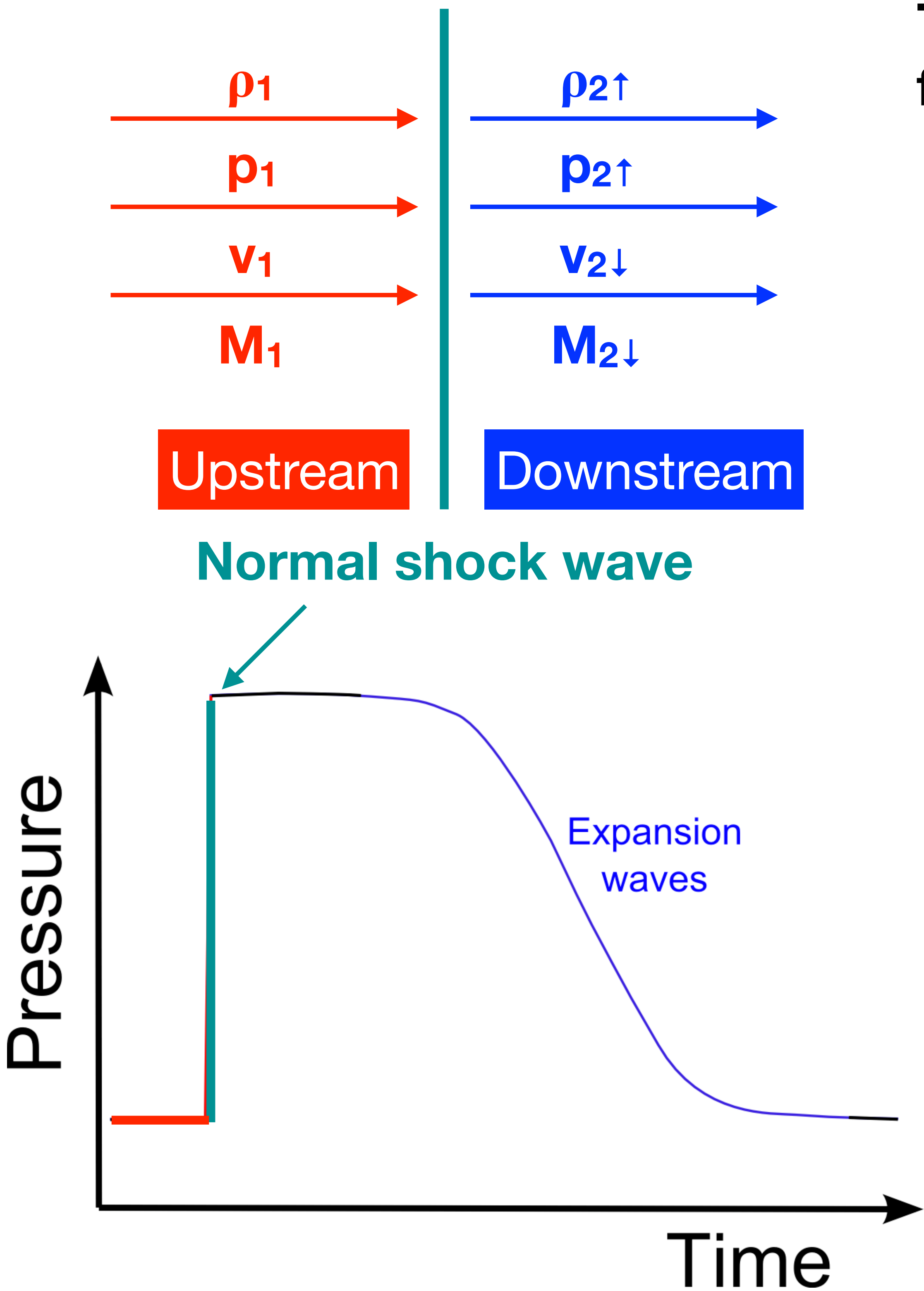
Section 6.5-6.6

2018/11/22

Yuzhu, Cui

6.5 The structure of shock waves



The mass flux, the momentum flux and the energy flux is conserved under the steady conditions:

$$\rho_1 v_1 = \rho_2 v_2 \tag{6.33}$$

$$p_1 + \rho_1 v_1^2 = p_2 + \rho_2 v_2^2 \tag{6.34}$$

$$\frac{1}{2}v_1^2 + \frac{\gamma p_1}{(\gamma - 1)\rho_1} = \frac{1}{2}v_2^2 + \frac{\gamma p_2}{(\gamma - 1)\rho_2} \tag{6.35}$$

