Quiz 1 Gravitational thermodynamics — originally [III-6] of the 2008 entrance exam of Department of Physics, Kyoto University

Let's discuss the total energy E of a star, which are composed of the internal energy U(>0) and the gravitational energy Ω(<0).

Assume:

- the star is spherically symmetric
- gravity is statically balanced with pressure gradient, achieving the steady state
- Pressure at distance r from the center: P(r)
- Mass density ρ(r)
- Stellar radius R
- Outside of the star is vacuum
- $\rho(r \geqq R) = 0$, $P(r \geqq R) = 0$
- The mass of the star inside of radius r: M(r)
- The total mass of the star is $M_0=M(R)$
- The adiabatic constant is γ (constant)
- The gravitational constant is G

1. Derive the relationship of Ω, U, and γ. You can assume the following relations. ε is the internal energy.

$$\Omega = -\int_0^R G\frac{M(r)}{r}\rho(r)\cdot 4\pi r^2 dr$$

$$P = (\gamma - 1)\varepsilon$$

2. The condition for the star to be stable is that the total energy E is negative. Derive the condition for γ for the star to be stable. In addition, prove that the star is stable if the star is composed of non-relativistic monoatomic gas (γ=5/3).

In the following discussion, assume that the star is composed of non-relativistic monoatomic gas

3. Derive the average temperature of the star T_0 by using U and M_0. Assume additional variables if needed and reasonable.

4. A star loses its energy by emitting radiation. If the total energy changed from E to E+⊿E, what happens for the internal energy and the gravitational energy? Also for the average temperature? In addition, derive the effective specific heat C= ⊿E/⊿T_0

5. Discuss the change of the average temperature and the expansion or contraction of the star when the star emits radiation.

Quiz 2 Astronomical shock — originally [III-3] of the 2009 entrance exam of Department of Physics, Kyoto University

Let's discuss the shock wave produced by a supernova. Assume that the shock-propagating speed is $V_{sh,}$ the interstellar medium has a constant density and are static, neglect the mass of the materials ejected from supernova, the Rankine-Hugoniot relation is satisfied (strong shock), neglect any spacial changes after the shock propagate, and neglect cooling.

1. Derive the total energy of the material inside of the shock wave front U(t). The internal energy is $\varepsilon_U=(\gamma-1)^{-1}(p/\rho)$.

2. The expansion speed of the materials inside the shock wave V_{mat} is slower than V_{sh}. Derive the total kinetic energy of the materials inside the shock wave K(t), and prove that K(t)=U(t).

3. Since we assume that the cooling process is negligible, the sum of the internal energy and the kinetic energy is conserved, and they are equal to the explosion energy E_0. Using this, derive the equation that R(t), which is the distance of the shock wave front from the center, and solve it. If the explosion energy is 10^{51} [erg] and the interstellar medium has the number density 1 [cm^{-3}] composed of atomic hydrogen, how long is R (t=10^4 [years]) in [pc]?