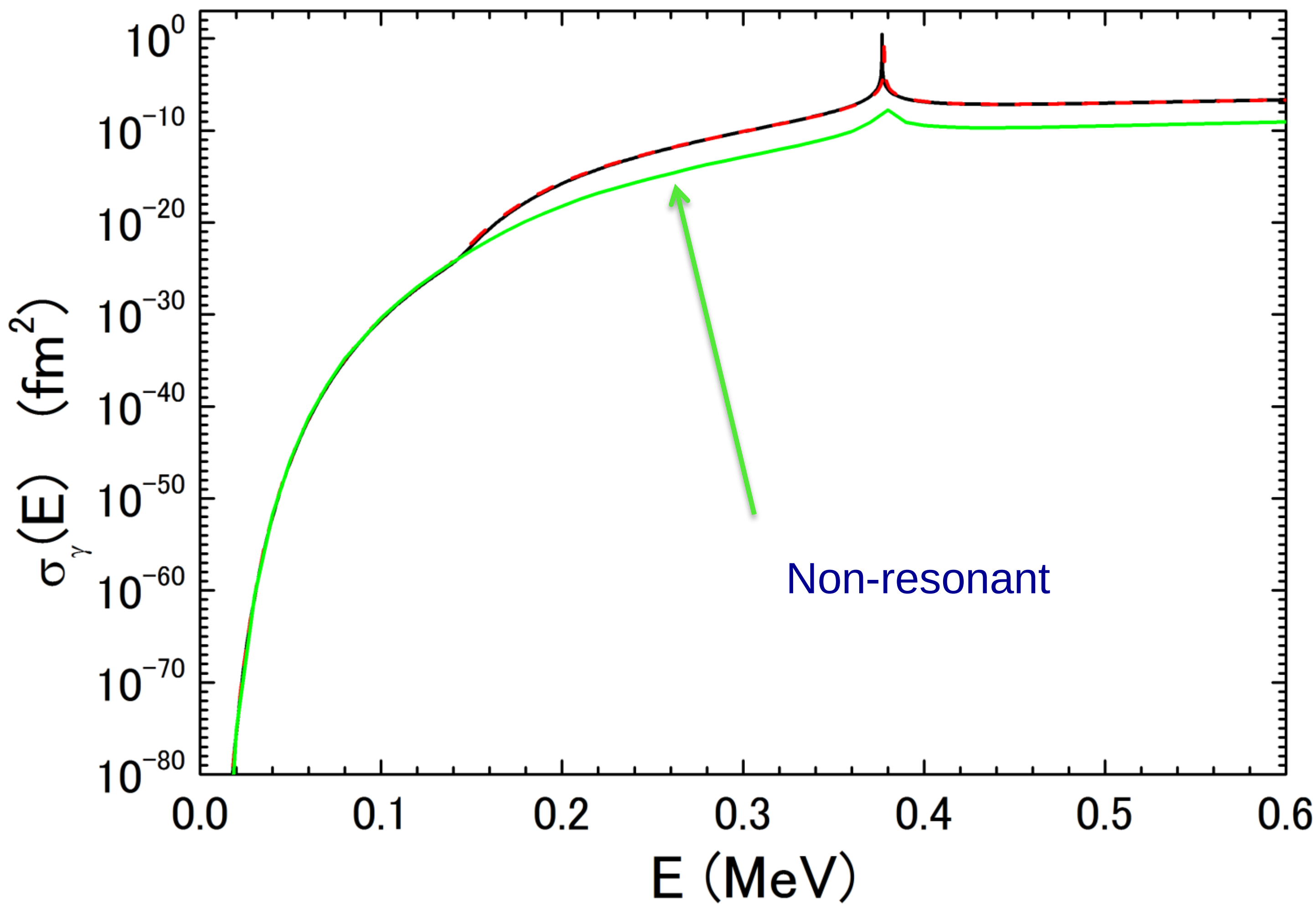
  --Faddeev: Sequential decay-dominant

  --CDCC: large contribution from Direct decay
  67% at E=380keV

Non-resonant contribution (CDCC)



Non-resonant contribution (Faddeev)



5. SUMMARY

- Quantum mechanical 3-body calculations of 3α-reactionas photodisintegration of $^{12}C(2^+)$
  Faddeev method, CDCC method
- Faddeev calculation: similar to the NACRE 3α rate
- CDCC calculations: Increase of the cross section at low energies (similar to Ogata's CDCC results)
- 3α-decay of Hoyle resonance
  Faddeev：Sequential decay (via ^{8}Be) dominant
  CDCC: A large contribution from the Direct decay
  → This may be tested by experiments.

- Future problem:
  ・Higher energies (theoretical calculations of ^{12}C-resonance other than the Hoyle state)
  . ^{9}Be(α-α-n), ^{6}He(α-n-n), n-n-n (3-n potential)
  ・4α problem, $^{12}C(\alpha,\gamma)^{16}O$