Mach number: $\mathscr{M} = \dfrac{v_1}{\sqrt{\gamma p_1/\rho_1}} = \dfrac{v_1}{c_{s,1}}$

$$\mathscr{M} < 1 \quad \boxed{\frac{p_2}{p_1} = \frac{2\gamma\mathscr{M}^2 - (\gamma - 1)}{\gamma + 1}} \tag{6.38}$$

$$\mathscr{M} > 1 \quad \boxed{\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)\mathscr{M}^2}{2 + (\gamma - 1)\mathscr{M}^2} = \frac{\gamma + 1}{\frac{2}{\mathscr{M}^2} + (\gamma - 1)}} \tag{6.36}$$

$$\mathscr{M} \to \infty \quad \boxed{\frac{\rho_2}{\rho_1} \to \frac{\gamma + 1}{\gamma - 1}} \tag{6.39}$$

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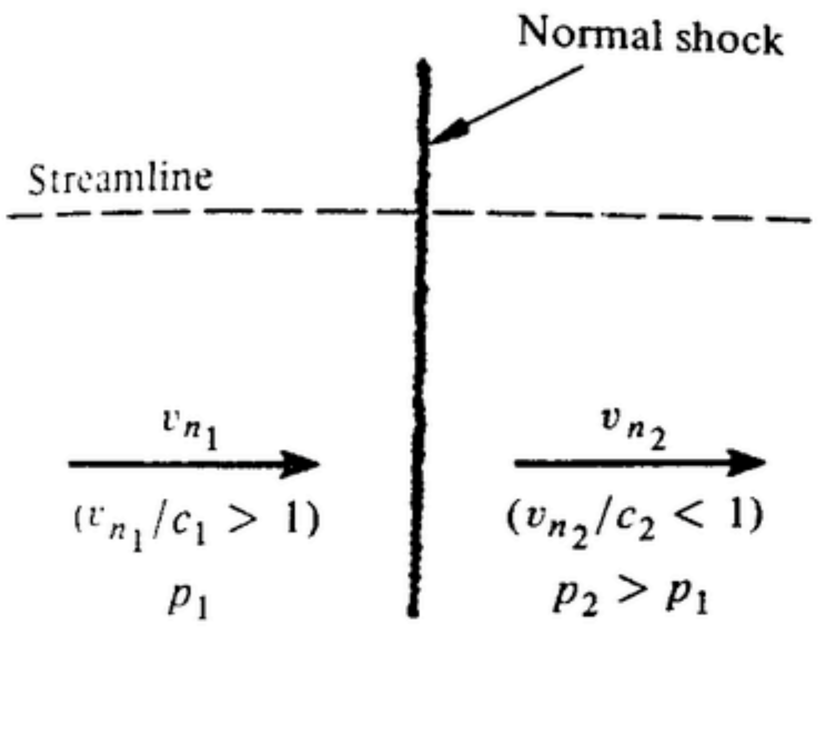
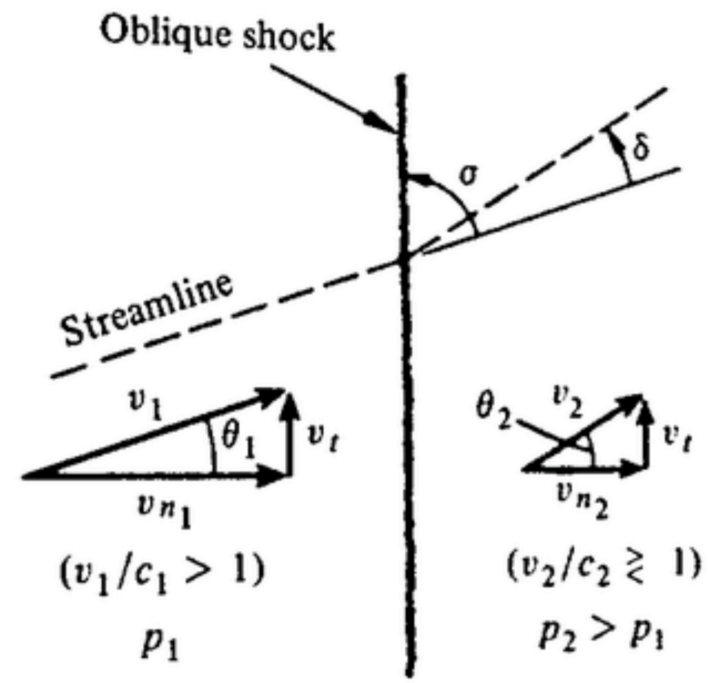
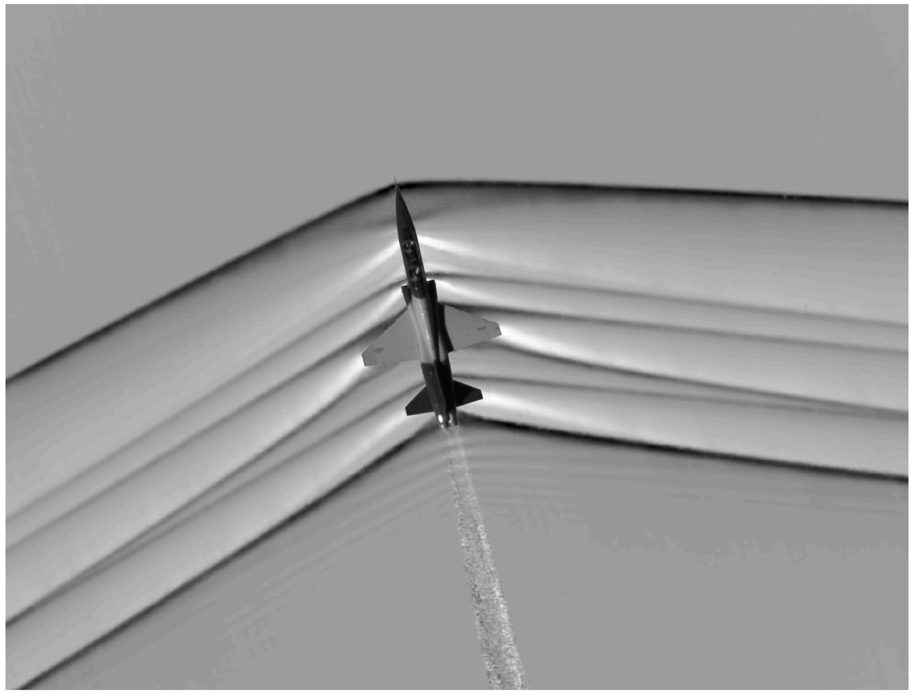


