Effects of Particle Size Distribution on Opacity Curves of Protoplanetary Disks around T Tauri Stars. (Miyake and Nakagawa, 1993)

① Introduction

- photometric observations of T Tauri stars at submillimeter and millimeter wavelength regions by Beckwith and Sargent (1991, ApJ) have revealed that power law indices β of frequency dependence of particle opacities in circumstellar disks around T Tauri stars are 0-1.

- As long as dust particles are small enough for the theory of Rayleigh scattering to be appropriate and the optical thickness is small, the flux of thermal radiation from dust particles does not depend on size distributions but does only on the total mass of dust particles. (Hilderand 1983)

- the mass of the dust particles ⟵ Photometric observation.

  dependent on the assumed magnitude of particle opacity k_ν $\xrightarrow{\text{But.}}$ We do not know enough about the abundance, composition, or structure of interstellar dust particle.

- The standard value of the particle opacity for the unit mass of gas.

$$k_\nu = 0.002 - 0.004 \left(\frac{\nu}{10^{11.5}\,[\mathrm{Hz}]}\right)^2 \,[\mathrm{cm^2\,g^{-1}}] \quad (\text{Hildebrand}, 1983)$$

- At submillimeter -and millimeter wavelength regions, particle opacities are usually assumed to follow a power law dependence on frequency.

$$\Rightarrow k_\nu \propto \nu^{\beta}$$

- The indices β for the circumstellar disks around T Tauri stars were determined for 29 objects by Beckwith and Sargent (1991) from the spectral energy distributions at $0.6\,mm \le \lambda \le 3\,mm$

$\rightarrow$ $\beta \sim -1.0 \sim +1.0$ (disk model fitting)

$\beta \sim -0.5 \sim +2.0$ (optically thin assumption)

- This paper investigates the possibility that just particle growth through coagulation can explain the observed frequency dependence of particle opacity in circumstellar disks around T Tauri stars.

- Dust particles can grow to be centimeters in size according to the numerical calcurations for the particle growth in the solar nebulae.
Such particle growth must have some effects on the opacities.

- We calculate the mass opacities of those particles from the Mie theory for many particle sizes or size distributions and show that the particle growth can explain the reduced β and also meet other observational constraint.

② [Procedure] We describe the procedure for determination of optical constraints of silicate and H_2O-ice mixture as well as our basic assumptions of dust shape, species, and size distribution.

2.1 shape.

- Dust particles are spherical (回転楕円体)
  → complicated-shaped particles do not survive in circumstellar disk around T Tauri stars and become more or less spherical and compact ones.

2.2. species.

- Dust particle are composed of only silicate and H_2O-ice as a first approximation. (temperature below about 150K)
- Dust species in the circumstellar disk are dependent on when and where dust particles are formed.
  ① All interstellar dust particles will evaporate once in the circumstellar disk and condense again with thermodynamic equilibrium composition.
  ② Interstellar dust particles will survive in the circumstellar disk without any metamorphism.
- Density silicate $\rho_{sil} = 3.3\ g/cm^3$. H_2O-ice $\rho_{ice} = 0.92\ g/cm^3$
- mass fractional abundance $\zeta_{sil} = 0.0043$, $\zeta_{ice} = 0.0094$.

2.3. Optical Properties of Intimate Mixture and Effects of Porosity.

- Effective dielectric constant medium theories.
  - Maxwell-Garnett theory → very small particles are embeded in a matrix.
  - Bruggeman theory → small particles of more than two kinds of materials and no distinction between "matrix" and "inclusion" material.

$\Downarrow$

The condition that the constituent dust particles are small compared with the wavelength will be realized especially for the long-wavelength regions.

- Intimate mixture of silicate and H_2O-ice

The effective dielectic function : ϵ_{eff} ⇒ silicate : ϵ_{sil} H_2O-ice : ϵ_{ice}.

$$f_{sil}\frac{\epsilon_{sil}-\epsilon_{eff}}{\epsilon_{sil}+2\epsilon_{eff}}+f_{ice}\frac{\epsilon_{ice}-\epsilon_{eff}}{\epsilon_{ice}+2\epsilon_{eff}}=0. \qquad (1)$$

volume function ; filling factor (充填率)

Mass fractional abundance with respect to gas.
Silicate : $\zeta_{sil} = 0.0043$
H_2O-ice : $\zeta_{ice} = 0.0094$.