(a)



(b)



6.6 Spherical blast waves. Supernova explosions

Suppose an energy E is suddenly released in an explosion producing a spherical blast wave:

Scale parameter of the blast wave at time t:

$$\lambda = (Et^2/\rho_1)^{1/5} \quad (6.40)$$

A dimensionless distance parameter:

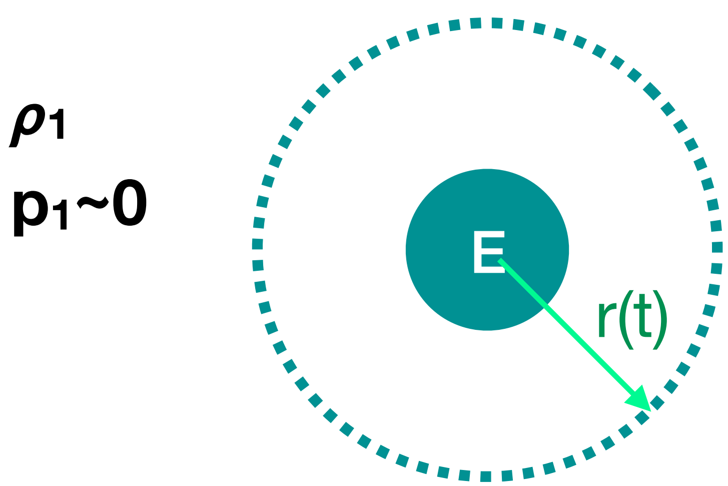
$$\xi = \frac{r}{\lambda} = r(\frac{\rho_1}{Et^2})^{1/5} \quad (6.41)$$

Radius of the spherical blast:

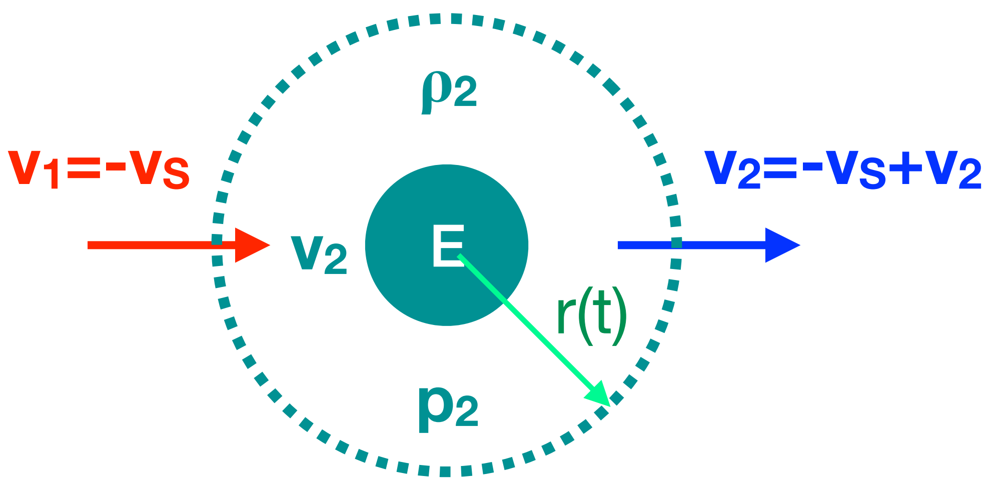
$$r_S(t) = \xi_0(\frac{Et^2}{\rho_1})^{1/5} \quad (6.42)$$