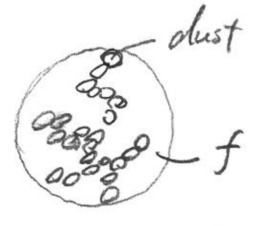
$$f_{sil}=\frac{\zeta_{sil}\cdot\rho_{ice}}{\zeta_{sil}\rho_{ice}+\zeta_{ice}\rho_{sil}}\cong 0.11. \qquad f_{ice}=1-f_{sil}=0.89.$$

$$\left(=\frac{0.0043\cdot 0.92}{0.0043\cdot 0.92+0.0094\cdot 3.3}\right)$$

filling factor

$$f\frac{\epsilon_{eff}-\epsilon_{eff}}{\epsilon_{eff}+2\epsilon_{eff}}+(1-f)\frac{\epsilon_{vac}-\epsilon_{eff}}{\epsilon_{vac}-2\epsilon_{eff}}=0. \qquad (2)$$

intimate dust material. vacuum



porous aggregate.

$0 \le f \le 1$ より。

$f \ll 1$ ほど、aggregate は スカスカである。

∘ complex refractive indices (複素屈折率) : $m_{eff.} = \underset{\text{屈折率}}{n} + i\underset{\text{消失係数 (消衰係数)}}{\underline{k}}$

$$m_{eff} = \varepsilon_{eff}^{2} \quad -(3)$$

∘ We distinguish between $\underline{\overset{\epsilon_{inc}}{\text{the inclusion}}}$ and $\underline{\overset{\epsilon_{mat}}{\text{the matrix}}}$ for the highly porous particles with $f \leqq 0.1$.

$$\varepsilon_{eff} = \epsilon_{mat}\left(1 + \frac{3fF}{1 - fF}\right) \quad -(4).$$

$$F = \frac{\epsilon_{inc} - \epsilon_{mat}}{\epsilon_{inc} + 2\epsilon_{mat}} \quad (5).$$

2.4. Opacity of single-sized Particles.

∘ We can get the absorption efficiency $Q_{abs}(a, \nu)$ as a function of particle radius "a" and frequency "ν" from straight forward Mie calculations employing the effective complex refractive indices ($m_{eff} = n + ik$).

∘ In terms of $Q_{abs}(a, \nu)$, the mass opacity coeffient $k_\nu(a)$ of dust particles with radius "a" and filling factor "f" is given by

$$k_\nu(a) = \frac{3}{4a}\frac{1}{f\rho} Q_{abs}(a, \nu) \cdot (\xi_{sil} + \xi_{ice}) \quad -(7).$$

ρ : density of silicate and H_2O-ice.

$$\rho = f_{sil} \cdot \rho_{sil} + f_{ice}\,\rho_{ice} \sim 1.18\ [g/cm^3] \quad -(8)$$

2.5. Growth and Size distribution

- In the circumstellar disk around T Tauri star, dust particles grow up to centimeters in radii by mutual collision.

- size distribution : $n(a)$.

$$n(a) = \begin{cases} n_0 \cdot a^{-p} & \text{for } 0.01\,\mu m \le a \le a_{max} \\ 0 & \text{otherwise.} \end{cases} \quad -(9).$$

n_0 : normalization constant.

p : constant power law exponent. → coaglation process : $p \sim 2.5$ / disruption process : $p \sim 3.5$. (分裂)

a_{max} : maximum cut-off radius of dust particles.

- Mass opacity : κ_ν of the emsemble of dust particle with the size distribution : $n(a)$.

$$\kappa_\nu = \frac{\int n(a) \cdot a^3 \cdot \kappa_\nu(a)\, da}{\int n(a) \cdot a^3\, da} \quad -(10)$$

2.6. Single Scattering Albedo.

- Mass scattering coeffient : $\sigma_\nu(a)$ for a particle with radius "a"

$$\sigma_\nu(a) = \frac{3}{4a}\frac{1}{f\rho}\,\underbrace{Q_{sca}(a,\nu)}_{\text{Scattering efficiency obtained from the Mie calucuration.}} \cdot (\zeta_{sil} + \zeta_{ice}) \quad -(11)$$

- Mass scattering coeffient σ_ν of the emsemble of dust particle with the size distribution $n(a)$.

$$\sigma_\nu = \frac{\int n(a) \cdot a^3 \cdot \sigma_\nu(a) \cdot da}{\int n(a) \cdot a^3\, da} \quad -(12)$$

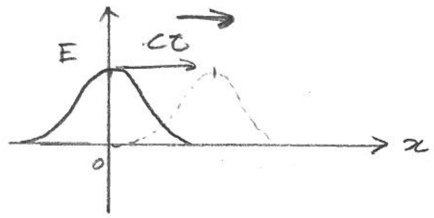
- single scattering albedo $\bar{\omega}_\nu$ of the emsemble of dust particle

$$\bar{\omega}_\nu = \frac{\sigma_\nu}{k_\nu + \sigma_\nu} \quad - (13).$$

連続媒質中の光の伝搬

光の電場ベクトル

$$E(x,t)=E_0\sin\underbrace{\frac{2\pi}{\lambda}}_{=k\ :\ \text{波数}}(x-ct)=E_0\sin k(x-ct)$$



$\omega=\frac{2\pi}{T}=2\pi f=2\pi\frac{c}{\lambda}=kc$ より.