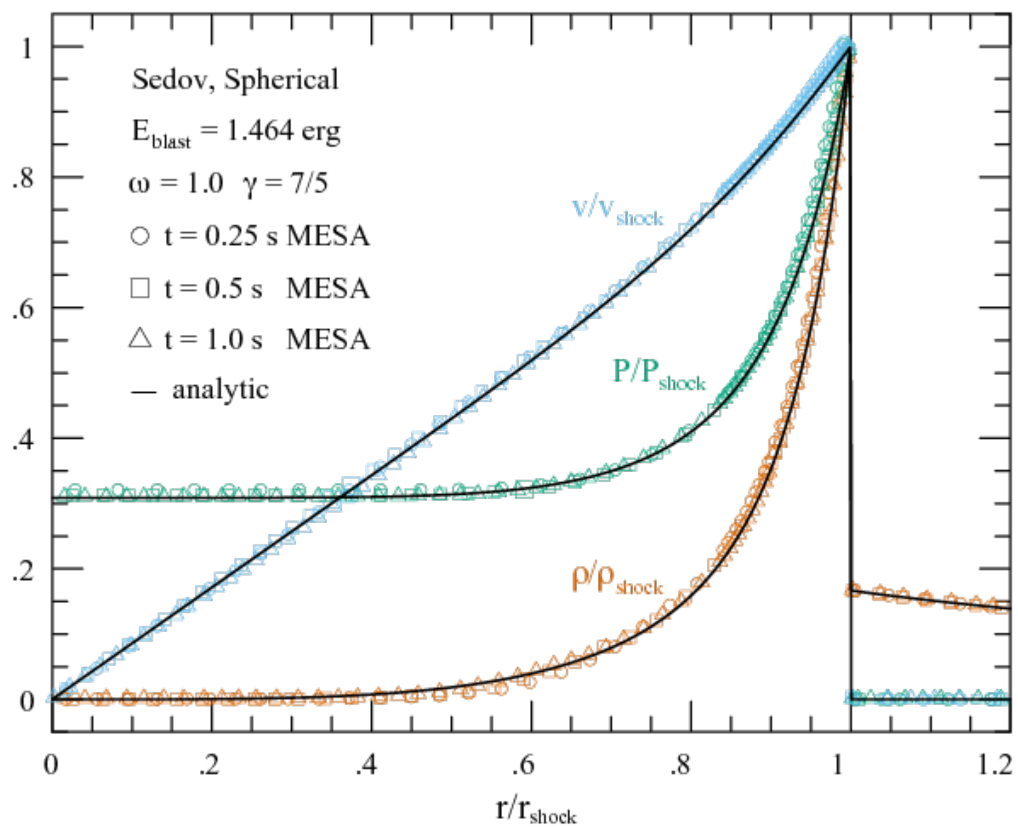
Velocity of expansion:

$$v_S(t) = \frac{dr_S}{dt} = \frac{2r_S}{5t} = \frac{2}{5}\xi_0(\frac{E}{\rho_1 t^3})^{1/5} \quad (6.43)$$



6.6 Spherical blast waves. Supernova explosions





In the frame of the shock:

$$\rho_2 = (\frac{\gamma+1}{\gamma-1})\rho_1 \quad (6.46)$$

$$v_2 = (\frac{2}{\gamma+1})v_S \quad (6.47)$$

$$p_2 = (\frac{2}{\gamma+1})\rho_1 v_S^2 \quad (6.48)$$

Substitute (6.49-6.51) into (6.53-6.55):

$$\frac{\partial}{\partial t} = -\frac{2\xi}{5t}\frac{d}{d\xi} \qquad \frac{\partial}{\partial r} = \frac{\xi}{r}\frac{d}{d\xi}$$

In terms of dimensionless variables:

$$\frac{32\pi}{25(\gamma^2-1)}\int_0^{\xi_0}[p' + \rho' v'^2]\xi^4 d\xi = 1 \quad (6.59)$$