$E(x,t)=E_0\sin(kx-\omega t)$ となる. ⟶ 余弦関数として表すこともできる.

オイラーの式より, $e^{i\theta}=\cos\theta+i\sin\theta$ から, $E(x,t)=E_0\cos(kx-\omega t)$

$=\mathrm{Re}\left[E_0 e^{i(kx-\omega t)}\right]$ となる. —①

媒質中では, $v=\frac{c}{n}$ なので, $k=\frac{\omega}{v}=\frac{n\omega}{c}$ —②

屈折率

○現実の媒質は吸収が存在する.

⟶ 吸収を表す光学定数が 消衰係数 K (extinction coeffient) が加わる。

⟶ 複素屈折率 $n_{eff}=n+iK$ に置きかえる.

つまり ② は, $k=\frac{n_{eff}\,\omega}{c}$ —③

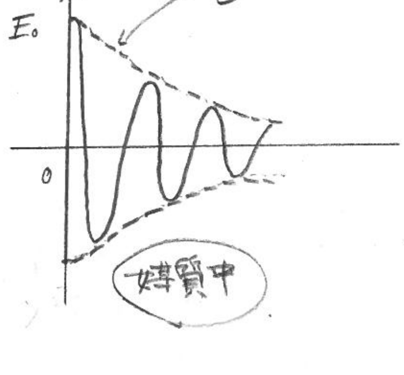
Point : 複素屈折率を導入すると波動を指数関数で表したときに都合が良くなる.

ここで ①, ③ から.

$$E(x,t)=E_0e^{ikx-i\omega t}=E_0e^{i\frac{n_{eff}\cdot\omega x}{c}-i\omega t}=E_0e^{i\frac{\omega x}{c}(n+iK)-i\omega t}$$

$$=\underbrace{E_0e^{-\frac{K\omega x}{c}}}_{E_0\text{が距離とともに減衰する。}}\cdot\underbrace{e^{-i\omega\left(t-\frac{nx}{c}\right)}}_{\text{波の伝搬}}\quad —④$$

⟶ 実部 $(t=0)$



光の強度は電場の振幅の絶対値の二乗に比例するので,

$I(x)\propto|E|^2=E\cdot E^*=E_0^2\cdot e^{-\frac{2K\omega x}{c}}$ —⑤ となる.

○ 消光係数と吸収係数について.

吸収係数：媒質による光の吸収の強さを表す ⇒ $\alpha\ [1/\mathrm{cm}]$

光の強度：$I(x) = I(0)\cdot e^{-\alpha x}$ — ⑥.

⑤ ↔ ⑥ を比較すると.

$$E_0^2\, e^{-\frac{2k\omega x}{c}} \leftrightarrow I(0)\cdot e^{-\alpha x} \Rightarrow \alpha = \frac{2k\omega}{c} = \frac{2k}{c}\cdot\overset{\omega}{\overset{\parallel}{\frac{2\pi c}{\lambda}}}$$

$$\therefore \boxed{\alpha = \frac{4\pi k}{\lambda}} \quad — ⑦.$$

となる。

○ マクスウェル方程式

$$\mathrm{rot}\, H = \frac{\partial D}{\partial t} + i, \quad \mathrm{rot}\, E = -\frac{\partial B}{\partial t} \quad — ⑧.$$

等方性媒体中の光の伝搬

$$D = \underbrace{\varepsilon_r}_{\text{比誘電率}}\cdot\underbrace{\varepsilon_0}_{\text{誘電率}} E, \quad B = \underbrace{\mu_r}_{\text{比透磁率}}\cdot\underbrace{\mu_0}_{\text{透磁率}} H, \quad i = \underbrace{\sigma}_{\text{導電率}}\cdot E \quad \left(\varepsilon_0\mu_0 = \frac{1}{c^2}\right) \quad — ⑨$$

比誘電率と比透磁率

$\varepsilon_r = \varepsilon_r' + i\,\varepsilon_r''$ のように複素数で表せる.

ここで ⑧⑨から.

$$\mathrm{rot\,rot}\, E = -\varepsilon_r \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} = -\left(\frac{\varepsilon_r}{c^2}\right)\frac{\partial^2 E}{\partial t^2}$$

ベクトル解析

$$\left(\begin{aligned}\mathrm{rot\,rot}\, E &= \mathrm{grad}(\mathrm{div}\, E) - \nabla^2 E \\ &= -\nabla^2 E \quad \text{より}\end{aligned}\right)$$

$E = E_0\cdot e^{-i\omega\left(t - \frac{m_{eff}\cdot x}{c}\right)}$ を代入.

$$\mathrm{rot\,rot}\, E = \left(\frac{m_{eff}^2\cdot\omega^2}{c^2}\right)\underbrace{E_0\, e^{-i\omega\left(t - \frac{m_{eff}\cdot x}{c}\right)}}_{=E}$$

$$-\left(\frac{\varepsilon_r}{c^2}\right)\frac{\partial^2 E}{\partial t^2} = \left(\frac{\varepsilon_r}{c^2}\right)\cdot\omega^2\cdot\underbrace{E_0\cdot e^{-i\omega\left(t - \frac{m_{eff}\cdot x}{c}\right)}}_{=E}$$

$$\frac{m_{eff}^2\cdot\omega^2}{c^2} E = \frac{\varepsilon_r\cdot\omega^2}{c^2} E$$

$$\rightarrow (m_{eff}^2 - \varepsilon_r) E = 0. \quad — ⑪.$$

↓ $E \neq 0$ ならば,

$m_{eff}^2 = \varepsilon_r$ となる. — ⑫

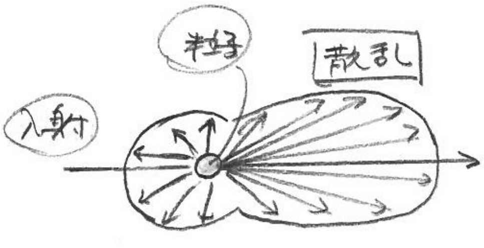
複素屈折率と複素誘電率

⑫：$m_{eff}^2 = \varepsilon_r \rightarrow (n+ik)^2 = \varepsilon_r' + i\varepsilon_r''$ より.

$$\left(= n^2 + 2ink - k^2 \right)$$

$\varepsilon_r' = n^2 - k^2$, $\varepsilon_r'' = 2nk$ となる ―⑬.

Mie 散乱

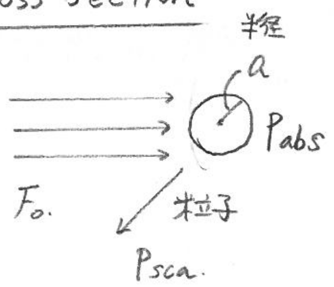


◦ 波長依存性はない. 全ての波長にわたって散乱がおきる。

◦ 粒子が大きくなると前方散乱がより強くなる.

$$\left(\begin{array}{c} \text{入射光} = \text{散乱光} \\ \nu_{rad} = \nu_{sca} \end{array} \right)$$

◦ Cross Section



F_0：進行方向に垂直な平面の単位面積を単位時間に通過するエネルギー

$P_{abs\,(sca)}$：粒子を入射した電磁波のうち単位時間あたりに吸収(散乱)されるエネルギー

$$\left. \begin{array}{l} \text{吸収断面積}\ \sigma_{abs} = \dfrac{P_{abs}}{F_0} \\ \text{散乱断面積}\ \sigma_{sca} = \dfrac{P_{sca}}{F_0} \end{array} \right\} \text{減光断面積}\ \sigma_{ext} = \sigma_{abs} + \sigma_{sca}$$

と表す。

↓ 球状粒子に対しては実効断面積を幾何学的な断面積 πa^2 で規格化した無次元量が用いられる

$$\left[\begin{array}{l} \text{複素屈折率}\quad m = n + ik \\ \text{サイズパラメータ}\quad x = \dfrac{2\pi a}{\lambda} \end{array} \right. \text{とおく.}$$

$$Q_{ext} = \frac{\sigma_{ext}}{\pi a^2}, \quad Q_{abs} = \frac{\sigma_{abs}}{\pi a^2}, \quad Q_{sca} = \frac{\sigma_{sca}}{\pi a^2}$$

↓ さらに Mie 理論によれば, 粒子の半径が入射光の波長にくらべて十分に小さいとき ($x \ll 1$)

$$Q_{sca} \simeq \frac{8}{3} x^4 \, \mathrm{Re} \left(\frac{m^2-1}{m^2+1} \right)^2, \quad Q_{abs} \simeq -4x \, \mathrm{Im} \left(\frac{m^2-1}{m^2+1} \right)^2$$

となる.